UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

MODULE: APM1B10
INTRODUCTION TO DYNAMICS
CAMPUS: APK

SUPPLEMENTARY EXAMINATION

DATE: 11/01/2017 **SESSION: 08:00 - 11:00**

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INTERNAL MODERATOR: **PROF C M VILLET**

DURATION: 2 HOURS **MARKS: 50**

NUMBER OF PAGES: 5 PAGES

INSTRUCTIONS:

SYMBOLS HAVE THEIR USUAL MEANING.
PHYSICAL QUANTITIES ARE IN SI UNITS AND ANGLES ARE IN RADIANS.
CALCULATORS ARE PERMITTED.
WORK TO A PRECISION OF AT LEAST THREE DECIMAL PLACES IN ALL NUMERICAL CALCULATIONS.
ANSWER EACH OF THE FIRST THREE THE QUESTIONS.
ANSWER EITHER QUESTION FOUR OR QUESTION FIVE, NOT BOTH.

QUESTION 1

(a) Show that

$$\widehat{\theta} \cdot \widehat{n} = \frac{v_r a_n}{v_r a_\theta - v_\theta a_r}.$$

(b) The equation of the trajectory of a particle is given by

$$\mathbf{r}(t) = 4t^3\widehat{x} - \cos(2t)\,\widehat{y} \equiv \left(4t^3, -\cos(2t)\right).$$

Find, for $t = \frac{1}{2}$,

(i) $\mathbf{v}$ and $\mathbf{a}$,

(ii) $\widehat{\tau}$ and $\widehat{n}$,

(iii) the tangential and normal components of $\mathbf{a}$,

(iv) the radius of curvature ρ, and the centre of curvature $\mathbf{C} = \mathbf{r} + \rho\widehat{n}$.

[12]

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QUESTION 2

A force $\mathbf{F} = (F_x, F_y)$ acts on a particle, of mass 0.3kg, for 3 seconds. Here, F_x and F_y are constants, not necessarily equal. Assume that $\mathbf{F}$ makes an angle θ with the horizontal axis. Assume that the *horizontal* component of the initial velocity $\mathbf{v}_i$ is half the vertical component. Assume that the final velocity $\mathbf{v}_f$ has the same magnitude as $\mathbf{v}_i$, but is opposite in direction.

(a) Determine θ.

(b) If the horizontal component of $\mathbf{v}_i$ is 10ms^{-1} in the *negative* x-direction, determine F_x and F_y.

[13]

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QUESTION 3

Calculate the directional derivative $\frac{\partial \psi}{\partial s}$ of

$$\psi = -yz + xz^2$$

in the direction

$$-\widehat{x} + 2\widehat{y} - 4\widehat{z}$$

at the point

$$(1, -1, 3)\,.$$

(a) Use the parameterization $\mathbf{r} = \mathbf{r}_0 + s\widehat{s}$.

(b) Use $\nabla\psi$.

[12]

THE PREVIOUS THREE QUESTIONS WERE COMPULSORY.
MAKE SURE THAT YOU HAVE ANSWERED ALL OF THEM.

QUESTIONS FOUR AND FIVE ARE ON THE NEXT PAGE.
ANSWER EITHER ONE OF THEM, NOT BOTH.

QUESTION 4

Calculate $\int_\Gamma \mathbf{a} \cdot d\mathbf{r}$ with $\mathbf{a} = \left(x^2y^2,\ y - 3z,\ 2x - z^2\right)$, and where the path Γ is defined by

$$\begin{aligned} y &= 1 - x^2 \\ z &= x^2 + 2y. \end{aligned}$$

In the integral, the lower limit is $(0,1,3)$, and the upper limit is $(1,0,1)$. You should find that the value of the integral is $\frac{533}{105}$.

[13]

OR

QUESTION 5

The coefficient of friction between the tyres of a braking car and the road is μ. At the moment the brakes are applied, the car has speed v_0. Assume that the road is inclined upwards at an angle θ, and let d denote the distance in which the car comes to a stop. Using the relationship

$$W = \int_\Gamma \mathbf{F} \cdot d\mathbf{r} = \Delta T,$$

show that, if $\theta = \frac{\pi}{4}$ rad and $\mu = \sqrt{2} - 1$,

$$d = \frac{v_0^2}{2g},$$

where g is the gravitational acceleration. In this problem, the friction is **not** to be treated as a conservative force, so that the principle of conservation of energy is **not** applicable.

[13]

INFORMATION

$$\begin{aligned}
\mathbf{r} &= (x, y) = (r\cos\theta, r\sin\theta) \\
\mathbf{v} &= v_r\widehat{r} + v_\theta\widehat{\theta} = v_\tau\widehat{\tau} + v_n\widehat{n} \\
&= \dot{r}\widehat{r} + r\dot{\theta}\widehat{\theta} = v\widehat{\tau} \\
\mathbf{a} &= a_r\widehat{r} + a_\theta\widehat{\theta} = a_\tau\widehat{\tau} + a_n\widehat{n} \\
&= \left(\ddot{r} - r\dot{\theta}^2\right)\widehat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\widehat{\theta} = \dot{v}\widehat{\tau} + \left(\frac{v^2}{\rho}\right)\widehat{n} \\
\widehat{r} &= (\cos\theta, \sin\theta) \qquad \widehat{\theta} = (-\sin\theta, \cos\theta) \\
\widehat{\tau} &= (\cos\psi, \sin\psi) \qquad \widehat{n} = (-\sin\psi, \cos\psi)
\end{aligned}$$

$$\begin{aligned}
\mathbf{I} &= \int_{t_i}^{t_f} \mathbf{F}dt = m\mathbf{v}_f - m\mathbf{v}_i \\
\ddot{x} &= -\omega^2 x \\
\Rightarrow x &= x_0 \sin(\omega t + B)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\Psi}{\partial s} &= \nabla\Psi\cdot\widehat{s} \\
d\mathbf{r} &= (dx, dy) \text{ in two dimensions} \\
d\mathbf{r} &= (dx, dy, dz) \text{ in three dimensions}
\end{aligned}$$

$$\begin{aligned}
T = \frac{m(\mathbf{v}\cdot\mathbf{v})}{2} &= \frac{mv^2}{2} \\
W = \int_\Gamma \mathbf{F}\cdot d\mathbf{r} &= \Delta T \\
\text{Newtonian gravity:}\quad \mathbf{F} &= -\frac{GMm}{r^2}\widehat{r}
\end{aligned}$$