UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACCALAUREUS INGENERIAE *MECHANICAL ENGINEERING*
SUBJECT	:	**STRENGTH OF MATERIALS 4A**
CODE	:	**SLR4A11**
DATE	:	JUNE 2016
DURATION	:	3 HOURS
WEIGHT	:	50 : 50
TOTAL MARKS	:	100

EXAMINER	:	PROF RF LAUBSCHER (UJ)
MODERATOR	:	PROF C POLESE (WITS)
NUMBER OF PAGES	:	8 PAGES

INSTRUCTIONS : QUESTION PAPERS MUST BE HANDED IN.

INSTRUCTIONS TO CANDIDATES:

PLEASE ANSWER ALL THE QUESTIONS.

QUESTION 1

1.1 You need to prepare a report about a finite element analysis you conducted to assess the structural integrity of a certain part for a customer. Present an appropriate list of contents of such a report. [10]

1.2 Derive the general case of Hooke's law in tensor notation. [10]

1.3 Derive the shape functions Ni and Nj for a 1D simplex bar element with nodes i and j. [5]

(25)

QUESTION 2

A strain gauge rosette is positioned on the bracket as shown in Figure 1.

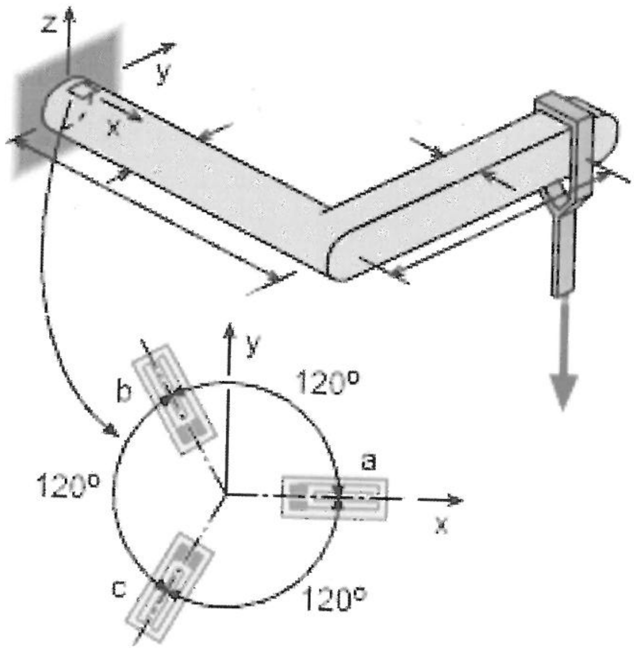

Figure 1 Rosette placement geometry

The following strain readings are obtained (E = 200 GPa, v = 0.3):

- $\varepsilon_a = 800\times10^{-6}$
- $\varepsilon_b = -200\times10^{-6}$
- $\varepsilon_c = 200\times10^{-6}$

2.1 Transform the strains obtained by the strain gauges to a general strain state relative to the coordinate system as shown in Figure 1. [5]

2.2 Calculate the principal strains. [5]

2.3 Calculate the general stress state (relative to axis X-Y). [5]

2.4 Calculate the principal stresses for the strain state measured by the rosette. [5]

2.5 For a SN Curve as shown (Figure 2) estimate the fatigue life (assume that the current stress distribution is fully reversed for each load cycle). [5]

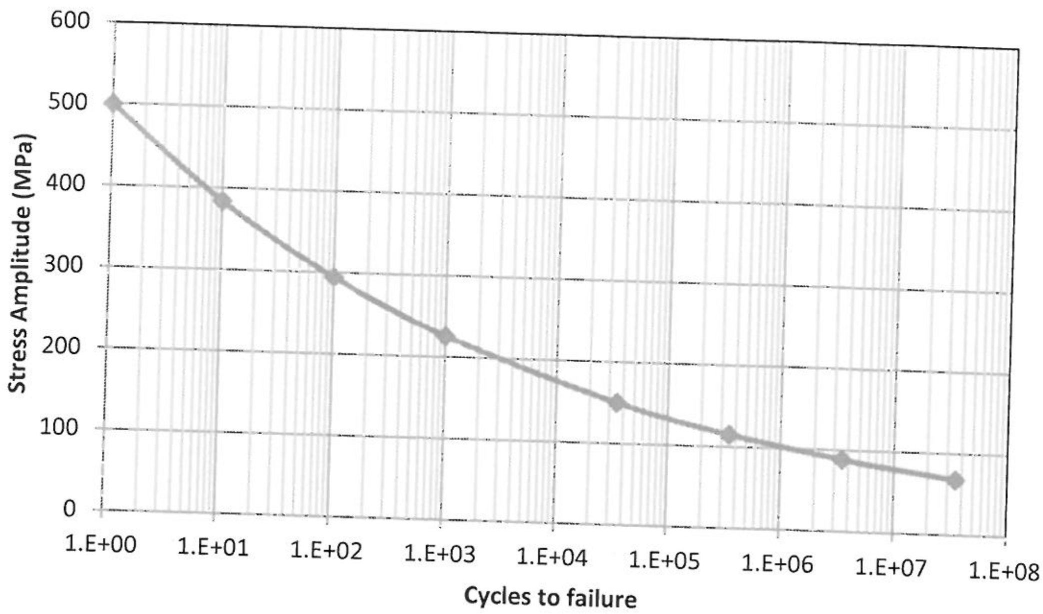

Figure 2 SN curve (Structural steel)

2.6 If the yield stress is σ_0 = 300 MPa. Check for yielding using both the Von Mises and Tresca Criteria.

[5]

(30)

QUESTION 3

A solid bar system (one dimensional) is shown in Figure 3. Use the finite element method to calculate the end displacement and the reaction force on the wall along with the stress states of the individual elements. L_1 = 30 mm, L_2 = 20 mm, F_1 = 1000 N, F_2 = 4000 N, A_1 = 100 mm^2, A_2 = 75 mm^2 and E_1 = 1 GPa, E_2 = 2 GPa.

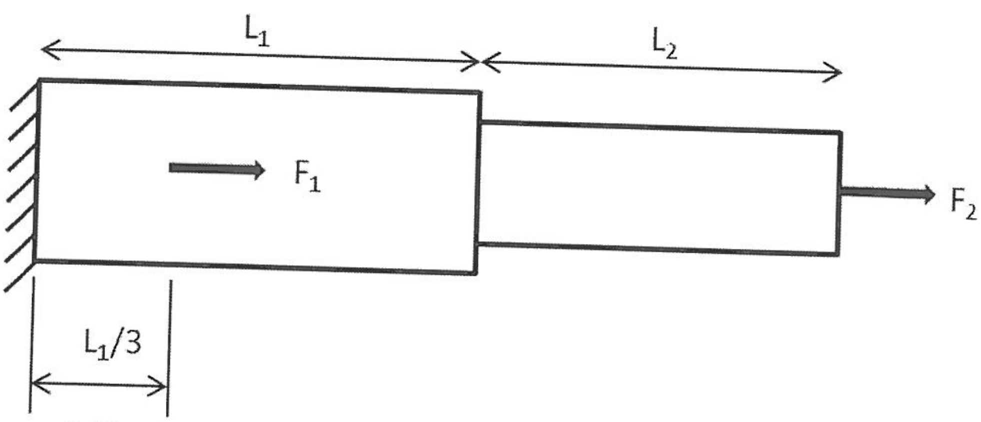

Figure 3 Simple bar system

(20)

QUESTION 4

Consider the 2D plane stress element in Figure 4.

4.1 Calculate the shape functions associated with the three nodes i, j and k. [6]

4.2 Calculate the appropriate load vectors. [4]

4.3 Calculate the total force vector. [2]

4.4 Calculate the material property matrix. [2]

4.5 Calculate the matrix that relates the strains to the displacements. [2]

4.6 If the nodal displacements of the element are as follows:

$$\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{Bmatrix} \times 10^{-3}\,\text{mm}$$

What are the displacements at the point (2,2)? [4]

4.7 Calculate the stress state of the element. [5]

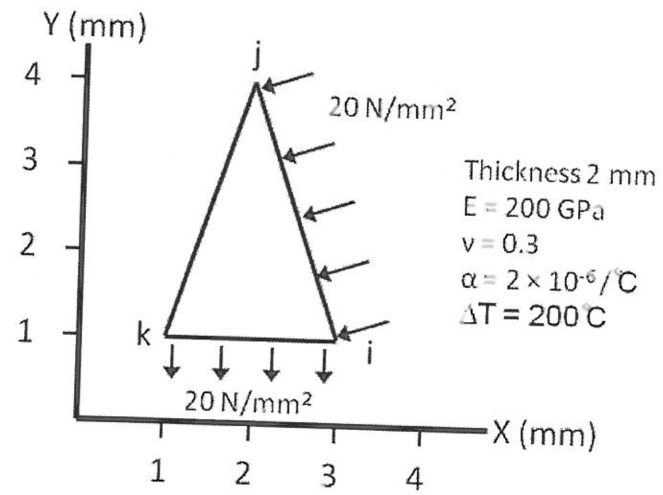

Figure 4 Plane stress triangular element

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Equation sheet

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \qquad \lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)} \qquad \frac{E}{1+\upsilon} = 2G$$

$$\varepsilon'_{kl} = a_{ki}a_{lj}\varepsilon_{ij} \qquad \sigma'_{kl} = a_{ki}a_{lj}\sigma_{ij}$$

$$\cos^2\theta = \frac{(\cos 2\theta + 1)}{2} \qquad \sin^2\theta = \frac{(1 - \cos 2\theta)}{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta \qquad \cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\sigma^3 - (\sigma_{11} + \sigma_{22} + \sigma_{33})\sigma^2 + (\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2)\sigma$$
$$-(\sigma_{11}\sigma_{22}\sigma_{33} + 2\tau\sigma_{12}\sigma_{23}\sigma_{31} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2) = 0$$

$$\varepsilon^3 - (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})\varepsilon^2 + (\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}\varepsilon_{33} + \varepsilon_{11}\varepsilon_{33} - \varepsilon_{12}^2 - \varepsilon_{23}^2 - \varepsilon_{13}^2)\varepsilon$$
$$-(\varepsilon_{11}\varepsilon_{22}\varepsilon_{33} + 2\varepsilon_{12}\varepsilon_{23}\varepsilon_{31} - \varepsilon_{11}\varepsilon_{23}^2 - \varepsilon_{22}\varepsilon_{13}^2 - \varepsilon_{33}\varepsilon_{12}^2) = 0$$

$$ax^3 + bx^2 + cx + d = 0$$

$$t^3 - pt + q = 0$$

$$x = t - \frac{b}{3a} \quad and \quad p = \frac{3ac - b^2}{3a^2} \quad and \quad q = \frac{2b^3 - 9abc + 27a^2d}{27a^3}$$

$$t_k = 2\sqrt{-\frac{p}{3}}\cos\left(\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - k\frac{2\pi}{3}\right) \quad \text{for} \quad k = 0,1,2.$$

$$\sigma_0 = \sigma_1 - \sigma_3$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)\right]^{1/2}$$

$$J_2 = \frac{1}{6}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2)\right]$$

$$[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad [k] = K\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad [k] = \frac{AK}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\sigma_{11'} = \frac{\sigma_{11}+\sigma_{22}}{2} + \frac{\sigma_{11}-\sigma_{22}}{2}\cos 2\theta + \sigma_{12}\sin 2\theta$$

$$\sigma_{22'} = \frac{\sigma_{11}+\sigma_{22}}{2} - \frac{\sigma_{11}-\sigma_{22}}{2}\cos 2\theta - \sigma_{12}\sin 2\theta$$

$$\sigma_{1'2'} = \frac{\sigma_{22}-\sigma_{11}}{2}\sin 2\theta + \sigma_{12}\cos 2\theta$$

$$\varepsilon_{11'} = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \frac{\varepsilon_{11}-\varepsilon_{22}}{2}\cos 2\theta + \varepsilon_{12}\sin 2\theta$$

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$$\left[[B]^T[D][B]tA\right]\begin{Bmatrix}u\\v\end{Bmatrix} = \frac{\alpha E t(\Delta T)}{2(1-\nu)}\begin{Bmatrix}b_i\\c_i\\b_j\\c_j\\b_k\\c_k\end{Bmatrix} + \frac{At}{3}\begin{Bmatrix}X\\Y\\X\\Y\\X\\Y\end{Bmatrix} + \frac{t}{2}\left\{H_{ij}\begin{Bmatrix}p_x\\p_y\\p_x\\p_y\\0\\0\end{Bmatrix} + H_{jk}\begin{Bmatrix}0\\0\\p_x\\p_y\\p_x\\p_y\end{Bmatrix} H_{ki}\begin{Bmatrix}p_x\\p_y\\0\\0\\p_x\\p_y\end{Bmatrix}\right\} + \{P\}$$

$$\{\sigma\} = [D]\{\varepsilon\} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \qquad \{\sigma\} = [D]\{\varepsilon\} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

$$\varepsilon_z = -\frac{\upsilon}{E}(\sigma_x+\sigma_y) + \alpha\Delta T \qquad \sigma_z = \nu(\sigma_x+\sigma_y) - E\alpha\Delta T$$

$$\varepsilon_0 = \alpha\Delta T\begin{Bmatrix}1\\1\\0\end{Bmatrix} \qquad \varepsilon_0 = (1+\nu)\alpha\Delta T\begin{Bmatrix}1\\1\\0\end{Bmatrix}$$

$$\{\varepsilon\} = [B]\{U\} \qquad \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \frac{1}{2A}\begin{bmatrix}b_i & 0 & b_j & 0 & b_k & 0\\ 0 & c_i & 0 & c_j & 0 & c_k\\ c_i & b_i & c_j & b_j & c_k & b_k\end{bmatrix}\begin{Bmatrix}u_{2i-1}\\u_{2i}\\u_{2j-1}\\u_{2j}\\u_{2k-1}\\u_{2k}\end{Bmatrix} = [B]\{U\}$$

$$A=\frac{1}{2}\left(x_i y_j + x_j y_k + x_k y_i - x_i y_k - x_j y_i - x_k y_j\right)=\frac{1}{2}\begin{vmatrix}1 & x_i & y_i\\ 1 & x_j & y_j\\ 1 & x_k & y_k\end{vmatrix}$$

$$\begin{array}{lll} a_i = x_j y_k - x_k y_j & b_i = y_j - y_k & c_i = x_k - x_j \\ a_j = x_k y_i - x_i y_k & b_j = y_k - y_i & c_j = x_i - x_k \\ a_k = x_i y_j - x_j y_i & b_k = y_i - y_j & c_k = x_j - x_i \end{array}$$

$$\phi = N_i\phi_i + N_j\phi_j + N_k\phi_k = [N]\{\Phi\}$$

$$N_i = \left(a_i + b_i x + c_i y\right)/2A$$
$$N_j = \left(a_j + b_j x + c_j y\right)/2A$$
$$N_k = \left(a_k + b_k x + c_k y\right)/2A$$

$$\begin{Bmatrix}u\\ v\end{Bmatrix}=\begin{bmatrix}N_i & 0 & N_j & 0 & N_k & 0\\ 0 & N_i & 0 & N_j & 0 & N_k\end{bmatrix}\begin{Bmatrix}u_i\\ v_i\\ u_j\\ v_j\\ u_k\\ v_k\end{Bmatrix}=[N]\{U\}$$

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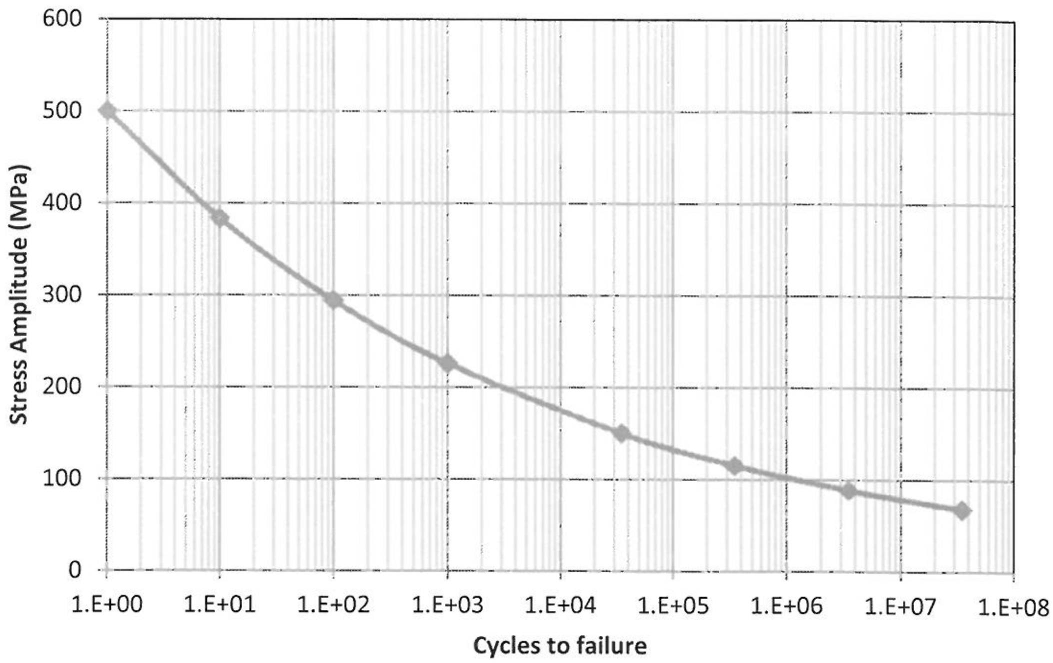

Figure 2 SN curve (Structural steel)

2.6 If the yield stress is σ_0 = 300 MPa. Check for yielding using both the Von Mises and Tresca Criteria. [5]

(30)

QUESTION 3

Two masses are suspended from two wires (1mm diameter). The two masses are interconnected to one another and the floor with springs. Calculate the displacements U_2 and U_3, the reaction forces on the walls and floor and stress state of the wires using the finite element method.

Given : $K_1=5\times10^4$ N/m, $K_2=5\times10^4$ N/m, E=200 GPa, μ=0.3 and g=10 m.s^{-2}

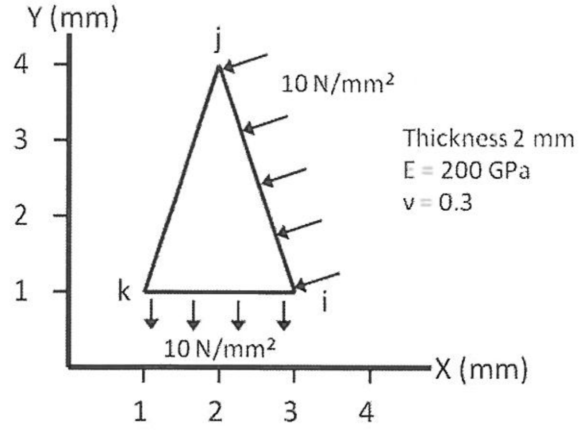

Figure 4 Plane strain triangular element

(25)

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$$\varepsilon_z = -\frac{\upsilon}{E}(\sigma_x + \sigma_y) + \alpha\Delta T \qquad \sigma_z = \nu(\sigma_x + \sigma_y) - E\alpha\Delta T$$

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