UNIVERSITY OF JOHANNESBURG

PROGRAM	:	NATIONAL DIPLOMA **MECHANICAL ENGINEERING**
SUBJECT	:	**APPLIED STRENGTH OF MATERIALS 3**
CODE	:	**ASM 301**
DATE	:	SUPPLEMENTARY EXAMINATION 10 JANUARY, 2017
DURATION	:	(SESSION 1) 08:00 – 11:00 HRS
WEIGHT	:	40:60
TOTAL MARKS	:	106
FINAL MARKS	:	100
ASSESSOR	:	A. MASHAMBA
MODERATOR	:	K. SITHOLE
NUMBER OF PAGES	:	6 PAGES + 4 ANNEXURE

INSTRUCTIONS

1. ANSWER ALL QUESTIONS.
2. SHOW ALL CALCULATIONS.
3. ANSWERS WITHOUT UNITS WILL BE IGNORED.
4. All DIMENSIONS ARE IN mm UNLESS STATED OTHERWISE.
5. FOR MISSING DATA, ASSUME TYPICAL ENGINEERING VALUES.

QUESTION 1

A semi-elliptical steel leaf spring ($E = 206\ GPa$) has a length of 1.2 m and is to be made from blades that are 12 mm thick and 120 mm wide. The semi-elliptical leaf spring must carry a proof load of 12 kN without exceeding a maximum deflection of 24 mm and a maximum stress of 240 MPa. Calculate the number of blades that must be used for this semi-elliptical leaf spring.

[13]

QUESTION 2

A simply supported steel beam carries a uniformly distributed load of 12 kN/m as shown in Figure Qn 2. The steel beam has a length of 3 m and a flexural rigidity (EI) of 4 MNm^2.

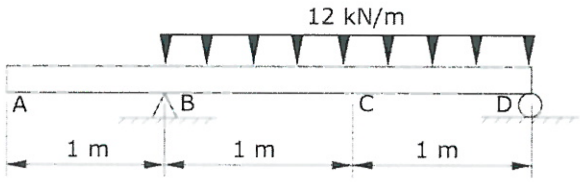


Figure Qn 2

Assuming that the steel beam has a negligible weight, calculate:

a) the reactions at Point B and D; (2)

b) the deflection at Point C and. (10)

[12]

QUESTION 3

A 40 mm × 40 mm rectangular steel rod ($E = 200\ GPa,\ S_y = 260\ MPa$) has a length of 3 m and is pin-jointed at both points A and B as shown in Figure Qn 3. A concentrated load P is suspended at point B. The Rankine constant for pinned joints is $\frac{1}{7000}$.

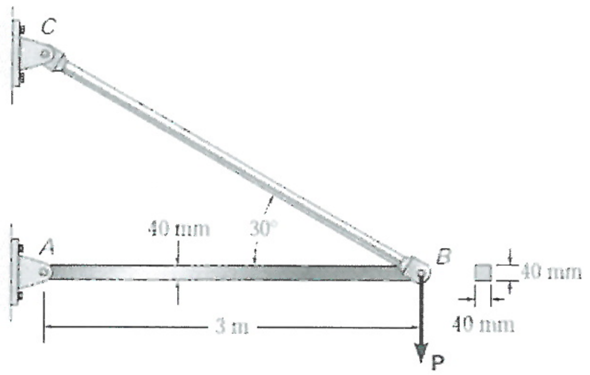


Figure Qn 3

a) Using the Euler buckling formula and a factor of safety of 1.5, find the maximum load P that can be suspended at Point B, without causing buckling.

(9)

b) Using the Rankine buckling formula and a factor of safety of 2, find the maximum load P that can be suspended at Point B, without causing buckling.

(6)

[15]

QUESTION 4

A 3 m long simply supported wooden beam has a 60 mm × 50 mm rectangular cross-section and carries a point load of 20 kN as shown in Figure Qn 4. The wooden beam has a knot (defect), which weakens its strength at Point A. Point A is located 1 m from one end of the beam and lies at 20 mm from the bottom as shown. Determine the principal stresses at Point A.

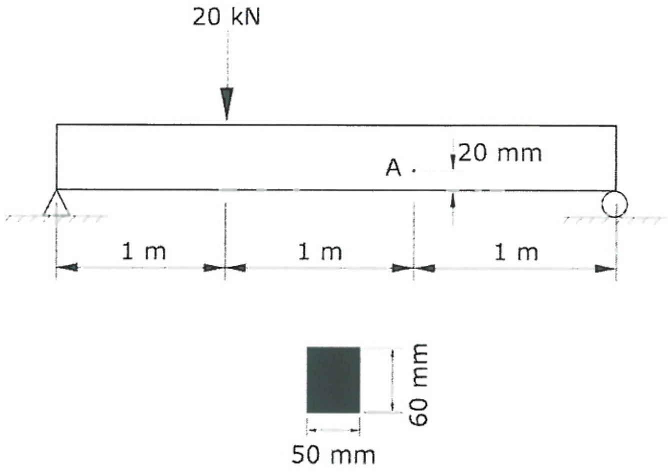


Figure Qn 4

[16]

QUESTION 5

A 12 mm strain gauge is rigidly attached on the surface of an unpressurised steel boiler ($E = 200\ GPa$; $\nu = 0.28$) as shown in Figure Qn 5. The unpressurised steel boiler has a mean internal diameter of 1.5 m and a wall thickness of 30 mm. An internal pressure then builds up in the steel boiler, which causes an elongation of 0.0006 mm to the strain gauge.

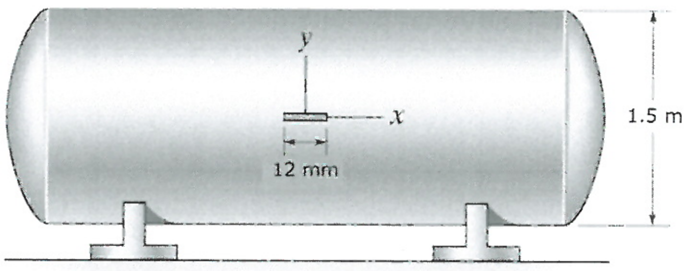


Figure Qn 5

a) Calculate:

   i) the internal pressure in the steel boiler that caused the 0.0006 mm elongation of the strain gauge; (4)

   ii) the volumetric strain of the steel boiler as a result of the pressure; (3)

   iii) the percentage change in volume of the steel boiler and; (2)

   ~~iv) the change in diameter (in mm) of the steel boiler.~~ ~~(3)~~

b) If the 12 mm strain gauge had been placed along the y- axis, what would have been the elongation of the strain gauge (in mm) after the same internal pressure build-up as in Qn 5 (a)? (2)

[14]

QUESTION 6

A bronze cylinder ($E_{bronze} = 100\ GPa, \upsilon = 0.35$) of 150 mm internal radius and 200 mm external radius is surrounded by a closely fitting steel sleeve ($E_{steel} = 200\ GPa, \upsilon = 0.28$) of 225 mm external radius to make a compound thick cylinder as shown in Figure Qn 6. When the compound thick cylinder is not subjected to any internal or external pressure, the contact pressure at the common surface is zero.

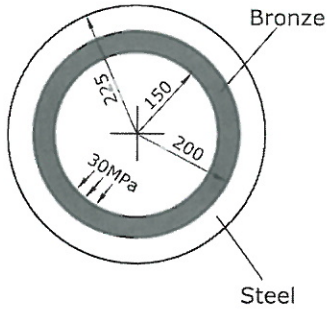


Figure Qn 6

If an internal pressure of 30 MPa is applied to the compound thick cylinder what is the contact pressure at the common surface?

Hint 1: The bronze cylinder experiences both the internal pressure of 30 MPa and the contact pressure at the common surface. The steel sleeve only experiences the contact pressure at the common surface.

Hint 2: The diametral strain of the bronze cylinder is equal to the diametral strain of the steel sleeve at the common surface.

[15]

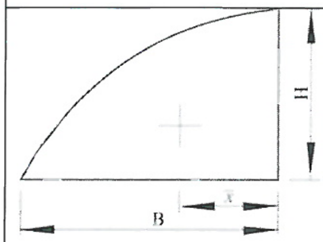
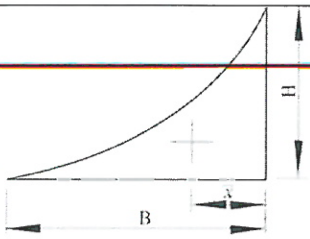
QUESTION 7

A solid steel drilling shaft ($E = 200\ GPa, S_y = 300\ MPa$) has a diameter of 80 mm and is rated to work at an axial compressive force of 90 kN, a torque of 4 kNm and a bending moment of 2 kNm. What are the factors of safety used in designing the drilling shaft for this working environment according to Tresca and Von Mises yield criteria?

[21]

TOTAL MARKS: 106
FINAL MARKS: 100

ANNEXTURE 1: FORMULA SHEET

1. Quarter elliptical leaf-spring	Maximum bending stress: $\sigma = \frac{6WL}{bnt^2}$ Maximum deflection: $\delta = \frac{6WL^3}{bEnt^3}$
2. Deflection of beams	Deflection of beam at position B relative to A: $y_{B/A} = \frac{\sum_{i=1}^{n} \int_A^B M(x)_i dx \cdot \bar{x}_i}{\sum_{i=1}^{n} EI_i}$ Slope of beam at position B relative to A: $\theta_{B/A} = \frac{\sum_{i=1}^{n} \int_A^B M(x)_i dx}{\sum_{i=1}^{n} EI_i}$ For $i = 1, 2, \ldots, n$ areas under the bending moment diagram. Where $\bar{x}_i$ is the centroidal distance of area i from position B.
	$\bar{x} = \frac{3}{8}B$ $Area = \frac{2}{3}BH$
	$\bar{x} = \frac{1}{4}B$ $Area = \frac{1}{3}BH$
3. Buckling of Struts	Euler Buckling: $P_E = \frac{\pi^2 EI}{L_e^2}$ Rankine Buckling: $P_R = \frac{S_y A}{\left[1+a\left(\frac{L_e}{K}\right)^2\right]}$ Validity Limit: $\left(\frac{L_e}{K}\right)_{lim} = \sqrt{\frac{2\pi^2 E}{S_y}}$
4. Transformation of Stress	Direct and shear plane stresses on an oblique plane θ degrees (anticlockwise) from the vertical axis: $\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)Cos2\theta + \tau_{xy}Sin2\theta$ $\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)Sin2\theta - \tau_{xy}Cos2\theta$ Maximum principal direct stresses: $\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ Maximum principal shear stress:

	$\tau_{max} = \pm\frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2} = \pm\frac{1}{2}(\sigma_1 - \sigma_2)$ Direction of maximum principals stresses: $tan\ 2\theta = \frac{2\tau_{xy}}{\left(\sigma_x - \sigma_y\right)}$
5. Analysis of Strain	Bi-axial strain: $\varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E} - \frac{\upsilon\sigma_y}{E};\ \sigma_x = \frac{E}{(1-\nu^2)}\left(\varepsilon_x + \nu\varepsilon_y\right)$ $\varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\upsilon\sigma_x}{E};\ \sigma_y = \frac{E}{(1-\nu^2)}\left(\varepsilon_y + \nu\varepsilon_x\right)$ $\varepsilon_A = \frac{\Delta A}{A} = \varepsilon_y + \varepsilon_x$ Tri-axial volumetric strain: $\varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E} - \frac{\upsilon\sigma_y}{E} - \frac{\upsilon\sigma_z}{E}$ $\varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\upsilon\sigma_x}{E} - \frac{\upsilon\sigma_z}{E}$ $\varepsilon_z = \frac{\Delta z}{z} = \frac{\sigma_z}{E} - \frac{\upsilon\sigma_x}{E} - \frac{\upsilon\sigma_y}{E}$ $\varepsilon_v = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\nu)$ Strain in circular shafts: $\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E}(\sigma_L - 2\upsilon\sigma_D)$ $\varepsilon_D = \frac{\Delta D}{D} = \frac{1}{E}(\sigma_D - \upsilon\sigma_D - \upsilon\sigma_L)$ $\varepsilon_v = \frac{\Delta V}{V} = \varepsilon_L + 2\varepsilon_D$ Strain in thin cylinders: $\varepsilon_L = \frac{\Delta L}{L} = \frac{\sigma_L}{E} - \frac{\upsilon\sigma_H}{E} = \frac{pd}{4tE}(1-2\upsilon)$ $\varepsilon_H = \frac{\Delta H}{H} = \frac{\sigma_H}{E} - \frac{\upsilon\sigma_L}{E} = \frac{pd}{4tE}(2-\upsilon)$ $\varepsilon_v = \varepsilon_L + 2\varepsilon_H = \frac{pd}{4tE}(5-4\upsilon)$ Strain in thin spheres: $\varepsilon_H = \frac{\Delta H}{H} = \frac{1}{E}(\sigma_H - \upsilon\sigma_H) = \frac{pd}{4tE}(1-\upsilon)$ $\varepsilon_v = 3\varepsilon_H = \frac{3pd}{4tE}(1-\upsilon)$

	Elastic constants: $E = 2G(1 + \upsilon);\ E = 3K(1 - 2\upsilon)$ Direct and shear plane strains on an oblique plane θ degrees (anticlockwise) from the vertical axis: $$\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)Cos2\theta + \frac{1}{2}\gamma_{xy}Sin2\theta$$ $$\gamma_\theta = -(\varepsilon_x - \varepsilon_y)Sin2\theta + \gamma_{xy}Cos2\theta$$ Maximum principal direct strains: $$\varepsilon_{1,2} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$ Direction of maximum principals strains: $$tan\ 2\theta = \frac{\gamma_{xy}}{(\varepsilon_x - \epsilon_y)}$$ Shear strain: $\gamma_{xy} = \frac{\tau_{xy}}{G}$
6. Thick Cylinders	Radial stress: $\sigma_r = A - \frac{B}{r^2}$ Hoop stress: $\sigma_c = A + \frac{B}{r^2}$ Stresses in thick cylinders due to an internal pressure P_i and external pressure, P_o: $$\sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right] - \frac{P_o r_o^2}{r_o^2 - r_i^2}\left[1 - \frac{r_i^2}{r^2}\right]$$ $$\sigma_c = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r^2}\right] - \frac{P_o r_o^2}{r_o^2 - r_i^2}\left[1 + \frac{r_i^2}{r^2}\right]$$ $$\sigma_a = \frac{P_i r_i^2 - P_o r_o^2}{(r_o^2 - r_i^2)}$$ Stresses in thick cylinders due to an internal pressure only (P_o= 0): $$\sigma_r = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}\left[1 - \frac{r_o^2}{r^2}\right]$$ $$\sigma_c = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}\left[1 + \frac{r_o^2}{r^2}\right]$$ $$\sigma_a = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}$$ Stresses in thick cylinders due to an external pressure only (P_i= 0): $$\sigma_r = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}\left[1 - \frac{r_i^2}{r^2}\right]$$

	$\sigma_c = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}\left[1 + \frac{r_i^2}{r^2}\right]$ $\sigma_a = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}$ Shrinkage allowance for compound thick cylinder: $s.a = 2r_{int}\left(\frac{1}{E_O}\left(\sigma_{c,O,int} + \upsilon_O P_{int}\right) - \frac{1}{E_I}\left(\sigma_{c,I,int} + \upsilon_I P_{int}\right)\right)$ Shrinkage allowance for shaft and hub: $s.a = 2r_{int}\left(\frac{1}{E_O}\left(\sigma_{c,O,int} - \upsilon_O P_{int}\right) - \frac{1}{E_I}\left(-P_{int} + \upsilon_I P_{int}\right)\right)$ Torque transmitted by a shrink fit: $T = 2\pi\mu r_{int}^2 L P_{int}$ Frictional force to separate a shrink fit: $F = 2\pi\mu r_{int} L P_{int}$
7. Failure theories	Ductile materials: Failure occurs when: Maximum shear stress (Tresca): $\sigma_1 - \sigma_3 \geq \frac{S_y}{n}$ Maximum shear strain energy (von Mises): $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\left(\frac{S_y}{n}\right)^2$ Brittle materials: Failure occurs when: Maximum principal stress (Rankine): $\sigma_1 \geq \frac{S_{ut}}{n} (if\ \sigma_1 > 0)\ or\ \sigma_3 \geq -\frac{S_{ut}}{n} (if\ \sigma_3 < 0)$ Modified Mohr: $Quadrant\ 1: \sigma_1 \geq \frac{S_{ut}}{n}$ $Quadrant\ 2: \frac{\sigma_3}{S_{ut}} - \frac{\sigma_1}{S_{uc}} \geq \frac{1}{n}$ $Quadrant\ 3: \sigma_3 \geq -\frac{S_{uc}}{n}$ $Quadrant\ 4: \frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} \geq \frac{1}{n}$
8. Temperature Stresses	$\Delta L = L\alpha\Delta T$