UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS

BACCALAUREUS OPTOMETRIAE:

MODULE MAT01A1
CALCULUS OF ONE-VARIABLE FUNCTIONS

CAMPUS DFC

JUNE EXAMINATION

DATE 31/05/2016 **SESSION 12:30 – 14:30**

ASSESSOR **MR IK LETLHAGE**

INTERNAL MODERATOR **MR J BRUYNS**

DURATION 2 HOURS **MARKS 70**

SURNAME AND INITIALS: _____

STUDENT NUMBER: _____

COURSE: _____

CONTACT NO: _____

NUMBER OF PAGES: 15

INSTRUCTIONS :

1. **ANSWER ALL THE QUESTIONS IN THE SPACE PROVIDED**
2. **USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE INK).**
3. **USE THE BLANK PAGES FOR ROUGH WORK. INDICATE IT AS SUCH.**
4. **SHOULD YOU NEED MORE SPACE FOR WRITING, USE THE BLANK PAGES.**

QUESTION 1 [3]

Solve for x and represent the solution on a number line: $|2x-3| \geq 3$ (3)

QUESTION 2 [3]

Use a truth table to establish the logical equivalence $\neg(p \leftrightarrow q) \equiv (p \wedge \neg q) \vee (q \wedge \neg p)$

QUESTION 3 [3]

(a) Translate the following statement into predicate (first-order) language: "For every real number x, there exists a real number y such that $xy = 1$." (1)

(b) Determine the truth value of A, given the following: (2)

$B \to \neg A$ is true, $\neg B \to \neg C$ is true and C is true.

QUESTION 4 [3]

Calculate the value of the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i+1}{n}\right)^3$ (3)

QUESTION 5 [8]

(a) Use De Moivre's Theorem to calculate $\frac{\left(1-\sqrt{3}i\right)^3\left(-1+i\right)^4}{\left(2\sqrt{3}+2i\right)^2}$. Give the final answer in polar form. (4)

(b) Find all the cube roots of -27 and express the final answers in the form $a+bi$. (4)

QUESTION 6 [3]

Use the direct proof method to prove that $n^2 - n$ is even $\forall n \in \mathbb{Z}$.

QUESTION 7 [2]

Sketch the graph of $y = 2\cos\left(x - \frac{\pi}{2}\right)$ on the interval $[-2\pi, 2\pi]$. Clearly show all the intercepts and asymptotes, if any.

QUESTION 8 [3]

Let $f(x) = \log_2(x-3) + 2$ Find the inverse of f : f^{-1}

QUESTION 9[6]

Consider the function $f(x)=\dfrac{x^2-4}{|x-2|}$.

(a) Find $\lim\limits_{x\to 2^+} f(x)$ (2)

(b) Find $\lim\limits_{x\to 2^-} f(x)$ (2)

(c) Is f continuous at $x=2$? Explain your answer. (2)

QUESTION 10[3]

Use the Intermediate Value Theorem to show that the equation $x^3 - x - 1 = 0$ has a root in the interval $(1, 2)$. (**DO NOT** attempt to find this root.)

QUESTION 11[5]

Consider the function $f(x)=\frac{2}{\sqrt{x+2}}$.

(a) Use the definition of the derivative of a function to find $f'(2)$. (3)

(b) Find the equation of the line through the point $(-1;2)$ that is perpendicular to the tangent line to the curve $y=f(x)$ at $(2;1)$ (2)

QUESTION 12[3]

Prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

QUESTION 13[11]

(a) Find $\frac{d^2y}{dx^2}$ if $y = x^2 \tanh^{-1} x.$ (4)

(b) Find $\frac{dy}{dx}$, in its simplest form, if $y = \frac{\left(\cos^{-1} x\right)^{x} \cdot \sqrt[5]{x^3+7}}{e^{\sqrt{x+1}}}$. (4)

(c) Use implicit differentiation to find $\frac{dy}{dx}$, given that $xy = \ln\left(x+y\right)$ (3)

QUESTION 14[3]

Use l'Hôspital's Rule to calculate $\lim_{x\to 0}\left(\cot x-\csc x\right)$

QUESTION 15[4]

Find f if $f''(x)=10\sin x+3\cos x,\ f(0)=0,\ f(2\pi)=12$.

QUESTION 16 [9]

Evaluate the following integrals. Show all the integration steps.

(a) $\int_1^3 \left(e^{\ln x} + \frac{5}{\sqrt{x}} - \frac{13}{x} \right) dx$ (3)

(b) $\int \cos^3 x dx$ (3)

(c) $\int \frac{\sec^2\theta}{\sqrt{\tan^2\theta - 1}} d\theta$ (3)

AVAILABLE MARKS : 72