UNIVERSITEIT VAN JOHANNESBURG
UNIVERSITY OF JOHANNESBURG

Department of Economics and Econometrics

November 2016 Examination

Course:	Econometrics 2B (EKM2B01 / ECM02B2)
Examiners:	Mr JJA Kouadio
Internal Moderator:	Prof JWM Mwamba
Time:	180 min
Marks:	70

SECTION A: Theory **[9 marks]**

Answer the following questions by providing a clear and concise answer.

1. What is Statistics? **(3)**
2. Is there a link between the Statistics and the concept of probability? Motivate your answer. **(3)**
3. State clearly the Uniqueness Theorem **(3)**

SECTION B: **[12 marks]**

Answer the following questions. Ensure to provide your detailed workings when necessary.

1. Let X_1 and X_2 be jointly continuous random variables with joint density function $g(x_1, x_2)$. Then the marginal density function of X_2 is given by

$$g_2(x_2) = ?$$

(2)

2. Let $X_1, X_2, \dots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Find the Maximum Likelihood Estimators of μ and σ^2. **(6)**

3. Using the identity

$$(\hat{\theta} - \theta) = [\hat{\theta} - E(\hat{\theta})] + [E(\hat{\theta}) - \theta] = [\hat{\theta} - E(\hat{\theta})] + B(\hat{\theta})$$

Show that

$$MSE(\hat{\theta}) = E\left[(\hat{\theta} - \theta)^2\right] = V(\hat{\theta}) + [B(\hat{\theta})]^2$$

(4)

SECTION C **[22 marks]**

1. Let the distribution function of a random variable Y be

$$\begin{cases} 0, & y \leq 0 \\ \dfrac{y}{8}, & 0 < y < 2 \\ \dfrac{y^2}{16}, & 2 \leq y < 4 \\ 1, & y > 4 \end{cases}$$

a) Find the density function of Y **(4)**

b) Find $P(1 \leq Y \leq 3)$ **(2)**

c) Find the mean of Y **(2)**

d) Find the variance of Y **(2)**

2. Let Y_1 and Y_2 have joint density function

$$f(y_1, y_2) = \begin{cases} e^{-(y_1+y_2)}, & y_1 > 0, y_2 > 0 \\ 0, & elsewhere \end{cases}$$

a) What is $P(Y_1 < 1, Y_2 > 5)$? **(3)**

b) What is $P(Y_1 + Y_2) < 3$? **(3)**

3. Consider a random variable M with probability density function given by

$$f(m) = \begin{cases} 2(1-m), & 0 \leq m \leq 1 \\ 0, & elsewhere \end{cases}$$

Use the method of transformation to find the density functions of

a) $U_1 = M^2$ **(3)**

b) $U_2 = 1 - 2M$ **(3)**

SECTION D [27 marks]

1. Contracts for two construction jobs are randomly assigned to one or more of three firms, A, B and C. Let Y_1 denote the number of contracts assigned to firm A and Y_2 the number of contracts assigned to firm B. Recall that each firm can receive 0, 1 or 2 contracts.
   a) Find the joint probability function for Y_1 and Y_2. **(4)**
   b) Find $F(1, 0)$. **(2)**
   c) Find the marginal probability distribution of Y_1 **(2)**
   d) Find $E(Y_1)$ **(2)**
   e) Find $E(Y_1 - Y_2)$ **(4)**

2. The manager of a construction job needs to figure prices carefully before submitting a bid. He also needs to account for uncertainty (variability) in the amounts of products he might need. To oversimplify the real situation, suppose that a project manager treats the amount of sand, in yards, needed for a construction project as a random variable Y_1, which is normally distributed with mean 10 yards and standard deviation 0.5 yard. The amount of cement mix needed, in hundreds of pounds, is a random variable Y_2, which is normally distributed with mean 4 and standard deviation 0.2. The sand costs R70 per yard, and the cement mix costs R30 per hundred pounds. Adding R1000 for other costs, he computes his total cost to be

$$\boldsymbol{U = 1000 + 70Y_1 + 30Y_2.}$$

   If Y_1 and Y_2 are independent, how much should the manager bid to ensure that the true costs will exceed the amount bid with a probability of only 0.02? **(6)**

3. The hourly wages in a particular industry are normally distributed with mean R132 and standard deviation R25. A company in this industry employs 40 workers, paying them an average of R122 per hour. Can this company be accused of paying substandard wages? Use an $\alpha = 0.01$ level test. (All necessary steps should be provided) **(7)**