UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS

BACCALAUREUS OPTOMETRIAE:

MODULE MAT01A1
CALCULUS OF ONE-VARIABLE FUNCTIONS

CAMPUS **DFC**

JULY SUPPLEMENTARY EXAMINATION

DATE 29/07/2016 **SESSION 08:00 – 10:00**

ASSESSOR **MR IK LETLHAGE**

INTERNAL MODERATOR **MR J BRUYNS**

DURATION 2 HOURS **MARKS 70**

SURNAME AND INITIALS: ______________________________

STUDENT NUMBER: ______________________________

CONTACT NO: ______________________________

NUMBER OF PAGES: 16

INSTRUCTIONS :

1. **ANSWER ALL THE QUESTIONS IN THE SPACE PROVIDED**
2. **USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE INK).**
3. **USE THE BLANK PAGES FOR ROUGH WORK. INDICATE IT AS SUCH.**
4. **SHOULD YOU NEED MORE SPACE FOR WRITING, USE THE BLANK PAGES.**

QUESTION 1 [4]

Solve for x and represent the solution in interval form: $(x-1)(x+3)(2x+4)<0$

QUESTION 2 [3]

Solve the equation $3\sin\theta = 2\cos^2\theta$, $0 \le \theta \le \pi$.

QUESTION 3 [3]

Evaluate the sum of the telescoping sum $\sum_{k=0}^{99}\left(\frac{1}{4^k}-\frac{1}{4^{k+1}}\right)$.

QUESTION 4 [5]

Find all the fourth roots of $-8\sqrt{3}+8i$. Express the roots in the form $a+bi$.

QUESTION 5 [4]

Use The Principle of Mathematical Induction to prove that

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1} \quad \forall n \in \mathbb{N}.$$

QUESTION 6 [2]

Use a truth table to show that $\neg(p \to q) \to p \wedge q$ is logically equivalent to $p \to q$

QUESTION 7[5]

In the table below, column A contains logical statements and column B contains the meanings of these statements. Match each statement in column A to its meaning in column B.

Column A	Column B
(i) $(p \wedge \neg q) \vee (q \wedge \neg p)$	(a) Tautology
(ii) $p \vee \neg p$	(b) Logical equivalence of $\neg p \vee q$
(iii) $q \to p$	(c) A contradiction
(iv) $p \wedge \neg p$	(d) The converse of $p \to q$
(v) $p \to q$	(e) Logical equivalence of $\neg(p \leftrightarrow q)$

Answers

	Answer
(i)	
(ii)	
(iii)	
(iv)	
(v)	

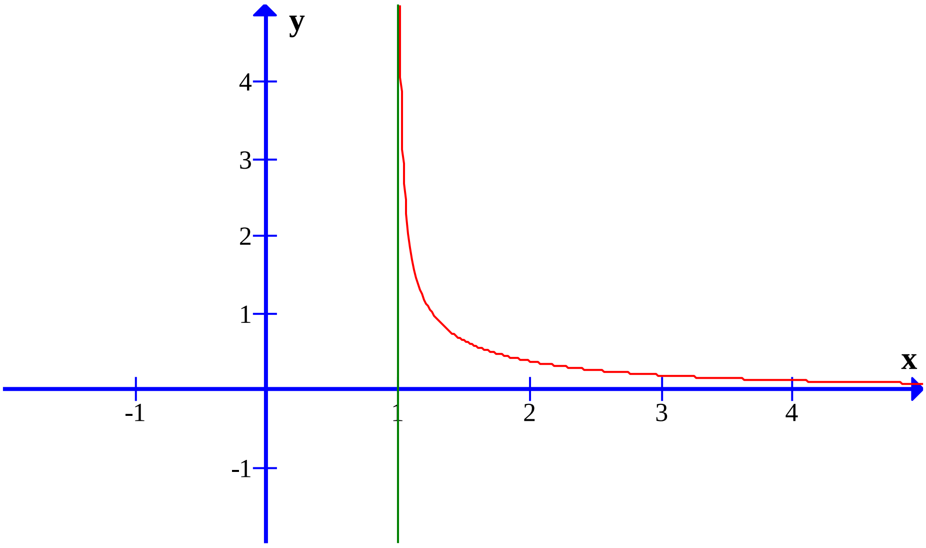
<u>QUESTION 8</u> [1]

Use predicate (first order) language to negate the following statement.

All real numbers are integers. (Use □ to denote the set of all real numbers and □ to denote the set of all integers.)

QUESTION 9 [3]

Let $f\left(x\right)=\dfrac{1}{\sqrt{x^3-1}}$. The graph of this function is as below.



Find the inverse of f : f^{-1} and use the given graph to sketch the graph of f^{-1} on the same set of axes.

<u>QUESTION 10</u>[4]

(a) State the **Squeeze Theorem**. (1)

(b) Use the Squeeze Theorem to find $\lim_{x\to 0}\left(x^4\cos\frac{2}{x}\right)$. (3)

QUESTION 11[6]

(a) Use the definition of the derivative of a function to find $f'(x)$ if $f(x)=\frac{1}{x}$.

(3)

(b) Find the equation of a line that is parallel to the tangent line to curve $y=\frac{1}{x}$ at $(1;1)$ that passes through the point $(0;1)$. (3)

<u>QUESTION 12</u>[5]

Use the result $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ to prove the following results.

(a) $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$ (2)

(b) $\frac{d}{dx}(\sin x) = \cos x,$ using the definition of the derivative of a function. (3)

QUESTION 13[10]

(a) Find $\dfrac{dy}{dx}$ if $y = \dfrac{2x - e^{\sqrt{x}}}{1 + \tan x}$ (3)

(b) Find $\dfrac{dy}{dx}$, in its simplest form, if $y = \dfrac{x \tan^{-1} x}{e^{x^2} \sin x}$. (4)

(c) Let $x^2 + y^2 + 2xy = 1$. Use implicit differentiation to find $\frac{dy}{dx}$. (3)

QUESTION 14[3]

Use l'Hôspital's Rule to calculate $\lim_{x \to 0^+} (\sin x \ln x)$

QUESTION 15[3]

Find f if $f'(x)=5x^4-3x^2+4,\quad f(-1)=2.$

QUESTION 16 [2]

Use the Fundamental Theorem of Calculus, Part 1, to evaluate $\frac{d}{dx}\int_0^{\tan x}\tan^{-1} t\,dt$

QUESTION 17[9]

Evaluate the following integrals. Show all the integration steps.

(a) $\int \sinh(\ln x)dx$ (3)

(b) $\int_0^{\pi} \tan^4 x dx$ (4)

(c) $\int \frac{2x}{x^2+1} d\theta$ (2)

AVAILABLE MARKS : 72