



PROGRAM : BACCALAUREUS INGENIERIAE
MECHANICAL ENGINEERING

SUBJECT : **THERMOMACHINES 4A**

CODE : **TRM4A11**

DATE : WINTER EXAMINATION
7 JUNE 2016

DURATION : (1-PAPER) 3 HOURS

WEIGHT : 50 : 50

TOTAL MARKS : 120, MARKED OUT OF 100

EXAMINER : Dr CR BESTER

MODERATOR : Mr JG BENADE

NUMBER OF PAGES : 2 PAGES INSTRUCTIONS
3 PAGES QUESTIONNAIRE
4 PAGES ANNEXURE (FORMULAE)
1 PAGE FUEL CHART

INSTRUCTIONS : SEE NEXT PAGE

REQUIREMENTS : NONE

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-

Question 1**(15 marks)**

Draw diagrammes showing the operation of the following RICE-electric motor hybrid drives for land vehicles:

- (i) Series-connected (5)
- (ii) Parallel-connected (4)
- (iii) Series-parallel-connected (6)

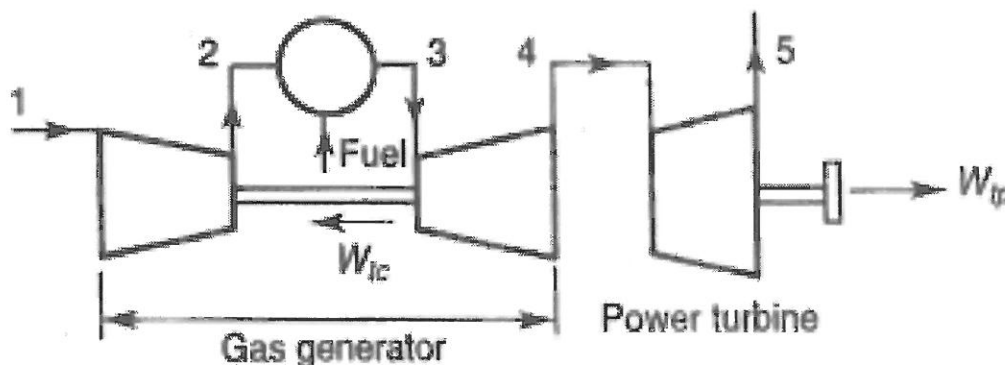
Question 2**(26 marks)**

Determine the specific work output, specific fuel consumption and cycle efficiency of a simple gas turbine with a free power turbine given the following specification:

Compressor pressure ratio:	12,0:1
Turbine inlet temperature:	1350 K
Isentropic efficiency η_C of compressor:	86%
Isentropic efficiency η_T of each turbine:	89%
Mechanical efficiency η_m of each shaft:	99%
Combustion efficiency η_b :	99%
Combustion chamber pressure loss:	6% of P_{02}
Exhaust pressure loss:	3 kPa
Ambient conditions:	100 kPa, 288K

Working substance properties are as follows:

$$C_{pa} = 1005 \text{ J/kgK}; \quad \gamma_a = 1,4; \quad C_{pg} = 1148 \text{ J/kgK}; \quad \gamma_g = 4/3;$$



Question 3**(34 marks)**

A 4,8 l V8 SI engine operates on a four-stroke GDI cycle at 4 200 RPM. The engine operates with two direct injections of gasoline in each cylinder during each cycle with an overall air-fuel ratio of 28:1. The first injection in each cylinder consists of one quarter of the total fuel input and occurs late in the intake stroke and early part of the compression stroke, 10° BBDC to 80° ABDC. The second injection inputs the rest of the fuel near the spark plug shortly before ignition, 70° BTDC to 30° BTDC. A supercharger gives the engine a volumetric efficiency of 98% at this speed. The atmospheric air density is 1,181 kg/m³.

Calculate:

1. "Steady-state" fuel mass flow into the engine (6)
2. Duration of one first fuel injection (7)
3. Fuel mass flow rate through injector during first injection (10)
4. Fuel mass flow rate through injector during second injection (11)

Question 4**(28 marks)**

An engine working on the air-standard Otto cycle has an inlet pressure of 100 kPa and a pressure of 1 400 kPa at the end of compression. The pressure at the end of combustion is 2,1 MPa. The γ -value of the working substance is 1,35.

Calculate:

- (i) The compression ratio of the engine (4)
- (ii) The clearance volume as a percentage of the cylinder volume (4)
- (iii) The ideal air-standard Otto efficiency (3)
- (iv) The compression work per unit displacement volume (8)
- (v) The expansion work per unit displacement volume (6)
- (vi) The net work per unit displacement volume (MEP) (3)

Question 5**(17 marks)**

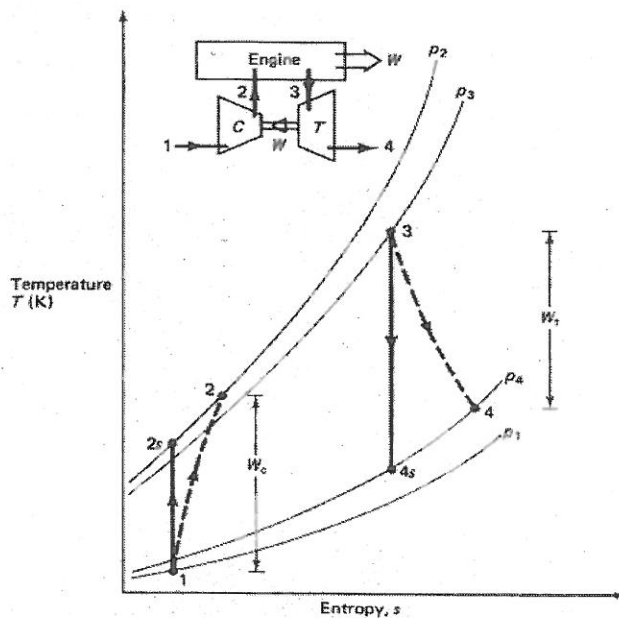
A diesel engine is fitted with a turbocharger, which comprises a centrifugal compressor driven by an inflow radial exhaust gas turbine. The air is drawn into the compressor at a pressure of 95 kPa and a temperature of 15°C, and is delivered to the engine at a pressure of 200 kPa. The engine is operating on an air-fuel ratio of 18:1. The exhaust gas leaves the engine at a temperature of 600°C and a pressure of 180 kPa, and leaves the turbine at 105 kPa. The isentropic efficiencies of the compressor and turbine are 70% and 80% respectively.

Calculate:

- (i) Temperature of the air leaving the compressor (6)
- (ii) Temperature of the gas leaving the turbine (6)
- (iii) Mechanical efficiency of the shaft connecting the compressor and turbine (5)

Working substance properties are as follows:

$$C_{pa} = 1010 \text{ J/kgK}; \quad \gamma_a = 1.4; \quad C_{pg} = 1150 \text{ J/kgK}; \quad \gamma_g = 4/3;$$



Annexure: Formula sheets

$$\eta = \frac{T'_{01} - T_a}{T_{01} - T_a}$$

$$\eta = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$\eta = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}}$$

$$\eta = \frac{(FA)_{theor}}{(FA)_{actual}}$$

$$\eta = \frac{T_{04} - T_5}{T_{04} - T'_5}$$

$$\eta = \frac{T_{04} - T_c}{T_{04} - T'_c}$$

$$\eta = \frac{P_{01} - P_a}{P_{0a} - P_a}$$

$$\eta = -\frac{w_C}{w_T} = -\frac{\dot{w}_C}{\dot{w}_T}$$

$$\eta = \frac{w_{net}}{q_m}$$

$$w_T = -\frac{w_C}{\eta} + \frac{w_{net}}{\eta}$$

$$\alpha = 1$$

$$\lambda = 1/\phi$$

$$SFC = \dot{m}_f / \dot{W}$$

$$\eta = \frac{m_a}{\rho_a V_d}$$

$$W_{61} = P_{in}(V_1 - V_6) = P_{in} V_d$$

$$(W_{pump})_{net} = (P_{in} - P_{ex}) V_d$$

$$C_{Dv} = A_{act} / A_{pass}$$

$$A_i = \frac{\pi}{4} d_v^2 = CB^2 \frac{\bar{y}_{max}}{c_i}$$

$$P_1 v_1^\gamma = P_2 v_2^\gamma$$

$$\dot{Q} = \dot{m} C_p \Delta T$$

$$\dot{Q} = \dot{m} (C_p + C_v) \Delta T$$

$$m_f q_m = (m_a + m_f) C_v \Delta T$$

$$c = \sqrt{\gamma R T}$$

$$M_a = c_a / c$$

$$P_{03} = P_{02} \left(1 - \frac{\Delta P_h}{P_{02}} - \frac{\Delta P_{ha}}{P_{02}} \right)$$

$$P_a = P_{04} \left(1 - \frac{\Delta P_{hg}}{P_{04}} \right)$$

$$w = C_p (T_{01} - T_{02})$$

$$w = C_p (T_{03} - T_{04})$$

$$C_p - C_v = R$$

$$\frac{R}{C_p} = \frac{\gamma - 1}{\gamma}$$

$$\frac{n-1}{n} = \frac{\gamma-1}{\eta_p \gamma}$$

$$\frac{n-1}{n} = \eta_p \frac{\gamma-1}{\gamma}$$

$$\phi = (AF)_{stoich} / (AF)_{act}$$

$$SFC = 3600 \dot{m}_f / \dot{W}$$

$$\eta = ISFC / BSFC$$

$$\eta = \frac{60 \dot{m}_a}{\alpha \rho_a V_d N} = \frac{60 n \dot{m}_a}{\rho_a V_d N}$$

$$W_{56} = P_{ex} (V_6 - V_5) = -P_{ex} V_d$$

$$l_{max} < d_v / 4$$

$$A_{pass} = \pi d_v l$$

$$C \approx 1,3$$

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$$

$$\dot{Q} = \dot{m} C_v \Delta T$$

$$m_f q_m = (m_a + m_f) C_p \Delta T$$

Formulae (continued)

$$dq = du + dw$$

$$dq = dh - v dP$$

$$dw = P dv$$

$$q_{in} + q_{out} = w_{net}$$

$$T(h) = T_{SL} - \lambda h$$

$$P(h) = P_{SL} \left(1 - \frac{\lambda}{T_{SL}} h \right)^{g/(\lambda R)}$$

$$\rho(h) = \rho_{SL} \left(1 - \frac{\lambda}{T_{SL}} h \right)^{\left(\frac{g}{\lambda R} - 1 \right)}$$

$$P = \rho R T$$

$$C_p T_0 = C_p T + c^2 / 2$$

$$\frac{T_0}{T} = \left(\frac{P_0}{P} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}} \right)^{(n-1)/n}$$

$$\frac{T_{03}}{T_{04}} = \left(\frac{P_{03}}{P_{04}} \right)^{(n-1)/n}$$

$$\frac{T'_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_{04}}{T_{03}} = \left(\frac{P_{04}}{P_{03}} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_c}{T_{04}} = \left(\frac{P_c}{P_{04}} \right)^{(\gamma-1)/\gamma}$$

$$T_c = \frac{2}{\gamma + 1} T_0$$

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$B = \frac{\dot{m}_c}{\dot{m}_h}$$

$$F_D = \dot{m} c_a = (\dot{m}_c + \dot{m}_h) c_a$$

$$w_{TLP} = -\frac{w_F}{\eta}$$

$$T_{SL} = 288,15 \text{ K}$$

$$t = T_3 / T_1$$

$$F = \dot{m}(c_j - c_a) + A_j (P_j - P_a)$$

$$F = \dot{m}(c_j - c_a)$$

$$SFC = \frac{3600(FA)}{F_s}$$

$$C_p = 1005 \text{ J/kgK}$$

$$C_p = 1148 \text{ J/kgK}$$

$$R = 287,1 \text{ J/kgK}$$

$$\gamma = 1,4$$

$$\gamma = 1,333$$

$$\lambda = -0,986 \text{ }^\circ\text{C/km}$$

$$\lambda = 0 \text{ }^\circ\text{C/km}$$

$$\lambda = 6,5 \text{ }^\circ\text{C/km}$$

$$g = 9,81 \text{ m/s}^2$$

$$\dot{m}_{fuel} = (FA) \dot{m}_a$$

$$\dot{m} = \frac{\dot{W}}{w}$$

$$\dot{m} = \rho c A$$

$$\dot{m} = F / F_s$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$F_N = F_G - F_D$$

$$T_{05} - T_{06} = (1 + B) \frac{C_{pa}}{\eta_m C_{pg}} (T_{02} - T_{01})$$

$$P_{SL} = 101,325 \text{ kPa}$$

Formulae (continued)

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$\dot{m}_h = \dot{m}/(1+B)$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$\dot{W}_{TLP} = \dot{m}_h C_{pg}(T_{05} - T_{06})$$

$$TP = \eta_{pr} SP + F c_a$$

$$EP = \frac{TP}{\eta_{pr}} = SP + \frac{F c_a}{\eta_{pr}}$$

$$EP_{take\ off} = \frac{TP}{\eta_{pr}} = SP + \frac{F}{8.5}$$

$$\eta_{T\ Diesel} > \eta_{T\ Dual} > \eta_{T\ Otto}$$

$$y = (r + a) - (a \cos \theta + r \cos \phi)$$

$$\dot{y} = a \dot{\theta} \sin \theta \left(1 + \frac{\cos \theta}{\sqrt{R^2 - \sin^2 \theta}} \right)$$

$$\bar{\dot{y}} = 2S \frac{N}{60}$$

$$V_d = V_{BDC} - V_{TDC}$$

$$V_d = N_c \pi B^2 S / 4$$

$$W = \int F dy$$

$$w = \int P dv$$

$$A_p = \pi B^2 / 4$$

$$w_b = w_i - w_f$$

$$MEP = W / V_d$$

$$BMEP = (IMEP)_{net} - FMEP$$

$$FMEP = (1 - \eta_m)(IMEP)_{net}$$

$$\dot{W} = 2\pi NT / 60$$

$$BMEP = 2\pi T_b / \alpha V_d$$

$$B = \dot{m}_c / \dot{m}_h$$

$$\dot{m}_c = \dot{m} B / (1 + B)$$

$$F_D = \dot{m} c_a = (\dot{m}_c + \dot{m}_h) c_a$$

$$\dot{W}_{TLP} = -\dot{W}_F / \eta_m$$

$$c = v_3 / v_x$$

$$c = v_3 / v_2$$

$$\alpha = P_x / P_2$$

$$s = a \cos \theta + \sqrt{r^2 - a^2 \sin^2 \theta}$$

$$R = r / a$$

$$\dot{y} \approx a \Omega \left(\sin \theta + \frac{\sin 2\theta}{2R} \right)$$

$$\frac{\dot{y}}{\dot{y}} \approx \frac{\pi}{2} \left(\sin \theta + \frac{\sin 2\theta}{2R} \right)$$

$$V_d = \pi B^2 S / 4$$

$$r_v = (V_c + V_d) / V_c$$

$$W = \int P dV$$

$$F = P A_p$$

$$w_{net} = w_{gross} + w_{pump}$$

$$\eta_m = W_b / W_i$$

$$(IMEP)_{net} = (IMEP)_{gross} + PMEP$$

$$BMEP = \eta_m (IMEP)_{net}$$

$$BMEP = 2\pi T_b / \alpha V_d$$

$$\dot{W}_{b\ engine} = (BMEP) V_d N \alpha / 60$$

$$\alpha = 1/2$$

Formulae (continued)

$$m_f q_{in} = (m_a + m_f) C_v \Delta T$$

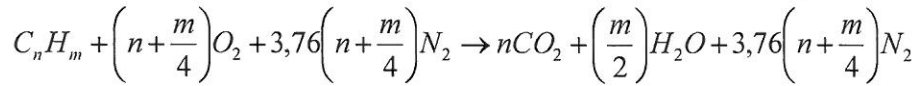
$$\eta = 1 - \frac{1}{r_T}$$

$$\eta = 1 - \frac{r_T}{t}$$

$$w_{net} / C_p T_1 = 2t - r_T + 1 - 2t / \sqrt{r_T}$$

$$\eta_T = \frac{w_{TP}}{Q_{HV} (FA)_{actual}}$$

$$W_{in} = \frac{P_2 V_2 - P_1 V_1}{1 - \gamma}$$



$$(AF)_{stoich} = 4,319 \frac{32n + 8m}{12n + 1,008m}$$

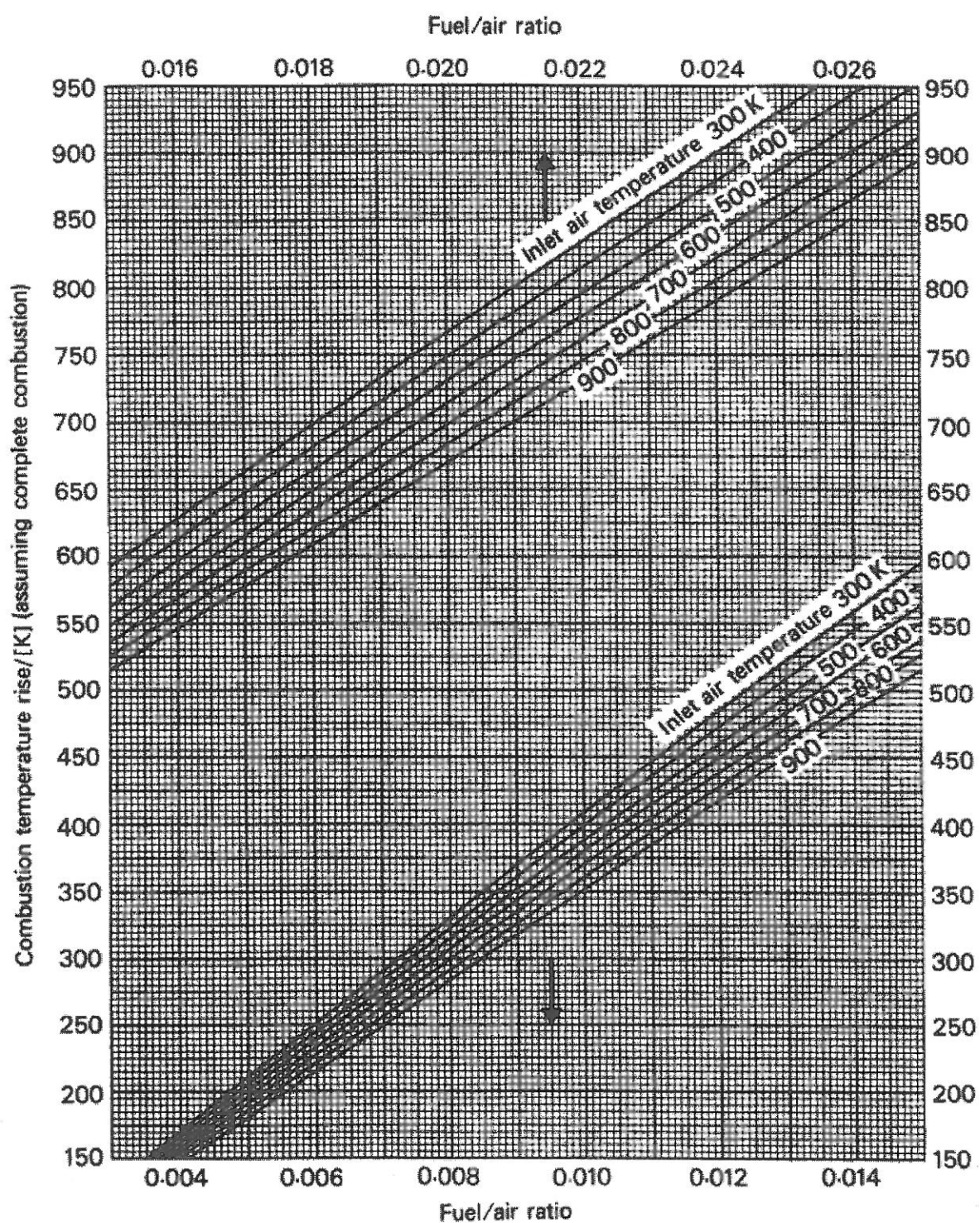
$$\eta = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$$

$$\eta = \frac{2t - r_T + 1 - 2t / \sqrt{r_T}}{2t - r_T - t / \sqrt{r_T}}$$

$$SFC = \frac{3600 (FA)_{actual}}{w_{TP}}$$

$$\eta = 1 - \frac{1}{r_v^{\gamma-1}}$$

$$W_{out} = \frac{P_4 V_4 - P_3 V_3}{1 - \gamma}$$





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Question 1**(17 marks)**

Using P - v - and T - s -diagrammes, compare the thermal efficiencies of the ideal air-standard Otto-, Diesel- and Dual cycles for two cases, i.e:

- (i) The same input conditions and compression ratio
 - (ii) The same input conditions, maximum pressure and maximum temperature
-

Question 2**(34 marks)**

The following data apply to the auxiliary power unit (APU) on the Boeing 777 (single-shaft unit, driving separate load compressor and generator – see Figure 1):

Load compressor:

Mass flow:	3,5 kg/s
Pressure ratio:	3,65:1
Isentropic efficiency:	0,88

Gas turbine:

Pressure ratio:	12,0:1
Compressor isentropic efficiency:	0,84
Turbine inlet temperature:	1 390 K
Turbine isentropic efficiency:	0,87
Relative combustion pressure loss $\Delta P/P_{02GT}$:	5%
Turbine exhaust pressure:	104 000 Pa

When operating at 35°C and 101 000 Pa at standstill, the unit must supply 200 kW electrical load. Calculate the airflow required for the gas turbine and the specific fuel consumption.

Working substance properties are:

$$C_{pa} = 1005 \text{ J/kgK}; \quad \gamma_a = 1,4; \quad C_{pg} = 1148 \text{ J/kgK}; \quad \gamma_g = 4/3$$

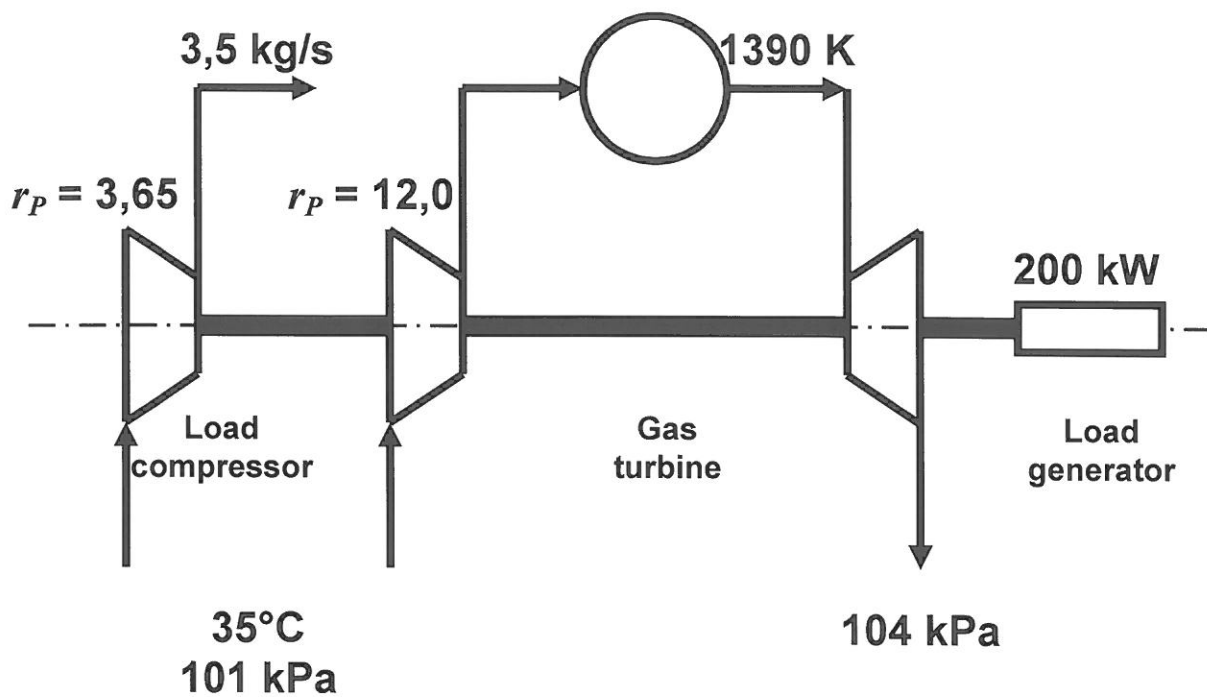


Figure 1: Auxiliary power unit

Question 3**(17 MARKS)**

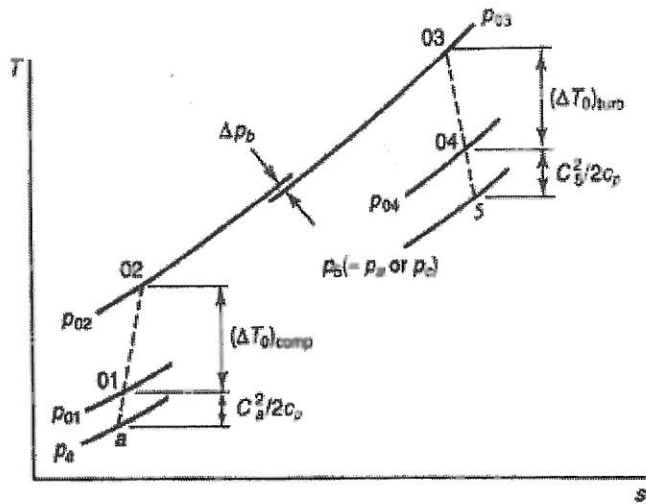
An aircraft powered by single-spool turbojets flies at a speed of 900 km/h at an altitude of 5 000 m ASL. The engines operate at a compressor pressure ratio of 8,0:1. The compressor- and turbine polytropic efficiencies are 92,5% and 85,5% respectively, and the combustion chamber pressure loss is 4% of the compressor delivery pressure. The inlet total pressure recovery is 97%. The air- and fuel mass flow rates per engine are 100 kg/s and 1,8 kg/s respectively. The engine mechanical efficiency and fuel heating value are 99% and 43 100 kJ/kgf respectively. Combustion efficiency is 98%. Assume the engines have convergent, isentropic nozzle flow. Calculate the turbine inlet temperature.

Sea-level static conditions are 288,15 K and 101 325 Pa. Working substance properties are:

$$C_{p_a} = 1005 \text{ J/kgK}; \quad \gamma_a = 1,4; \quad C_{p_g} = 1148 \text{ J/kgK}; \quad \gamma_g = 4/3$$

A T - s -diagramme of a turbojet cycle is shown in Figure 2.

SIMPLE TURBOJET CYCLE

Figure 2: Turbojet T - s -diagramme

Question 4

(14 MARKS)

From the P - V -diagramme of an engine working on the ideal air-standard Otto cycle, it is found that the pressure in the cylinder after the piston has been displaced by $1/8^{\text{th}}$ of the stroke, i.e. from BDC on the compression stroke, is, 140 kPa. After a piston displacement of $5/8^{\text{th}}$ of the compression stroke, the pressure is 350 kPa. Calculate the compression ratio, pressure P_1 at BDC and the ideal air standard thermal efficiency of the cycle.

The γ -value of the working substance is 1,4.

Sections through a cylinder, for piston displacements of $1/8^{\text{th}}$ and $5/8^{\text{th}}$ of the stroke S , are shown in Figure 3.

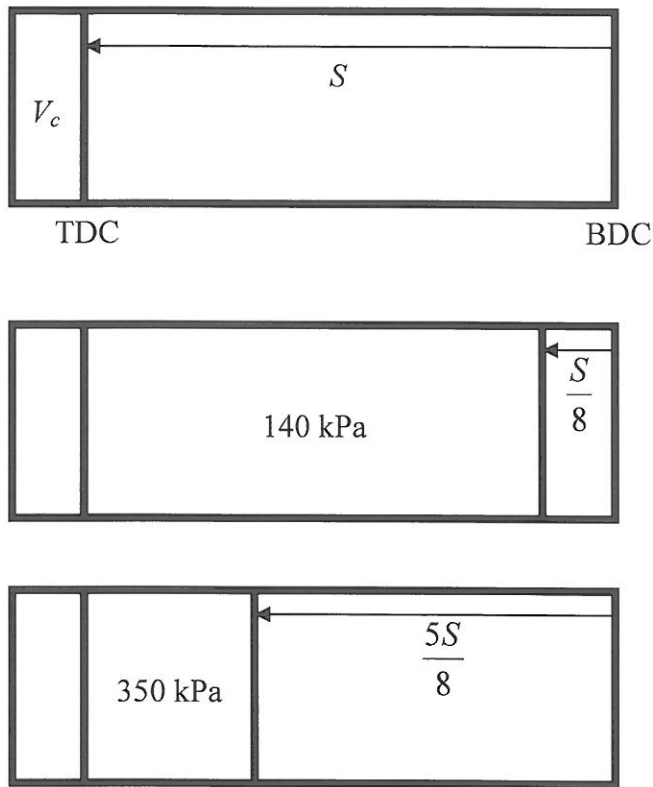


Figure 3: Sections through cylinder for piston displacements of $1/8^{\text{th}}$ and $5/8^{\text{th}}$ of stroke S

Question 5

(38 MARKS)

A 4-stroke, 4cylinder RICE with indirect mechanical petrol injection and a displacement volume of 1,6 l runs at WOT, and a speed of 6500 RPM, where its volumetric efficiency and air-fuel ratio are 90% and 12:1 respectively.

The fuel tank is situated 3 m from the plenum chamber containing the injectors. The fuel is supplied to the plenum chamber by means of a pump at the tank outlet, via a straight, smooth steel pipe with an inner diameter of 4 mm. Fuel flow through the pipe is constant at the above operating conditions.

Assume the surface of the fuel in the tank is approximately level with the injectors, and the tank is relatively shallow so that its depth may be neglected. Also assume the plenum and fuel surface in the tank are at atmospheric pressure. The pressure drop across the injectors is 300 kPa. The pump behaves like a positive displacement pump, i.e. its pressure is almost constant with flow.

Ambient conditions of 101 325 Pa and 15°C apply during the test. The density and viscosity of petrol are 700 kg/m³ and 542 μPa.s respectively.

Calculate the required pump power.

Annexure: Formula sheets

$$\eta = \frac{T'_{01} - T_a}{T_{01} - T_a}$$

$$\eta = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$\eta = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}}$$

$$\eta = \frac{(FA)_{theor}}{(FA)_{actual}}$$

$$\eta = \frac{T_{04} - T_5}{T_{04} - T'_5}$$

$$\eta = \frac{T_{04} - T_c}{T_{04} - T'_c}$$

$$\eta = \frac{P_{01} - P_a}{P_{0a} - P_a}$$

$$\eta = -\frac{w_C}{w_T} = -\frac{\dot{w}_C}{\dot{w}_T}$$

$$\eta = \frac{w_{net}}{q_{in}}$$

$$w_T = -\frac{w_C}{\eta} + \frac{w_{net}}{\eta}$$

$$\alpha = 1$$

$$\lambda = 1/\phi$$

$$SFC = \dot{m}_f / \dot{W}$$

$$\eta = \frac{m_a}{\rho_a V_d}$$

$$W_{61} = P_{in}(V_1 - V_6) = P_{in}V_d$$

$$(W_{pump})_{net} = (P_{in} - P_{ex})V_d$$

$$C_{Dv} = A_{act} / A_{pass}$$

$$A_i = \frac{\pi}{4} d_v^2 = CB^2 \frac{\bar{y}_{max}}{c_i}$$

$$P_1 v_1^\gamma = P_2 v_2^\gamma$$

$$\dot{Q} = \dot{m} C_p \Delta T$$

$$\dot{Q} = \dot{m} (C_p + C_v) \Delta T$$

$$m_f q_{in} = (m_a + m_f) C_v \Delta T$$

$$c = \sqrt{\gamma RT}$$

$$M_a = c_a / c$$

$$P_{03} = P_{02} \left(1 - \frac{\Delta P_b}{P_{02}} - \frac{\Delta P_{ha}}{P_{02}} \right)$$

$$P_a = P_{04} \left(1 - \frac{\Delta P_{hg}}{P_{04}} \right)$$

$$w = C_p (T_{01} - T_{02})$$

$$w = C_p (T_{03} - T_{04})$$

$$C_p - C_v = R$$

$$\frac{R}{C_p} = \frac{\gamma - 1}{\gamma}$$

$$\frac{n-1}{n} = \frac{\gamma-1}{\eta_p \gamma}$$

$$\frac{n-1}{n} = \eta_p \frac{\gamma-1}{\gamma}$$

$$\phi = (AF)_{stoich} / (AF)_{act}$$

$$SFC = 3600 \dot{m}_f / \dot{W}$$

$$\eta = ISFC / BSFC$$

$$\eta = \frac{60 \dot{m}_a}{\alpha \rho_a V_d N} = \frac{60 n \dot{m}_a}{\rho_a V_d N}$$

$$W_{56} = P_{ex} (V_6 - V_5) = -P_{ex} V_d$$

$$l_{max} < d_v / 4$$

$$A_{pass} = \pi d_v l$$

$$C \approx 1,3$$

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$$

$$\dot{Q} = \dot{m} C_v \Delta T$$

$$m_f q_{in} = (m_a + m_f) C_p \Delta T$$

Formulae (continued)

$$dq = du + dw$$

$$dq = dh - v dP$$

$$dw = P dv$$

$$q_{in} + q_{out} = w_{net}$$

$$T(h) = T_{SL} - \lambda h$$

$$P(h) = P_{SL} \left(1 - \frac{\lambda}{T_{SL}} h \right)^{g/(\lambda R)}$$

$$\rho(h) = \rho_{SL} \left(1 - \frac{\lambda}{T_{SL}} h \right)^{\left(\frac{g}{\lambda R} - 1 \right)}$$

$$P = \rho R T$$

$$C_p T_0 = C_p T + c^2/2$$

$$\frac{T_0}{T} = \left(\frac{P_0}{P} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}} \right)^{(n-1)/n}$$

$$\frac{T_{03}}{T_{04}} = \left(\frac{P_{03}}{P_{04}} \right)^{(n-1)/n}$$

$$\frac{T'_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_{04}}{T_{03}} = \left(\frac{P_{04}}{P_{03}} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_c}{T_{04}} = \left(\frac{P_c}{P_{04}} \right)^{(\gamma-1)/\gamma}$$

$$T_c = \frac{2}{\gamma + 1} T_0$$

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$B = \frac{\dot{m}_c}{\dot{m}_h}$$

$$F_D = \dot{m} c_a = (\dot{m}_c + \dot{m}_h) c_a$$

$$w_{TLP} = -\frac{w_F}{\eta}$$

$$T_{SL} = 288,15 \text{ K}$$

$$t = T_3/T_1$$

$$F = \dot{m}(c_j - c_a) + A_j(P_j - P_a)$$

$$F = \dot{m}(c_j - c_a)$$

$$SFC = \frac{3600(FA)}{F_s}$$

$$C_p = 1005 \text{ J/kgK}$$

$$C_p = 1148 \text{ J/kgK}$$

$$R = 287,1 \text{ J/kgK}$$

$$\gamma = 1,4$$

$$\gamma = 1,333$$

$$\lambda = -0,986 \text{ °C/km}$$

$$\lambda = 0 \text{ °C/km}$$

$$\lambda = 6,5 \text{ °C/km}$$

$$g = 9,81 \text{ m/s}^2$$

$$\dot{m}_{fuel} = (FA)\dot{m}_a$$

$$\dot{m} = \frac{\dot{W}}{w}$$

$$\dot{m} = \rho c A$$

$$\dot{m} = F/F_s$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$F_N = F_G - F_D$$

$$T_{05} - T_{06} = (1 + B) \frac{C_{pa}}{\eta_m C_{pg}} (T_{02} - T_{01})$$

$$P_{SL} = 101,325 \text{ kPa}$$

Formulae (continued)

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$\dot{m}_h = \dot{m}/(1+B)$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$\dot{W}_{TLP} = \dot{m}_h C_{pg}(T_{05} - T_{06})$$

$$TP = \eta_{pr} SP + F c_a$$

$$EP = \frac{TP}{\eta_{pr}} = SP + \frac{F c_a}{\eta_{pr}}$$

$$EP_{take\ off} = \frac{TP}{\eta_{pr}} = SP + \frac{F}{8,5}$$

$$\eta_{T\ Diesel} > \eta_{T\ Dual} > \eta_{T\ Otto}$$

$$y = (r+a) - (a \cos \theta + r \cos \phi)$$

$$\dot{y} = a \dot{\theta} \sin \theta \left(1 + \frac{\cos \theta}{\sqrt{R^2 - \sin^2 \theta}} \right)$$

$$\bar{\dot{y}} = 2S \frac{N}{60}$$

$$V_d = V_{BDC} - V_{TDC}$$

$$V_d = N_c \pi B^2 S / 4$$

$$W = \int F \, dy$$

$$w = \int P \, dv$$

$$A_p = \pi B^2 / 4$$

$$w_b = w_i - w_f$$

$$MEP = W / V_d$$

$$BMEP = (IMEP)_{net} - FMEP$$

$$FMEP = (1 - \eta_m)(IMEP)_{net}$$

$$\dot{W} = 2\pi NT / 60$$

$$BMEP = 2\pi T_b / \alpha V_d$$

$$B = \dot{m}_c / \dot{m}_h$$

$$\dot{m}_c = \dot{m} B / (1+B)$$

$$F_D = \dot{m} c_a = (\dot{m}_c + \dot{m}_h) c_a$$

$$\dot{W}_{TLP} = -\dot{W}_F / \eta_m$$

$$c = v_3 / v_x$$

$$c = v_3 / v_2$$

$$\alpha = P_x / P_2$$

$$s = a \cos \theta + \sqrt{r^2 - a^2 \sin^2 \theta}$$

$$R = r/a$$

$$\dot{y} \approx a\Omega \left(\sin \theta + \frac{\sin 2\theta}{2R} \right)$$

$$\frac{\dot{y}}{\dot{y}} \approx \frac{\pi}{2} \left(\sin \theta + \frac{\sin 2\theta}{2R} \right)$$

$$V_d = \pi B^2 S / 4$$

$$r_v = (V_c + V_d) / V_c$$

$$W = \int P \, dV$$

$$F = PA_p$$

$$W_{net} = W_{gross} + W_{pump}$$

$$\eta_m = W_b / W_i$$

$$(IMEP)_{net} = (IMEP)_{gross} + PMEP$$

$$BMEP = \eta_m (IMEP)_{net}$$

$$BMEP = 2\pi T_b / \alpha V_d$$

$$\dot{W}_{b\ engine} = (BMEP) V_d N \alpha / 60$$

$$\alpha = 1/2$$

Formulae (continued)

$$m_f q_{in} = (m_a + m_f) C_v \Delta T$$

$$\eta = 1 - \frac{1}{r_T}$$

$$\eta = 1 - \frac{r_T}{t}$$

$$w_{\text{net}} / C_p T_1 = 2t - r_T + 1 - 2t / \sqrt{r_T}$$

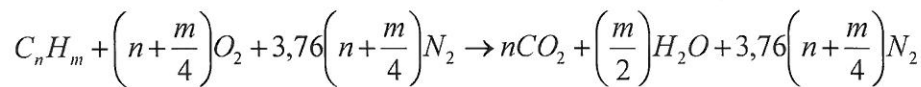
$$\eta_T = \frac{w_{TP}}{Q_{HV} (FA)_{\text{actual}}}$$

$$W_{\text{in}} = \frac{P_2 V_2 - P_1 V_1}{1 - \gamma}$$

$$\dot{m} = \rho Q$$

$$\text{Re} = \rho D V / \mu$$

$$P = \rho g h_L$$



$$(AF)_{\text{stoich}} = 4,319 \frac{32n + 8m}{12n + 1,008m}$$

$$\eta = 1 - \frac{1}{r_P^{(\gamma-1)/\gamma}}$$

$$\eta = \frac{2t - r_T + 1 - 2t / \sqrt{r_T}}{2t - r_T - t / \sqrt{r_T}}$$

$$SFC = \frac{3600 (FA)_{\text{actual}}}{w_{TP}}$$

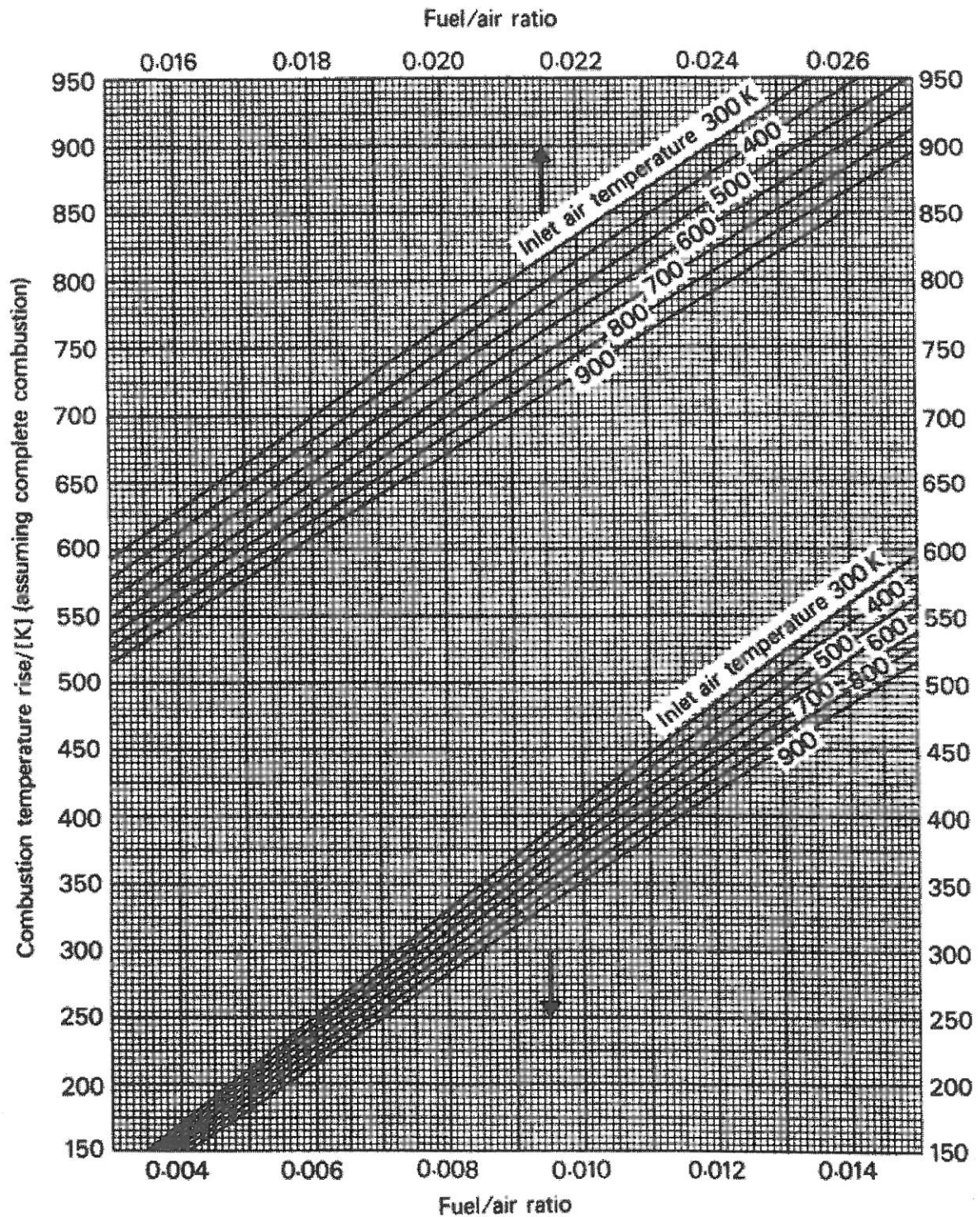
$$\eta = 1 - \frac{1}{r_V^{\gamma-1}}$$

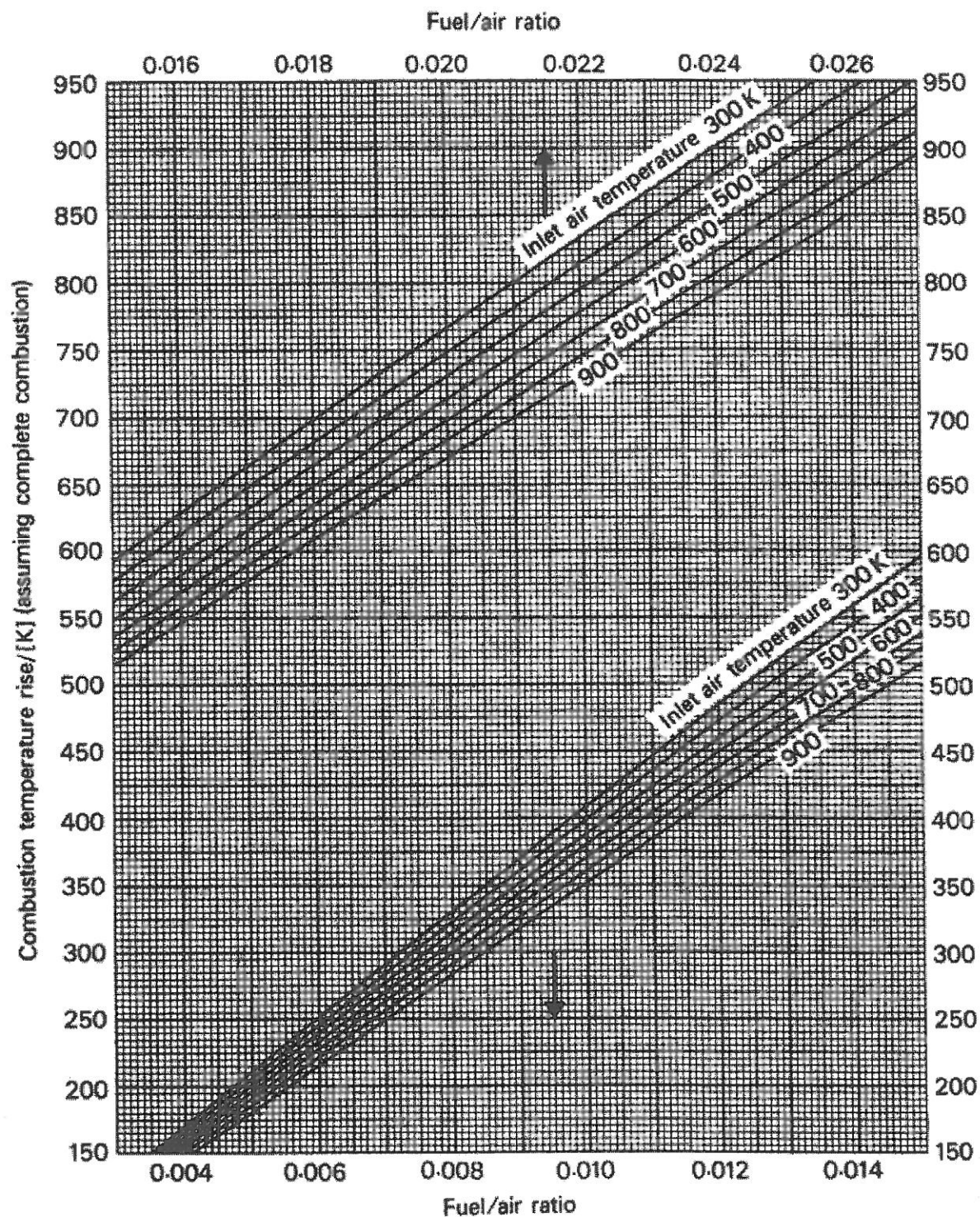
$$W_{\text{out}} = \frac{P_4 V_4 - P_3 V_3}{1 - \gamma}$$

$$Q = AV$$

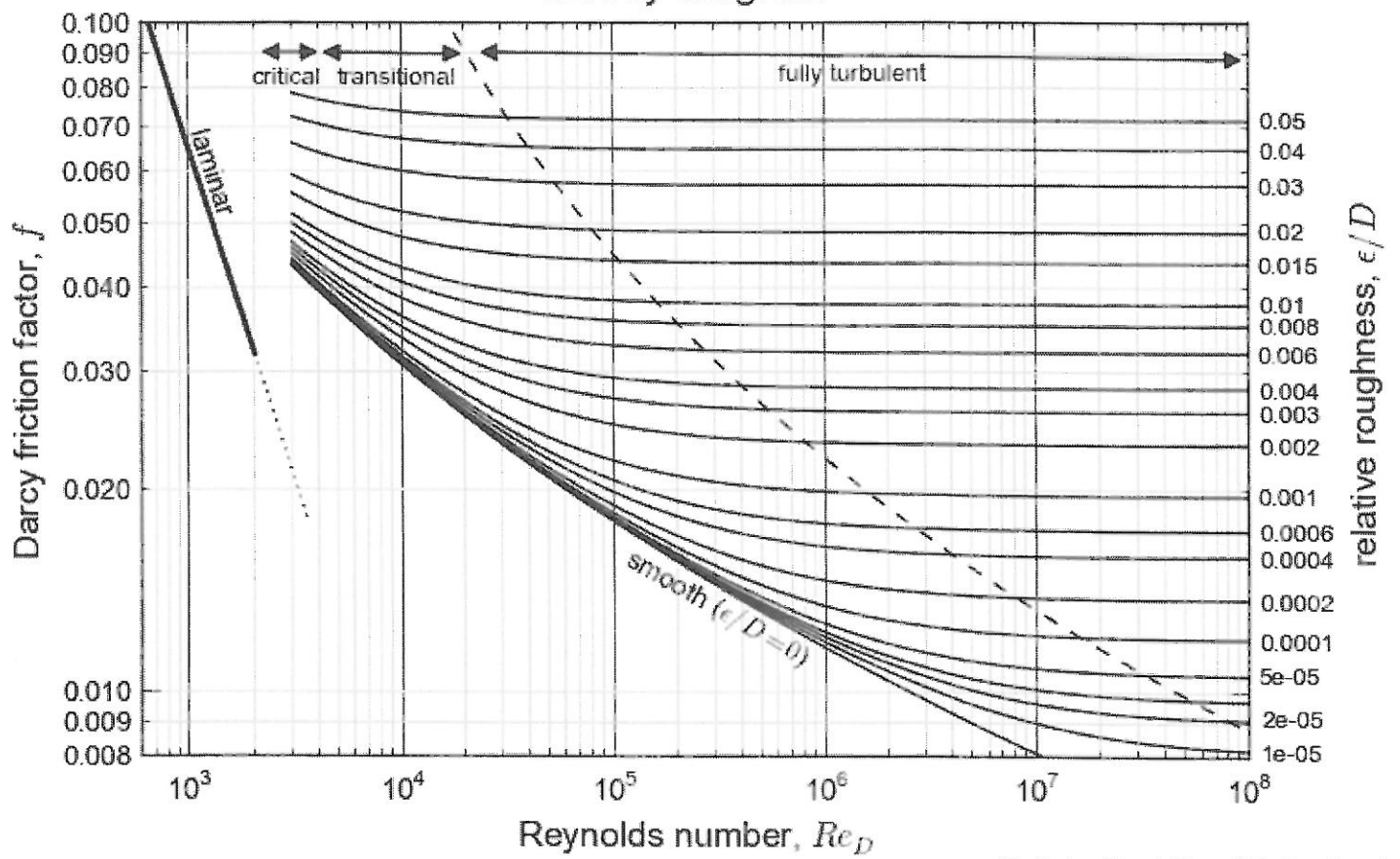
$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

$$\dot{W} = PQ$$





Moody Diagram



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PROGRAM : BACCALAUREUS INGENERIAE
MECHANICAL ENGINEERING

SUBJECT : THERMOMACHINES 4A

CODE : TRM4A11

DATE : SPECIAL EXAMINATION
22 AUGUST 2016

DURATION : (1-PAPER) 3½ HOURS (HANDICAPPED STUDENT)

WEIGHT : 50 : 50

TOTAL MARKS : 120 AVAILABLE, FULL MARKS 100

EXAMINER : Dr CR BESTER PrEng

MODERATOR : Mr JG BENADE PrEng

NUMBER OF PAGES : 2 PAGES INSTRUCTIONS
3 PAGES QUESTIONS AND SOLUTIONS
4 PAGES ANNEXURE (FORMULAE)
2 PAGES FUEL CHARTS

INSTRUCTIONS : SEE NEXT PAGE

REQUIREMENTS : NONE

INSTRUCTIONS TO CANDIDATES:

- FORMULA SHEETS AND FUEL CHARTS ATTACHED
 - NO BOOKS, LECTURE NOTES, STUDY-, HOMEWORK- OR TUTORIAL MATERIAL ALLOWED
 - UJ APPROVED CALCULATORS ALLOWED
 - NO ANSWERS IN PENCIL OR RED INK WILL BE ACCEPTED
 - ANSWER ALL THE QUESTIONS IN ENGLISH
 - SMOKING IS PROHIBITED DURING THE DURATION OF THE EXAM
-

Question 1**(19 marks)**

Consider steady, incompressible fluid flow through a pump. Derive the Euler pump equation from first principles, in terms of the control volume parameters shown in Figure 1. Obtain expressions for the ideal torque, power, head, specific work and pressure of an incompressible fluid pump.

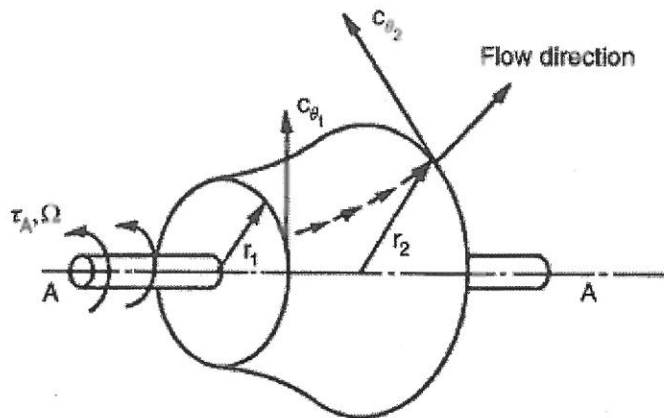


Figure 1: Control volume for Euler pump equation

Question 2**(23 marks)**

A single-shaft gas turbine for electric power generation (Figure 2) has the following data:

Compressor pressure ratio	12,0:1
Compressor polytropic efficiency	88%
Combustion chamber pressure loss	5%
Combustion efficiency	99%
Turbine entry temperature ("TET")	1400 K
Turbine polytropic efficiency	88%
Mechanical efficiency of compressor drive shaft	99%
Airflow	80,0 kg/s

Inlet stagnation conditions are 101 325 kPa and 288,15 K. Working fluid properties are:

$$C_{pa} = 1005 \text{ J/kgK}; \quad \gamma_a = 1,4;$$

$$C_{pg} = 1148 \text{ J/kgK}; \quad \gamma_g = 4/3$$

Ignore inlet- and exhaust pressure losses and **load** drive shaft losses and calculate:

- All the pressures and temperatures of the cycle
- Net power of the gas turbine
- Specific fuel consumption

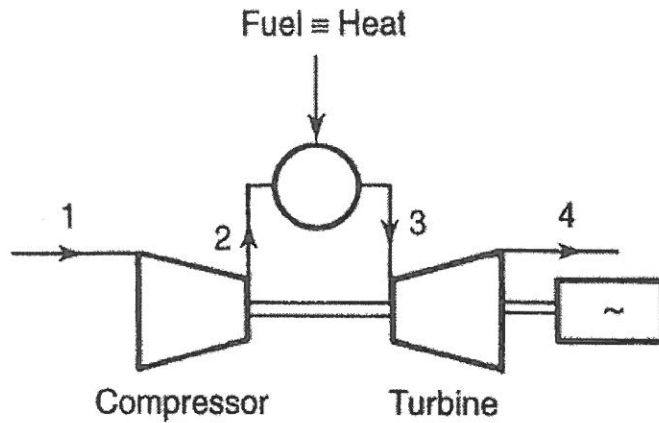


Figure 2: Single shaft gas turbine for electric power generation

Question 3

(28 marks)

The following data apply to a free turbine engine for aircraft propulsion (Figure 3):

Compressor pressure ratio	15,0:1
Compressor isentropic efficiency	81%
Combustion chamber pressure loss	5%
Combustion efficiency	99%
HP turbine entry temperature ("TET")	1600 K
HP turbine isentropic efficiency	87%
Free turbine isentropic efficiency	89%
Mechanical efficiency of both drive shafts	99%
Mechanical efficiency of gearbox	98,5%
Airflow	11,0 kg/s

Assume 3% bleed air for cooling of the HP turbine blades

Calculate:

- The output power of the free turbine for engine intake conditions of 101 300 Pa and 303 K
- The specific fuel consumption

Working fluid properties are:

$$C_{Pa} = 1005 \text{ J/kgK}; \quad \gamma_a = 1,4;$$

$$C_{Pg} = 1148 \text{ J/kgK}; \quad \gamma_g = 4/3$$

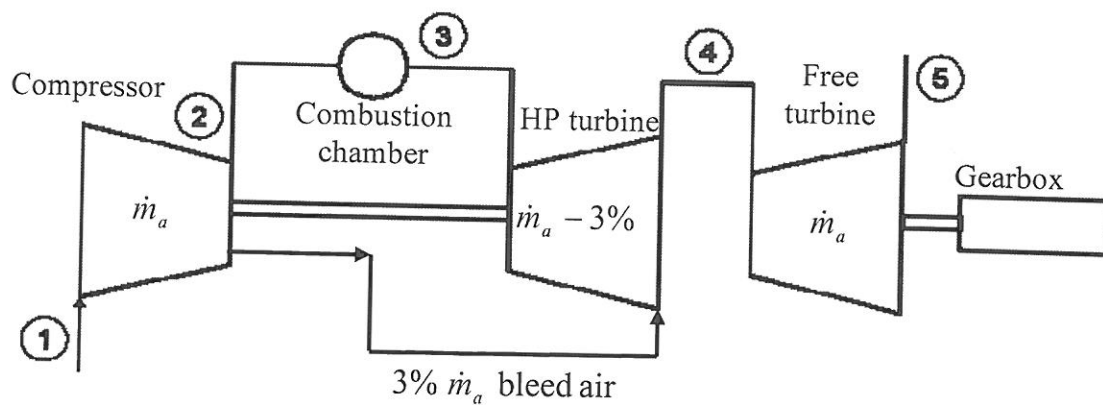


Figure 3: Aircraft gas turbine schematic with bleed air and free turbine

Question 4

(27 marks)

A 4-cylinder 4-stroke reciprocating internal combustion engine with a compression ratio of 11:1, bore and stroke of 84 mm and 88,5 mm respectively runs at 5500 RPM, where the volumetric efficiency is 88%. Assume all the combustion gas is flushed out of the engine during the exhaust stroke, i.e. from both the cylinder and combustion chamber. A direct fuel injection system feeds the fuel into the engine after the intake valves have closed. If the actual air-fuel ratio is 12:1, calculate the fuel mass flow into the engine. Intake conditions are 84 kPa and 50°C.

Question 5

(23 marks)

An engine operating on an air standard Diesel cycle has a compression ratio of 16. The temperature before compression is 27°C and the temperature after expansion is 627°C.

Calculate:

- (i) The temperature after compression (T_2)
- (ii) The temperature before expansion (T_3)
- (iii) Heat input into the cycle per unit mass of air (q_{in})
- (iv) Heat rejected to atmosphere per unit mass of air (q_{out})
- (v) The net work output per unit mass of air (w_{net})
- (vi) Cycle thermal efficiency (η_T)

Working fluid properties are: $C_p = 1005 \text{ J/kgK}$;

$\gamma = 1,4$

Annexure: Formula sheets

$$\eta = \frac{T'_{01} - T_a}{T_{01} - T_a}$$

$$\eta = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$\eta = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}}$$

$$\eta = \frac{(FA)_{theor}}{(FA)_{actual}}$$

$$\eta = \frac{T_{04} - T_5}{T_{04} - T'_5}$$

$$\eta = \frac{T_{04} - T_c}{T_{04} - T'_c}$$

$$\eta = \frac{P_{01} - P_a}{P_{0a} - P_a}$$

$$\eta = -\frac{w_c}{w_T} = -\frac{\dot{w}_c}{\dot{w}_T}$$

$$\eta = \frac{w_{net}}{q_{in}}$$

$$w_T = -\frac{w_c}{\eta} + \frac{w_{net}}{\eta}$$

$$\alpha = 1$$

$$\lambda = 1/\phi$$

$$SFC = \dot{m}_f / \dot{W}$$

$$\eta = \frac{m_a}{\rho_a V_d}$$

$$W_{61} = P_{in}(V_1 - V_6) = P_{in}V_d$$

$$(W_{pump})_{net} = (P_{in} - P_{ex})V_d$$

$$C_{Dv} = A_{act} / A_{pass}$$

$$A_i = \frac{\pi}{4} d_v^2 = CB^2 \frac{\dot{y}_{max}}{c_i}$$

$$P_1 v_1^\gamma = P_2 v_2^\gamma$$

$$\dot{Q} = \dot{m} C_p \Delta T$$

$$\dot{Q} = \dot{m}(C_p + C_v) \Delta T$$

$$m_f q_{in} = (m_a + m_f) C_v \Delta T$$

$$c = \sqrt{\gamma RT}$$

$$M_a = c_a / c$$

$$P_{03} = P_{02} \left(1 - \frac{\Delta P_b}{P_{02}} - \frac{\Delta P_{ha}}{P_{02}} \right)$$

$$P_a = P_{04} \left(1 - \frac{\Delta P_{hg}}{P_{04}} \right)$$

$$w = C_p (T_{01} - T_{02})$$

$$w = C_p (T_{03} - T_{04})$$

$$C_p - C_v = R$$

$$\frac{R}{C_p} = \frac{\gamma - 1}{\gamma}$$

$$\frac{n-1}{n} = \frac{\gamma-1}{\eta_p \gamma}$$

$$\frac{n-1}{n} = \eta_p \frac{\gamma-1}{\gamma}$$

$$\phi = (AF)_{stoich} / (AF)_{act}$$

$$SFC = 3600 \dot{m}_f / \dot{W}$$

$$\eta = ISFC / BSFC$$

$$\eta = \frac{60 \dot{m}_a}{\alpha \rho_a V_d N} = \frac{60 n \dot{m}_a}{\rho_a V_d N}$$

$$W_{56} = P_{ex}(V_6 - V_5) = -P_{ex} V_d$$

$$l_{max} < d_v / 4$$

$$A_{pass} = \pi d_v l$$

$$C \approx 1,3$$

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$$

$$\dot{Q} = \dot{m} C_v \Delta T$$

$$m_f q_{in} = (m_a + m_f) C_p \Delta T$$

Formulae (continued)

$$dq = du + dw$$

$$dq = dh - v dP$$

$$dw = P dv$$

$$q_{in} + q_{out} = w_{net}$$

$$T(h) = T_{SL} - \lambda h$$

$$P(h) = P_{SL} \left(1 - \frac{\lambda}{T_{SL}} h \right)^{g/(\lambda R)}$$

$$\rho(h) = \rho_{SL} \left(1 - \frac{\lambda}{T_{SL}} h \right)^{\left(\frac{g}{\lambda R} - 1 \right)}$$

$$P = \rho R T$$

$$C_p T_0 = C_p T + c^2/2$$

$$\frac{T_0}{T} = \left(\frac{P_0}{P} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}} \right)^{(n-1)/n}$$

$$\frac{T_{03}}{T_{04}} = \left(\frac{P_{03}}{P_{04}} \right)^{(n-1)/n}$$

$$\frac{T'_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_{04}}{T_{03}} = \left(\frac{P_{04}}{P_{03}} \right)^{(\gamma-1)/\gamma}$$

$$\frac{T'_c}{T_{04}} = \left(\frac{P_c}{P_{04}} \right)^{(\gamma-1)/\gamma}$$

$$T_c = \frac{2}{\gamma + 1} T_0$$

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$B = \frac{\dot{m}_c}{\dot{m}_h}$$

$$F_D = \dot{m} c_a = (\dot{m}_c + \dot{m}_h) c_a$$

$$w_{TLP} = - \frac{w_F}{\eta}$$

$$T_{SL} = 288,15 \text{ K}$$

$$t = T_3/T_1$$

$$F = \dot{m}(c_j - c_a) + A_j(P_j - P_a)$$

$$F = \dot{m}(c_j - c_a)$$

$$SFC = \frac{3600(FA)}{F_s}$$

$$C_p = 1005 \text{ J/kgK}$$

$$C_p = 1148 \text{ J/kgK}$$

$$R = 287,1 \text{ J/kgK}$$

$$\gamma = 1,4$$

$$\gamma = 1,333$$

$$\lambda = -0,986 \text{ }^\circ\text{C/km}$$

$$\lambda = 0 \text{ }^\circ\text{C/km}$$

$$\lambda = 6,5 \text{ }^\circ\text{C/km}$$

$$g = 9,81 \text{ m/s}^2$$

$$\dot{m}_{fuel} = (FA)\dot{m}_a$$

$$\dot{m} = \frac{\dot{W}}{w}$$

$$\dot{m} = \rho c A$$

$$\dot{m} = F/F_s$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$F_N = F_G - F_D$$

$$T_{05} - T_{06} = (1 + B) \frac{C_{pa}}{\eta_m C_{pg}} (T_{02} - T_{01})$$

$$P_{SL} = 101,325 \text{ kPa}$$

Formulae (continued)

$$\dot{m} = \dot{m}_h + \dot{m}_c$$

$$\dot{m}_h = \dot{m}/(1+B)$$

$$F_G = \dot{m}_c c_{jc} + \dot{m}_h c_{jh}$$

$$\dot{W}_{TLP} = \dot{m}_h C_{pg} (T_{05} - T_{06})$$

$$TP = \eta_{pr} SP + F c_a$$

$$EP = \frac{TP}{\eta_{pr}} = SP + \frac{F c_a}{\eta_{pr}}$$

$$EP_{take\ off} = \frac{TP}{\eta_{pr}} = SP + \frac{F}{8,5}$$

$$\eta_{T\ Diesel} > \eta_{T\ Dual} > \eta_{T\ Otto}$$

$$y = (r+a) - (a \cos \theta + r \cos \phi)$$

$$\dot{y} = a \dot{\theta} \sin \theta \left(1 + \frac{\cos \theta}{\sqrt{R^2 - \sin^2 \theta}} \right)$$

$$\bar{\dot{y}} = 2S \frac{N}{60}$$

$$V_d = V_{BDC} - V_{TDC}$$

$$V_d = N_c \pi B^2 S / 4$$

$$W = \int F \, dy$$

$$w = \int P \, dv$$

$$A_p = \pi B^2 / 4$$

$$w_b = w_i - w_f$$

$$MEP = W / V_d$$

$$BMEP = (IMEP)_{net} - FMEP$$

$$FMEP = (1 - \eta_m)(IMEP)_{net}$$

$$\dot{W} = 2\pi NT / 60$$

$$BMEP = 2\pi T_b / \alpha V_d$$

$$B = \dot{m}_c / \dot{m}_h$$

$$\dot{m}_c = \dot{m} B / (1+B)$$

$$F_D = \dot{m} c_a = (\dot{m}_c + \dot{m}_h) c_a$$

$$\dot{W}_{TLP} = -\dot{W}_F / \eta_m$$

$$c = v_3 / v_x$$

$$c = v_3 / v_2$$

$$\alpha = P_x / P_2$$

$$s = a \cos \theta + \sqrt{r^2 - a^2 \sin^2 \theta}$$

$$R = r/a$$

$$\dot{y} \approx a \Omega \left(\sin \theta + \frac{\sin 2\theta}{2R} \right)$$

$$\frac{\dot{y}}{\bar{\dot{y}}} \approx \frac{\pi}{2} \left(\sin \theta + \frac{\sin 2\theta}{2R} \right)$$

$$V_d = \pi B^2 S / 4$$

$$r_v = (V_c + V_d) / V_c$$

$$W = \int P \, dV$$

$$F = P A_p$$

$$w_{net} = w_{gross} + w_{pump}$$

$$\eta_m = W_b / W_i$$

$$(IMEP)_{net} = (IMEP)_{gross} + PMEP$$

$$BMEP = \eta_m (IMEP)_{net}$$

$$BMEP = 2\pi T_b / \alpha V_d$$

$$\dot{W}_{b\ engine} = (BMEP) V_d N \alpha / 60$$

$$\alpha = 1/2$$

Formulae (continued)

$$m_f q_{in} = (m_a + m_f) C_v \Delta T$$

$$\eta = 1 - \frac{1}{r_T}$$

$$\eta = 1 - \frac{r_T}{t}$$

$$w_{net}/C_p T_1 = 2t - r_T + 1 - 2t/\sqrt{r_T}$$

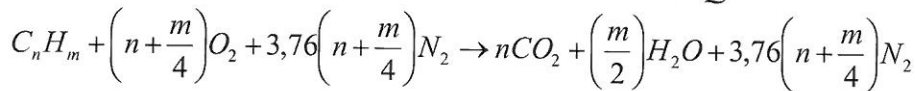
$$\eta_T = \frac{w_{TP}}{Q_{HV} (FA)_{actual}}$$

$$W_{in} = \frac{P_2 V_2 - P_1 V_1}{1 - \gamma}$$

$$\dot{m} = \rho Q$$

$$Re = \rho D V / \mu$$

$$P = \rho g h_L$$



$$(AF)_{stoich} = 4,319 \frac{32n + 8m}{12n + 1,008m}$$

$$\eta = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$$

$$\eta = \frac{2t - r_T + 1 - 2t/\sqrt{r_T}}{2t - r_T - t/\sqrt{r_T}}$$

$$SFC = \frac{3600 (FA)_{actual}}{w_{TP}}$$

$$\eta = 1 - \frac{1}{r_p^{\gamma-1}}$$

$$W_{out} = \frac{P_4 V_4 - P_3 V_3}{1 - \gamma}$$

$$Q = AV$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

$$\dot{W} = PQ$$

