FACULTY OF SCIENCE

DEPARTMENT:	**PURE AND APPLIED MATHEMATICS**
MODULE:	**APM2A10**
	INTRODUCTION TO DIFFERENTIAL EQUATIONS
CAMPUS:	**AUCKLAND PARK KINGSWAY**

FINAL EXAMINATION

DATE: 26/05/2016	**SESSION: 08:30 – 11:30**
ASSESSORS	**MR KD ANDERSON**
	DR MV VISAYA
MODERATOR	**PROF M KHUMALO**
DURATION: 2 HOURS 30 MINUTES	**MARKS: 50**

NUMBER OF PAGES	1 COVER PAGE
	1 QUESTION PAGE(S)
	1 FORMULA PAGE
INSTRUCTIONS	ANSWER ALL THE QUESTIONS.
	SHOW ALL CALCULATIONS.
	POCKET CALCULATORS MAY BE USED.
	SYMBOLS HAVE THEIR USUAL MEANING.

QUESTION 1 [12 MARKS]

Solve

$$4y'' + 4y' + y = 4e^{-x/2}\left(\frac{1}{x} + x\right)$$

using the method of undetermined coefficients.

QUESTION 2 [13 MARKS]

Solve

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

using the method of variation of parameters.

QUESTION 3 [10 MARKS]

Use the Laplace transform to solve the initial value problem

$$y'' + y = 1, \quad y(0) = 0, \quad y'(0) = 0.$$

Verify that your answer satisfies the initial conditions.

QUESTION 4 [10 MARKS]

Let f be a piecewise linear function given by

$$f(t) = \begin{cases} t-1 & 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Use the definition of the Laplace transform to find $\mathcal{L}(f)$.

QUESTION 5 [5 MARKS]

An object with a mass of 16 kilogram stretches a spring $\frac{8}{9}$ metres. The spring is initially displaced $\frac{1}{2}$ metres upwards from its equilibrium position and given an initial velocity of 1 m/s downward. Let g = 32 m/s^2. Write down, *without solving*, the initial value problem to find the object's displacement $y(t)$ at any time t. Assume that there is no damping and no external forces acting on the system.

END OF QUESTION PAPER