UNIVERSITY OF JOHANNESBURG

PROGRAM	:	NATIONAL DIPLOMA *INDUSTRIAL ENGINEERING TECHNOLOGY*
SUBJECT	:	**OPERATIONS RESEARCH**
CODE	:	**BOA 321**
DATE	:	YEAR END EXAMINATIONS 2016 3 DECEMBER 2016
DURATION	:	8:30-11:30
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TOTAL MARKS	:	104

EXAMINER	:	MRS STEENKAMP
MODERATOR	:	MR T NENZHELELE
NUMBER OF PAGES	:	4 PAGES AND 1 ANNEXURES

INSTRUCTIONS	:	PLEASE ANSWER ALL THE QUESTIONS.
REQUIREMENTS	:	STUDENTS MAY USE CALCULATORS

QUESTION 1

Cliff Colby wants to determine whether or not his South Japan oil fields will yield oil. He has hired geologist Digger Barnes to run tests on the field. If there is oil in the field there is a 95% chance that Digger's tests will indicate that there is oil in the field. If the field contains no oil there is a 7% chance that Diggers test will indicate that there is oil in the field? If Diggers tests indicate that there is no oil in the field, what is the probability that the field contains oil? Before Digger conducts the tests, Cliff believes there is a 10% chance that the field will yield oil. Use Bayes to calculate posterior probabilities.

[6]

QUESTION 2

AFC is about to launch its new Wings and Things fast food nationally. The research department is convinced that Wings and Things will be a great success and wants to introduce it in all AFC outlets without advertisement. The marketing department has a different approach. The marketing department wants to unleash an intensive advertisement campaign. The advertisement campaign will cost R 500 000 and if successful and customers are receptive will produce R1800 000 revenue. If the advertisement campaign is successful there is a 90% probability that the customers will be receptive. If customers are not receptive the company will lose R 700 000. There is a 30% chance that the campaign is unsuccessful. If the campaign is unsuccessful there is a probability of 80% that customers will be unreceptive. If no advertisement is used there is a probability of 0.8 that customers will be receptive.
2.1 Draw a decision tree and find the optimal solution

[25]

QUESTION 3

Western National bank is considering opening a drive-in window for customer service. Management estimates that customers will arrive at a rate of 15 per hour. The teller who will staff the window can service at the rate of one every 3 minutes. Arrivals are poisson and service is exponential. Determine the following

a) Utilisation of the teller.
b) How many are currently in the system?
c) Average number in the waiting line
d) What is the probability that there is one machine in the system?
e) What is the average waiting time in the queue?
f) What is the average wait in the system?

[12]

QUESTION 4

The Puck and Pawn company manufactures hockey sticks and chess sets. Each hockey stick yields a profit of R25 and each chess set R20. A hockey stick requires one hour on machine 1 and four hours on machine 2. A chess set requires one hour on machine 1 and one hour on machine two. Machine one has 200 hours available and Machine two has 400 hours available. Market research indicates that at least 20 chess set need to be manufactured. Formulate this as a linear programming problem and solve it graphically. Find the optimal solution.

[15]

QUESTION 5

Rent A Car is a multi-site.car rental company in Johannesburg. It is trying a new policy of "return the car to the location most convenient to you" to improve customer service. This means that cars need to be moved constantly to maintain required levels of service. Supply and demand as well as the cost is indicated below.

5.1. Set up a transportation problem using the Northwest corner rule.

5.2. Use the stepping stone method to determine the next table to solve this problem (one iteration)

	Airport	Sandton	Linden	Melville	Supply
Bryanston	9	8	6	5	60
Germiston	9	8	8	3	40
Rosebank	5	3	3	10	65
Demand	50	60	25	30	

[14]

QUESTION 6

Sunco Oil wants to ship the maximum amount of oil (per hour) via pipeline from node 0 to node 5 in the table below. The various arcs in the figure represent pipelines of different diameters. The maximum number of barrels oil (millions of barrels per hour) that can be pumped through each arc is shown in table. Each of these numbers is called the arc capacity. Draw the network and determine maximum flow

Arc	Capacity in direction of flow	Capacity in opposite direction
0,1	2	1
0,2	3	3
1,2	3	2
1,3	4	2
3,4	1	0
2,4	2	5
3,5	2	3
4,5	4	2
2,5	3	1

[11]

QUESTION 7

Set up a simplex table for the following LP problem and indicate the pivot point
Maximise =15X +30Y
Subject to: $4X+6Y \leq 120$
$2X + 6Y \leq 72$
$Y \leq 25$
$X, Y \geq 0$

[8]

QUESTION 8

Scot Summers produces sunglasses in batches. The estimated demand for the year is 50 000 units. It costs R 1000 to setup the process. Carrying cost is R2.00 per unit per year. Once set up has been done 180 units can be manufactured daily. The demand during the production run is 150 units a day. The company manufactures for 220 days in a year. How many sunglasses should be manufactured in each batch?

[5]

QUESTION 9

Over any given month PNA loses 10% of their customers to CNA and 15 % to Forum. CNA loses 10% to PNA and 20% to Forum. Forum loses 5% to CNA and 10% to PNA. At present CAN has 25% of the market, PNA has 35% and Forum has 40% of the market. Determine what the market shares will be next month.

[6]

TOTAL : 104
FULL MARKS : 100

(2-1) $0 \leq P(\text{event}) \leq 1$

A basic statement of probability.

(2-2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Probability of the union of two events.

(2-3) $P(A|B) = \dfrac{P(AB)}{P(B)}$

Conditional probability.

(2-4) $P(AB) = P(A|B)P(B)$

Probability of the intersection of two events.

(2-5) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' theorem in general form.

(2-6) $E(X) = \sum_{i=1}^{n} X_i P(X_i)$

An equation that computes the expected value (mean) of a discrete probability distribution.

(2-7) $\sigma^2 = \text{Variance} = \sum_{i=1}^{n} [X_i - E(X)]^2 P(X_i)$

An equation that computes the variance of a discrete probability distribution.

(2-8) $\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$

An equation that computes the standard deviation from the variance.

(2-9) Probability of r successes in n trials $= \dfrac{n!}{r!(n-r)!} p^r q^{n-r}$

A formula that computes probabilities for the binomial probability distribution.

(2-10) Expected value (mean) $= np$

The expected value of the binomial distribution.

(2-11) Variance $= np(1-p)$

The variance of the binomial distribution.

(2-12) $f(X) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

The density function for the normal probability distribution.

(2-13) $Z = \dfrac{X - \mu}{\sigma}$

An equation that computes the number of standard deviations, Z, the point X is from the mean μ.

(2-14) $f(X) = \mu e^{-\mu x}$

The exponential distribution.

(2-15) Expected value $= \dfrac{1}{\mu}$

The expected value of an exponential distribution.

(2-16) Variance $= \dfrac{1}{\mu^2}$

The variance of an exponential distribution.

(2-17) $P(X \leq t) = 1 - e^{-\mu t}$

Formula to find the probability that an exponential random variable (X) is less than or equal to time t.

(2-18) $P(X) = \dfrac{\lambda^x e^{-\lambda}}{X!}$

The Poisson distribution.

(2-19) Expected value $= \lambda$

The mean of a Poisson distribution.

(2-20) Variance $= \lambda$

The variance of a Poisson distribution.

(3-1) EMV(alternative i) $= \sum X_i P(X_i)$

An equation that computes expected monetary value.

(3-2) EVwPI $= \sum$(best payoff in state of nature i) $\times$ (probability of state of nature i)

An equation that computes the expected value with perfect information.

(3-3) EVPI = EVwPI − Best EMV

An equation that computes the expected value of perfect information.

(3-4) EVSI = (EV with SI + cost) − (EV without SI)

An equation that computes the expected value (EV) of sample information (SI).

(3-5) Efficiency of sample information $= \dfrac{\text{EVSI}}{\text{EVPI}} 100\%$

An equation that compares sample information to perfect information.

(3-6) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' theorem—the conditional probability of event A given that event B has occurred.

(3-7) Utility of other outcome $= (p)(1) + (1-p)(0) = p$

An equation that determines the utility of an intermediate outcome.

Solved Problems

Key Equations

Equations 6-1 through 6-8 are associated with the economic order quantity (EOQ).

(6-1) Average inventory level $= \dfrac{Q}{2}$

(6-2) Annual ordering cost $= \dfrac{D}{Q}C_o$

(6-3) Annual holding cost $= \dfrac{Q}{2}C_h$

(6-4) $\text{EOQ} = Q^* = \sqrt{\dfrac{2DC_o}{C_h}}$

(6-5) $TC = \dfrac{D}{Q}C_o + \dfrac{Q}{2}C_h$

Total relevant inventory cost.

(6-6) Average dollar level $= \dfrac{(CQ)}{2}$

(6-7) $Q = \sqrt{\dfrac{2DC_o}{IC}}$

EOQ with C_h expressed as percentage of unit cost.

(6-8) $\text{ROP} = d \times L$

Reorder point: d is the daily demand and L is the lead time in days.

Equations 6-9 through 6-13 are associated with the production run model.

(6-9) Average inventory $= \dfrac{Q}{2}\left(1 - \dfrac{d}{p}\right)$

(6-10) Annual holding cost $= \dfrac{Q}{2}\left(1 - \dfrac{d}{p}\right)C_h$

(6-11) Annual setup cost $= \dfrac{D}{Q}C_s$

(6-12) Annual ordering cost $= \dfrac{D}{Q}C_o$

(6-13) $Q^* = \sqrt{\dfrac{2DC_s}{C_h\left(1 - \dfrac{d}{p}\right)}}$

Optimal production quantity.

Equation 6-14 is used for the quantity discount model.

(6-14) Total cost $= DC + \dfrac{D}{Q}C_o + \dfrac{Q}{2}C_h$

Total inventory cost (including purchase cost).

Equations 6-15 to 6-20 are used when safety stock is requi

(6-15) ROP = (Average demand during lead time) + SS

General reorder point formula for determining v safety stock (SS) is carried.

(6-16) ROP = (Average demand during lead time) + Zσ

Reorder point formula when demand during lead is normally distributed with a standard deviatio σ_{dLT}.

(6-17) $\text{ROP} = \bar{d}L + Z(\sigma_d\sqrt{L})$

Formula for determining the reorder point when da demand is normally distributed but lead time is cor stant, where $\bar{d}$ is the average daily demand, L is the constant lead time in days, and σ_d is the standard d viation of daily demand.

(6-18) $\text{ROP} = d\bar{L} + Z(d\sigma_L)$

Formula for determining the reorder point when da demand is constant but lead time is normally distri uted, where $\bar{L}$ is the average lead time in days, d is constant daily demand, and σ_L is the standard devi of lead time.

(6-19) $\text{ROP} = \bar{d}\bar{L} + Z(\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2})$

Formula for determining reorder point when both demand and lead time are normally distributed; wh $\bar{d}$ is the average daily demand, $\bar{L}$ is the average lea time in days, σ_L is the standard deviation of lead ti and σ_d is the standard deviation of daily demand.

(6-20) $\text{THC} = \dfrac{Q}{2}C_h + (\text{SS})C_h$

Total annual holding cost formula when safety stc carried.

Equation 6-21 is used for marginal analysis.

(6-21) $P \geq \dfrac{\text{ML}}{\text{ML} + \text{MP}}$

Decision rule in marginal analysis for stocking tional units.

λ = mean number of arrivals per time period
μ = mean number of people or items served per time period

Equations 12-1 through 12-7 describe operating characteristics in the single-channel model that has Poisson arrival and exponential service rates.

(12-1) L = average number of units (customers) in the system

$$= \frac{\lambda}{\mu - \lambda}$$

(12-2) W = average time a unit spends in the system (Waiting time + Service time)

$$= \frac{1}{\mu - \lambda}$$

(12-3) L_q = average number of units in the queue

$$= \frac{\lambda^2}{\mu(\mu - \lambda)}$$

(12-4) W_q = average time a unit spends waiting in the queue

$$= \frac{\lambda}{\mu(\mu - \lambda)}$$

(12-5) ρ = utilization factor for the system $= \frac{\lambda}{\mu}$

(12-6) P_0 = probability of 0 units in the system (i.e., the service unit is idle)

$$= 1 - \frac{\lambda}{\mu}$$

(12-7) $P_{n>k}$ = probability of more than k units in the system

$$= \left(\frac{\lambda}{\mu}\right)^{k+1}$$

Equations 12-8 through 12-12 are used for finding the costs of a queuing system.

(12-8) Total service cost $= mC_s$

where

m = number of channels

C_s = service cost (labor cost) of each channel

(12-9) Total waiting cost per time period $= (\lambda W)C_w$

C_w = cost of waiting

Waiting time cost based on time in the system.

(12-10) Total waiting cost per time period $= (\lambda W_q)C_w$

Waiting time cost based on time in the queue.

(12-11) Total cost $= mC_s + \lambda W C_w$

Waiting time cost based on time in the system.

(12-12) Total cost $= mC_s + \lambda W_q C_w$

Waiting time cost based on time in the queue.

Equations 12-13 through 12-18 describe operating characte tics in multichannel models that have Poisson arrival and ex nential service rates, where m = *the number of open channe*

(12-13) $$P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu - \lambda}}$$

for $m\mu > \lambda$

Probability that there are no people or units in system

(12-14) $$L = \frac{\lambda\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

Average number of people or units in the system.

(12-15) $$W = \frac{\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

Average time a unit spends in the waiting line or be serviced (namely, in the system).

(12-16) $$L_q = L - \frac{\lambda}{\mu}$$

Average number of people or units in line waiting service.

(12-17) $$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

Average time a person or unit spends in the queue w ing for service.

(12-18) $$\rho = \frac{\lambda}{m\mu}$$

Utilization rate.

Equations 12-19 through 12-22 describe operating characteristics in single-channel models that have Poisson arrivals and constant service rates.

(12-19) $L_q = \dfrac{\lambda^2}{2\mu(\mu - \lambda)}$

Average length of the queue.

(12-20) $W_q = \dfrac{\lambda}{2\mu(\mu - \lambda)}$

Average waiting time in the queue.

(12-21) $L = L_q + \dfrac{\lambda}{\mu}$

Average number of customers in the system.

(12-22) $W = W_q + \dfrac{1}{\mu}$

Average waiting time in the system.

Equations 12-23 through 12-28 describe operating characteristics in single-channel models that have Poisson arrivals and exponential service rates and a finite calling population.

(12-23) $P_0 = \dfrac{1}{\sum\limits_{n=0}^{N} \dfrac{N!}{(N-n)!}\left(\dfrac{\lambda}{\mu}\right)^n}$

Probability that the system is empty.

(12-24) $L_q = N - \left(\dfrac{\lambda + \mu}{\lambda}\right)(1 - P_0)$

Average length of the queue.

(12-25) $L = L_q + (1 - P_0)$

Average number of units in the system.

(12-26) $W_q = \dfrac{L_q}{(N - L)\lambda}$

Average time in the queue.

(12-27) $W = W_q + \dfrac{1}{\mu}$

Average time in the system.

(12-28) $P_n = \dfrac{N!}{(N-n)!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 \quad \text{for } n = 0, 1, \ldots, N$

Probability of n units in the system.

Equations 12-29 to 12-31 are Little's Flow Equations, which can be used when a steady state condition exists.

(12-29) $L = \lambda W$

(12-30) $L_q = \lambda W_q$

(12-31) $W = W_q + 1/\mu$

14-1) $\pi(i) = (\pi_1, \pi_2, \pi_3, \ldots, \pi_n)$

Vector of state probabilities for period i.

14-2) $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & & & & \vdots \\ P_{m1} & & \cdots & & P_{mn} \end{bmatrix}$

Matrix of transition probabilities, that is, the probability of going from one state into another.

14-3) $\pi(1) = \pi(0)P$

Formula for calculating the state 1 probabilities, given state 0 data.

14-4) $\pi(n + 1) = \pi(n)P$

Formula for calculating the state probabilities for the period $n + 1$ if we are in period n.

14-5) $\pi(n) = \pi(0)P^n$

Formula for computing the state probabilities for period n if we are in period 0.

(14-6) $\pi = \pi P$

Equilibrium state equation used to derive equilibrium probabilities.

(14-7) $P = \left[\begin{array}{c|c} I & O \\ \hline A & B \end{array}\right]$

Partition of the matrix of transition for absorbing state analysis.

(14-8) $F = (I - B)^{-1}$

Fundamental matrix used in computing probabilities ending up in an absorbing state.

(14-9) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} \dfrac{d}{r} & \dfrac{-b}{r} \\ \dfrac{-c}{r} & \dfrac{a}{r} \end{bmatrix}$ where $r = ad - bc$

Inverse of a matrix with 2 rows and 2 columns.

Solved Problems