UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACCALAUREUS INGENERIAE *MECHANICAL ENGINEERING*
SUBJECT	:	**STRENGTH OF MATERIALS 3B**
CODE	:	**SLR 3B21**
DATE	:	SUMMER EXAMINATION NOVEMBER 2016
DURATION	:	3 HOURS
WEIGHT	:	50:50
TOTAL MARKS	:	100

EXAMINER	:	MR D M MADYIRA
MODERATOR	:	PROF R F LAUBSCHER
NUMBER OF PAGES	:	6 PAGES

INSTRUCTIONS	:	QUESTION PAPERS MUST **NOT** BE HANDED IN.
REQUIREMENTS	:	ANSWER SHEETS

INSTRUCTIONS TO CANDIDATES:

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2. Explain answers and give all the necessary steps to arrive at the answer – simply giving the answer is not sufficient.
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4. Do all the questions in the answer scripts.

QUESTION 1 [25]

The AM1004-T61 magnesium alloy tube AB in Figure Q1 is capped with a rigid plate. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C.

1.1 Determine the temperature change that will lead to closure of the 0.2 mm gap. [10]

1.2 Determine the highest temperature to which the assembly can be raised without causing yielding either in the tube or the rod. [15]

Neglect the thickness of the rigid cap. For AM1004-T61 magnesium alloy, E = 44.8 GPa and $\alpha = 25.1 \times 10^{-6}$ /°C. For aluminum E = 70 GPa and $\alpha = 23.4 \times 10^{-6}$ /°C.

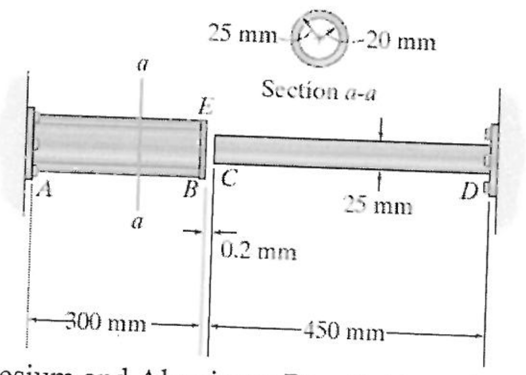

Figure Q1: Magnesium and Aluminum Bars Subjected to Thermal Loading

QUESTION 2 [25]

2.1 Briefly discuss the application of the secant formula for buckling. [5]

2.2 The hoisting arrangement for lifting a large pipe is shown in the Figure Q2. The spreader is a steel tubular section with outer diameter 70 mm and inner diameter 57 mm. Its length is 2.6 m and its modulus of elasticity is 200 GPa. Based upon a factor of safety of 2.25 with respect to Euler buckling of the spreader, what is the maximum

weight of pipe that can be lifted? (Assume pinned conditions at the ends of the spreader).
[20]

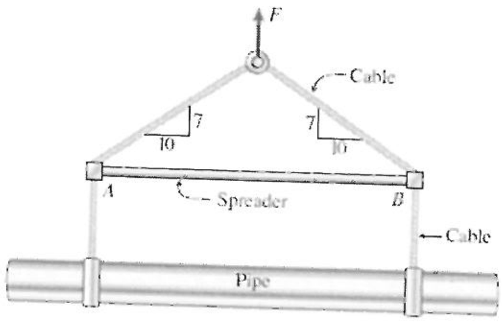

Figure Q2: A hoisting arrangement for lifting large pipes

QUESTION 3 [25]

A shaft of length L = 1.8 m is subjected to torques T = 5 kN-m at either end (Figure Q3). Segment AB (L_1 = 900 mm) is made of brass (G_b = 41 GPa) and has a square cross section (a = 75 mm). Segment BC (L_2 = 900 mm) is made of steel (G_s = 74 GPa) and has a circular cross section (d = a = 75 mm). Ignore stress concentrations near B.

3.1 Find the maximum shear stress and angle of twist for each segment of the shaft. [5]

3.2 Find a new value for the dimension a of bar AB if the maximum shear stress in AB and BC are to be equal. [8]

3.3 Repeat part 3.2 if the angles of twist of segments AB and BC are to be equal. [6]

3.4 If dimension a is reset to a = 75 mm and bar BC is now a hollow pipe with outer diameter d_2 = a, find the inner diameter d_1 so that the angles of twist of segments AB and BC are equal. [6]

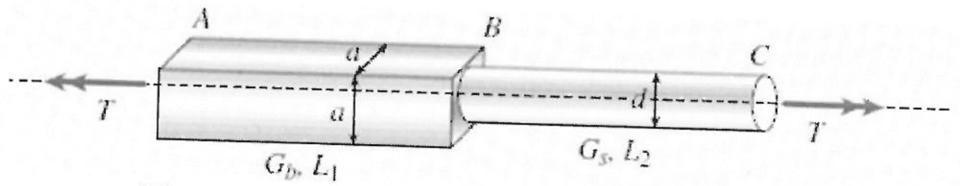

Figure Q3: Combined circular and non-circular shaft

QUESTION 4 [25]

4.1 Briefly describe the Castigliano's first principle. [5]

4.2 The members of a truss shown in Figure Q4 are all made of steel with E = 209 GPa. Each member has a cross sectional area of 400 mm^2. Use Castigliano's second principle to determine the vertical displacement at point C and B. [20]

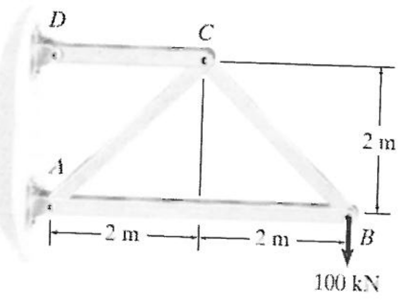

Figure Q4: Space frame carrying at end point load

THE END!

Formula Sheet

Buckling Equations

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad P_{cr} = \frac{\pi^2 EI}{L^2} \qquad P_{cr} \approx \frac{2\pi^2 EI}{L^2} \qquad P_{cr} = \frac{4\pi^2 EI}{L^2}$$

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

Initially Curved Sections

$$R = A \Big/ \int_A \frac{dA}{r} \qquad \sigma = \frac{M(R-r)}{rA(\bar{r}-R)}$$

Bending Stress Equations:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \qquad \tau_{xy} = \tau_{yx} = \frac{Q}{b \cdot I} \cdot \int_A y \cdot dA = \frac{Q \cdot A \cdot \bar{y}_A}{b \cdot I}$$

Springs

$$\tau_{\max} = \frac{8 \cdot F \cdot D}{\pi \cdot d^3} + \frac{4F}{\pi \cdot d^2} \qquad y = \alpha \cdot \frac{D}{2} = \frac{8 \cdot F \cdot D^3 \cdot N}{d^4 \cdot G} \qquad k = \frac{d^4 \cdot G}{8 \cdot D^3 \cdot N}$$

$$\sigma = \frac{M}{I/c} + \frac{F}{A} = K \cdot \frac{32 \cdot F \cdot r_m}{\pi \cdot d^3} + \frac{4 \cdot F}{\pi \cdot d^2} \qquad K \approx \frac{r_m}{r_i} \qquad N = N_T - N_D$$

Thick Cylinders

$$\sigma_r = \frac{1}{k^2 - 1} \cdot \left[p_i \cdot \left(1 - \frac{r_o^2}{r^2}\right) - p_o \cdot k^2 \cdot \left(1 - \frac{r_i^2}{r^2}\right)\right] \qquad \sigma_r = \frac{p_i}{k^2 - 1} \cdot \left(1 - \frac{r_o^2}{r^2}\right)$$

$$\sigma_\theta = \frac{1}{k^2 - 1} \cdot \left[p_i \cdot \left(1 + \frac{r_o^2}{r^2}\right) - p_o \cdot k^2 \cdot \left(1 + \frac{r_i^2}{r^2}\right)\right] \qquad \sigma_\theta = \frac{p_i}{k^2 - 1} \cdot \left(1 + \frac{r_o^2}{r^2}\right)$$

$$k = \frac{r_o}{r_i} \qquad \sigma_r = A - \frac{B}{r^2} \qquad \sigma_\theta = A + \frac{B}{r^2}$$

$$\delta = -u' + u'' = r_m \cdot (\varepsilon_\theta'' - \varepsilon_\theta')$$

$$\varepsilon_\theta'' = \frac{1}{E}\left(\sigma_\theta'' - \nu\sigma_r''\right) \quad \textit{outer cylinder}$$

$$\varepsilon_\theta' = \frac{1}{E}\left(\sigma_\theta' - \nu\sigma_r'\right) \quad \textit{inner cylinder}$$

Rotating Components

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \rho \cdot \omega^2 \cdot r = 0 \qquad \sigma_r = A - \frac{B}{r^2} - \left(\frac{3+\nu}{8}\right) \cdot \rho \cdot \omega^2 \cdot r^2$$

$$\omega_Y = \frac{1}{r_e} \cdot \sqrt{\frac{8 \cdot \sigma_Y}{(3+\nu) \cdot \rho}} \qquad \sigma_\theta = A + \frac{B}{r^2} - \left(\frac{1+3\cdot\nu}{8}\right) \cdot \rho \cdot \omega^2 \cdot r^2$$

$$\omega_Y = \sqrt{\frac{4 \cdot \sigma_Y}{\rho \cdot \left[(3+\nu) \cdot r_e^{\,2} + (1-\nu) \cdot r_i^{\,2}\right]}} \qquad \sigma_{r1} \cdot z_1 = \sigma_{r2} \cdot z_2 \qquad F_c = m \cdot \omega^2 \cdot r$$

Torsion of Non-circular sections

$$T = \alpha b t^2 \frac{G\theta}{L} = \beta b t^3 \frac{G\theta}{L} \qquad \tau_{\max} = \frac{G\theta t}{L} \qquad T = \frac{1}{3} b t^2 \frac{G\theta}{L}$$

b/t	**1**	**1.5**	**2**	**2.5**	**3**	**4**	**6**	**10**	∞
α	0.208	0.231	0.246	0.256	0.267	0.282	0.299	0.312	0.333
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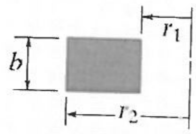
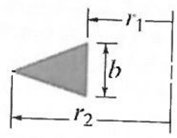
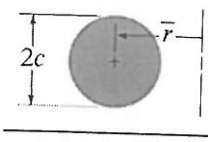
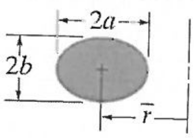
Shape	$\int_A \frac{dA}{r}$
	$b \ln \frac{r_2}{r_1}$
	$\frac{b\, r_2}{(r_2 - r_1)} \left(\ln \frac{r_2}{r_1}\right) - b$
	$2\pi \left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$
	$\frac{2\pi b}{a} \left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$

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QUESTION 1 [25]

A compound tube is formed by shrinking together two tubes with common radius 150 mm and thickness 25 mm. The shrinkage allowance is such that when an internal pressure of 30 MPa is applied to the compound cylinder, the maximum stress in each tube is the same.

3.1 Determine the value of this maximum stress [15]
3.1 What must have been the difference in diameters of the tubes before shrinkage? [10]

Both tubes are made of steel with E = 210 GPa and v = 0.3.

QUESTION 2 [25]

A close coiled helical compression spring made from round wire fits over the spindle of a plunger and has to work inside a tube. The spindle diameter is 12 mm and the tube is 25 mm outside diameter and 0.15 mm thick. The maximum working length of the spring has to be 120 mm and the minimum length 90 mm. The maximum force exerted by the spring has to be 350 N and the minimum 240 N. If the shear stress in the spring is not to exceed 600 MPa, find:

2.1 The free length of the spring [5]
2.2 The mean coil diameter [10]
2.3 The wire diameter [5]
2.4 The number of free coils [5]

For the spring material, E = 210 GPa, ν = 0.3 and G = 70 GPa.

QUESTION 3 [25]

The American Standard rolled-steel beam shown in Figure Q3 has been rein-forced by attaching to it two 16 × 200-mm plates, using 18-mm diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force. For the bolt material, E = 209 GPa, G = 70 GPa and ν = 0.3. [25]

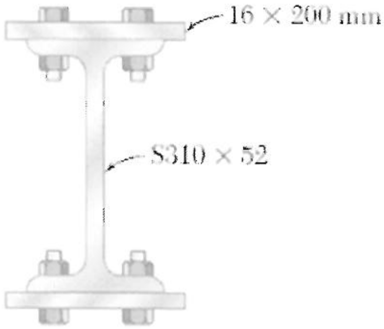

Figure Q3: Rein-forced rolled steel beam section

QUESTION 4 [25]

A ceiling suspended C-arm used to support an X-ray camera for medical diagnosis is illustrated in Figure Q4. The camera has a mass of 150 kg with its center of mass located at point G.

4.1 Determine the induced stress at the inner surface point A. [15]

4.1 Determine the induced stress at the outer surface point on the same horizontal plane as A. [10]

The arm is made of steel with E = 209 GPa, ν = 0.3 and G = 70 GPa.

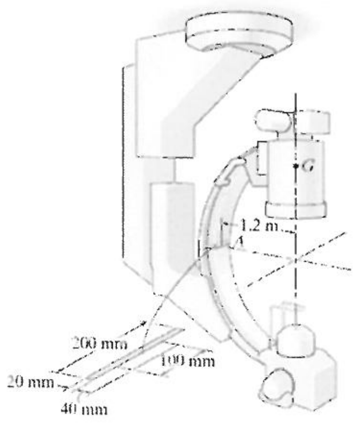

Figure Q4: Ceiling suspended X-ray camera C-arm

THE END!

Formula Sheet

Buckling Equations

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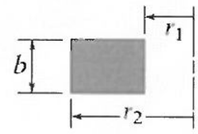
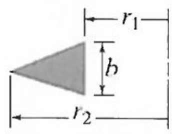
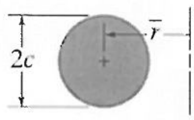
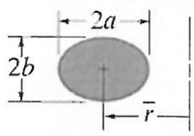
	$b \ln \frac{r_2}{r_1}$
	$\frac{b\, r_2}{(r_2 - r_1)}\left(\ln \frac{r_2}{r_1}\right) - b$
	$2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$
	$\frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$