UNIVERSITY
OF
JOHANNESBURG

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|---|---|---|
| **PROGRAM** | : | BACCALAUREUS INGENERIAE<br>*MECHANICAL ENGINEERING* |
| **SUBJECT** | : | **Design (Mechanical) 2A** |
| **CODE** | : | **OWM2A11** |
| **DATE** | : | WINTER EXAMINATION<br>02 June 2016 |
| **DURATION** | : | (1-PAPER) 12:30 - 15:30 |
| **WEIGHT** | : | 50 : 50 |
| **TOTAL MARKS** | : | 80 |

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|---|---|---|
| **EXAMINER** | : | Dr BW Botha |
| **MODERATOR** | : | Dr A Maneschijn |
| **NUMBER OF PAGES** | : | 4 PAGES AND 1 ANNEXURE |

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| **INSTRUCTIONS** | : | QUESTION PAPERS MUST BE HANDED IN. |
| **REQUIREMENTS** | : | ANSWER BOOKLET. |

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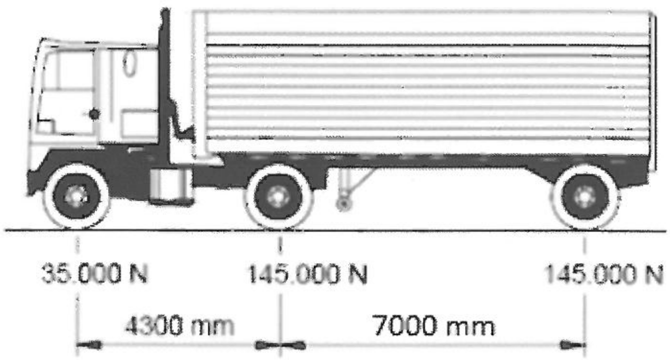
**INSTRUCTIONS TO CANDIDATES:**

PLEASE ANSWER ALL THE QUESTIONS.

## QUESTION 1

**25 Marks**

A truck with dimensions and loading similar to that in the figure has to pass over a lightweight bridge with a span of 25m between supports. The bridge itself has a weight of 4500 N/m. Determine the shear force and bending moment diagrams for the loading of the bridge when the middle wheel is in the centre of the bridge. Determine the maximum bending moment that the bridge structure has to withstand.

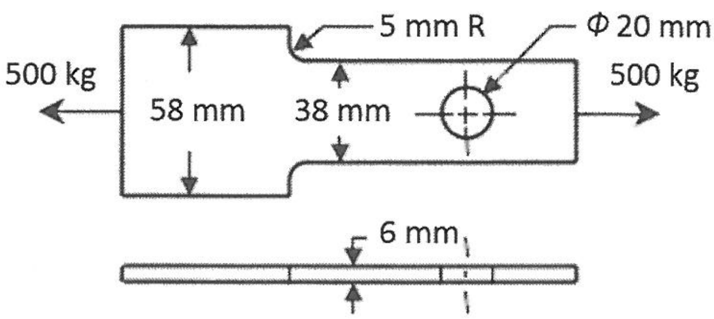



## QUESTION 2

**15 Marks**

A bar is machined from an ASTM No. 20 cast iron and subjected to a static load. Assume the stress concentrations do not interact and determine:

a) The localized effect of each stress concentration separately
b) The actual stress in the shoulder resulting from the fillet radius
c) The actual stress resulting from inserting the transverse hole
d) Determine the critical section in the bar

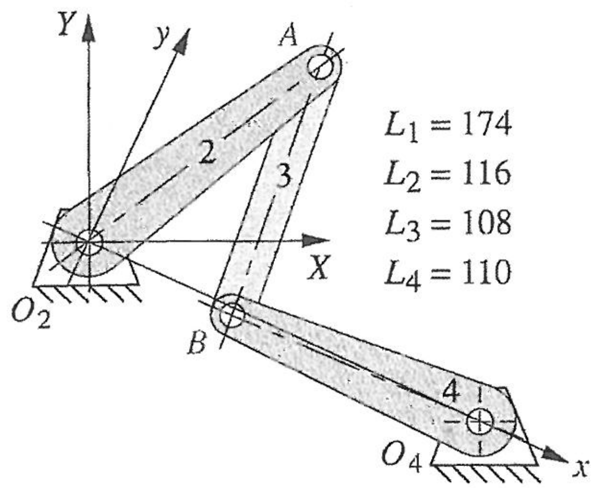


## QUESTION 3

**20 Marks**

The angle between the X and x axes of the mechanism in the figure is 25°. Using the graphical method indicate

(a) the possible positions
(b) if there are any infliction point(s)
(c) realistic limits of movement to ensure predictable movement
(d) the type of mechanism




## QUESTION 4

**25 Marks**

The layout of a single degree-of-freedom planar link supported in single shear at a hinge point is shown in Fig. 1. The link, a solid bar of 40 mm wide and 20 mm thick, is driven in a clockwise direction with an angular speed of +20 rad/s and an angular acceleration of -5 rad/s$^2$. It can be assumed that the centre of mass of the link is at the geometrical centre (i.e. mid-point of length AB) with the mass and inertia (about the centre of mass) of the link being 1.7 kg and 11000 kg-mm$^2$ respectively. An external force of 7500 N oriented at 20 degrees (relative to the X-axis) is acting on the link at Point B. Note that the link is pivoted at Point A and had a length of AB = 250 mm. The bar and pin is made of:

Bar : Mild steel, density: 7860 kg/m$^3$, Yield Strength: 220 MPa, Ultimate Tensile Strength: 400 MPa.

Pin: High Tensile Steel, density: 7860 kg/m$^3$, Yield Strength: 400 MPa, Ultimate Tensile Strength: 630 MPa,

Calculate:

a. the torque required in order to drive the link when the link is oriented at 40 degrees (from X-axis). Show all the steps in your calculations and draw the free body diagrams, wherever necessary.
b. The pin diameter needed to support the beam at Point A using a factor of safety of 1.5 on the yield stress.

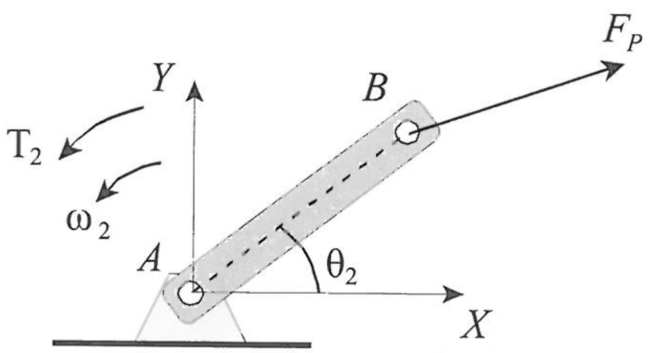



Fig. 1

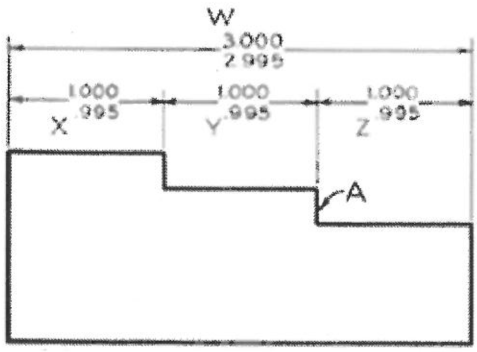
## QUESTION 5

**15 Marks**

a) Explain the difference between hole based and shaft based dimensioning system (4)

b) Explain the difference between geometric and dimensional tolerance and why they are important. (4)

c) Comment on the tolerancing given to a component as shown below and indicate alternatives if necessary. (4)




d) Define the following: (3)

i. Nominal Dimension
ii. Basic size
iii. Actual size

# ADDITIONAL INFORMATION
# OWM2A

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi$$

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\nu\frac{\sigma_x}{E}$$

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\frac{1}{\phantom{E}}\left[\quad \sigma_x + \sigma_y)\right]$$

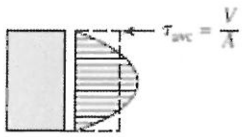
$$E = 2G(1+\nu)$$

$$\sigma_x = -\frac{My}{I} \qquad I = \int y^2 dA$$

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

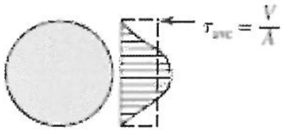
$$Q = \int_{y_1}^{c} y\,dA = \bar{y}'A'$$

$$\tau = \frac{VQ}{Ib}$$




Rectangular

$$\tau_{max} = \frac{3V}{2A}$$




$$\tau_{max} = \frac{4V}{3A}$$

$$\theta = \frac{Tl}{GJ} \qquad \tau_{max} = \frac{Tr}{J}$$

$$J = \frac{\pi d^4}{32} \qquad \theta = \frac{Tl}{\beta bc^3 G}$$

$$\tau_{max} = \frac{T}{\alpha bc^2} \doteq \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right)$$

$$T = \int \tau t r\,ds = (\tau t)\int r\,ds = \tau t(2A_m) = 2A_m t\tau$$

$$\tau = \frac{T}{2A_m t} \qquad \theta_1 = \frac{TL_m}{4GA_m^2 t}$$

$$\tau = G\theta_1 c = \frac{3T}{Lc^2}$$

$$K_t = \frac{\sigma_{max}}{\sigma} \qquad K_{ts} = \frac{\tau_{max}}{\tau}$$

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2(p_o - p_i)/r^2}{r_o^2 - r_i^2} \qquad \sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2(p_o - p_i)/r^2}{r_o^2 - r^2} \qquad \sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

$$(\sigma_t)_{av} = \frac{pd_i}{2t} \qquad (\sigma_t)_{max} = \frac{p(d_i + t)}{2t} \qquad \sigma_l = \frac{pd_i}{4t}$$

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right)$$

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]}$$

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right]$$

$$\sigma = -\epsilon E = -\alpha(\Delta T)E \qquad \sigma = -\frac{\alpha(\Delta T)E}{1-\nu}$$

| $b/c$ | 1.00 | 1.50 | 1.75 | 2.00 | 2.50 | 3.00 | 4.00 | 6.00 | 8.00 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.208 | 0.231 | 0.239 | 0.246 | 0.258 | 0.267 | 0.282 | 0.299 | 0.307 | 0.313 | 0.333 |
| $\beta$ | 0.141 | 0.196 | 0.214 | 0.228 | 0.249 | 0.263 | 0.281 | 0.299 | 0.307 | 0.313 | 0.333 |

## ADDITIONAL INFORMATION (OWM2A) (cntd.)

$$k = \frac{F}{y} \qquad \frac{1}{\rho} = \frac{M}{EI} \qquad U = \frac{F^2 l}{2AE} \qquad U = \frac{F^2 l}{2AG} \qquad U = \frac{CV^2 l}{2AG}$$

$$\delta = \frac{Fl}{AE} \qquad \frac{q}{EI} = \frac{d^4 y}{dx^4} \qquad U = \int \frac{F^2}{2AE}\,dx \qquad U = \int \frac{F^2}{2AG}\,dx \qquad U = \int \frac{CV^2}{2AG}\,dx$$

$$k = \frac{AE}{l} \qquad \frac{V}{EI} = \frac{d^3 y}{dx^3} \qquad U = \frac{T^2 l}{2GJ} \qquad U = \frac{M^2 l}{2EI}$$

$$\theta = \frac{Tl}{GJ} \qquad \frac{M}{EI} = \frac{d^2 y}{dx^2} \qquad U = \int \frac{T^2}{2GJ}\,dx \qquad U = \int \frac{M^2}{2EI}\,dx$$

$$k = \frac{T}{\theta} = \frac{GJ}{l} \qquad \theta = \frac{dy}{dx} \qquad P_{cr} = \frac{\pi^2 EI}{l^2} \qquad P_{cr} = \frac{C\pi^2 EI}{l^2} \qquad \frac{P_{cr}}{A} = \frac{C\pi^2 E}{(l/k)^2}$$

$$y = f(x) \qquad \sigma_c = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{PecA}{IA} = \frac{P}{A}\left(1 + \frac{ec}{k^2}\right)$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \quad \text{or} \quad \sigma_1 - \sigma_3 \geq S_y$$

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2}$$

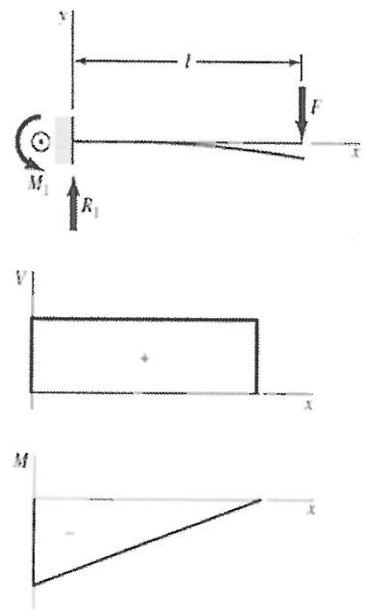
$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2}$$

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2} \qquad \sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \qquad K_I = \beta\sigma\sqrt{\pi a}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases}$$

1 Cantilever—end load



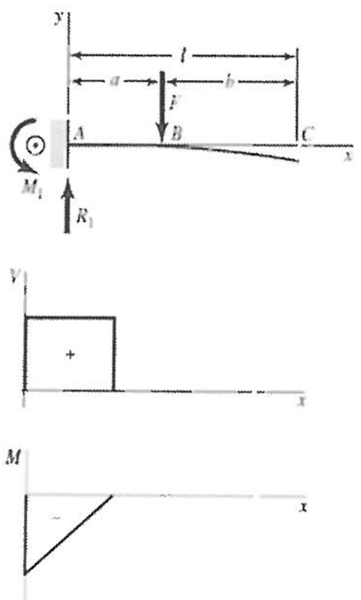
$$R_1 = V = F \qquad M_1 = Fl$$

$$M = F(x - l)$$

$$y = \frac{Fx^2}{6EI}(x - 3l)$$

$$y_{max} = -\frac{Fl^3}{3EI}$$

2 Cantilever—intermediate load



$$R_1 = V = F \qquad M_1 = Fa$$

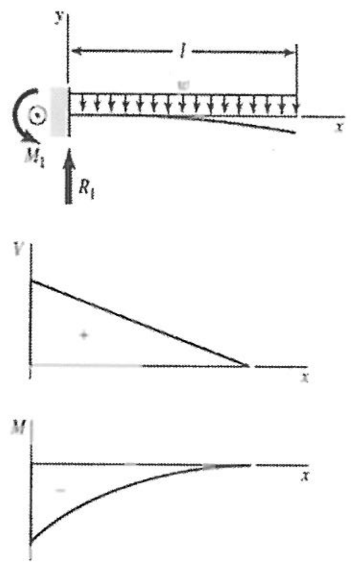
$$M_{AB} = F(x - a) \qquad M_{BC} = 0$$

$$y_{AB} = \frac{Fx^2}{6EI}(x - 3a)$$

$$y_{BC} = \frac{Fa^2}{6EI}(a - 3x)$$

$$y_{max} = \frac{Fa^2}{6EI}(a - 3l)$$

3 Cantilever—uniform load



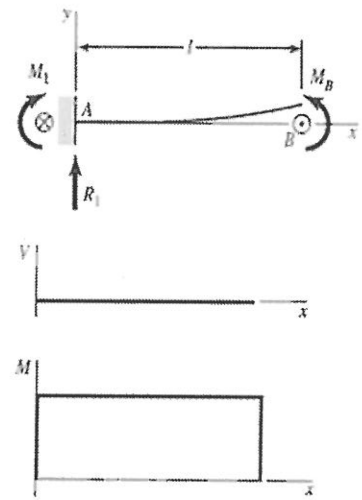
$$R_1 = wl \qquad M_1 = \frac{wl^2}{2}$$

$$V = w(l - x) \qquad M = -\frac{w}{2}(l - x)^2$$

$$y = \frac{wx^2}{24EI}(4lx - x^2 - 6l^2)$$

$$y_{max} = -\frac{wl^4}{8EI}$$

4 Cantilever—moment load

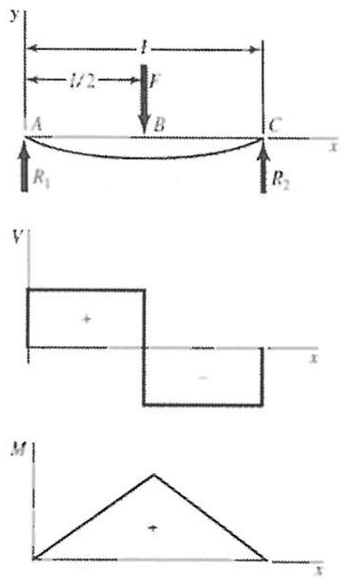


$$R_1 = V = 0 \qquad M_1 = M = M_B$$

$$y = \frac{M_B x^2}{2EI} \qquad y_{max} = \frac{M_B l^2}{2EI}$$

(continued)

## 5 Simple supports—center load




$$R_1 = R_2 = \frac{F}{2}$$

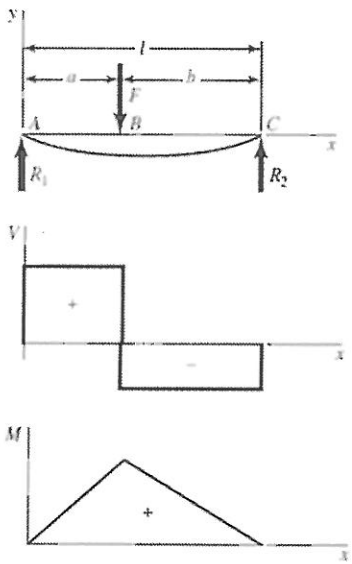
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fx}{2} \qquad M_{BC} = \frac{F}{2}(l - x)$$

$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$y_{max} = -\frac{Fl^3}{48EI}$$

## 6 Simple supports—intermediate load




$$R_1 = \frac{Fb}{l} \qquad R_2 = \frac{Fa}{l}$$

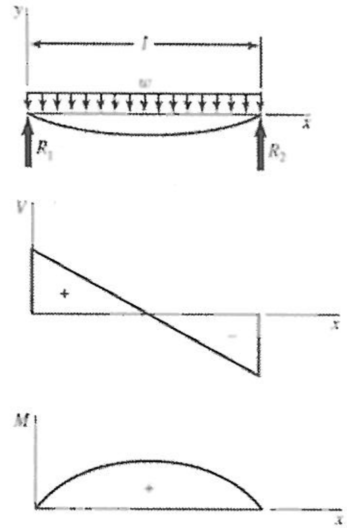
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fbx}{l} \qquad M_{BC} = \frac{Fa}{l}(l - x)$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$y_{BC} = \frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

## 7 Simple supports—uniform load



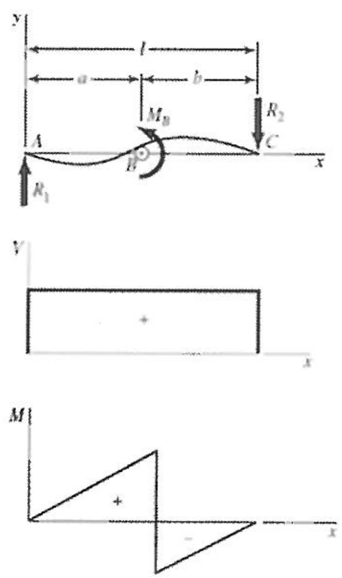

$$R_1 = R_2 = \frac{wl}{2} \qquad V = \frac{wl}{2} - wx$$

$$M = \frac{wx}{2}(l - x)$$

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3)$$

$$y_{max} = -\frac{5wl^4}{384EI}$$

## 8 Simple supports—moment load




$$R_1 = R_2 = \frac{M_B}{l} \qquad V = \frac{M_B}{l}$$

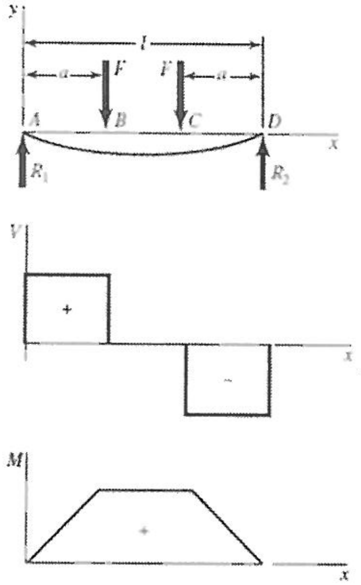
$$M_{AB} = \frac{M_B x}{l} \qquad M_{BC} = \frac{M_B}{l}(x - l)$$

$$y_{AB} = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$y_{BC} = \frac{M_B}{6EIl}[x^3 - 3lx^2 + x(2l^2 + 3a^2) - 3a^2 l]$$

(continued)

9 Simple supports—twin loads




$$R_1 = R_2 = F \qquad V_{AB} = F \qquad V_{BC} = 0$$

$$V_{CD} = -F$$

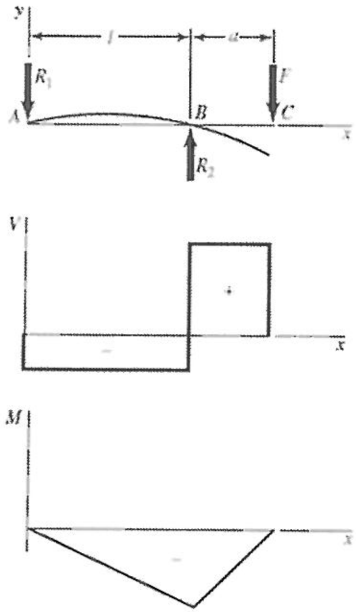
$$M_{AB} = Fx \qquad M_{BC} = Fa \qquad M_{CD} = F(l - x)$$

$$y_{AB} = \frac{Fx}{6EI}(x^2 + 3a^2 - 3la)$$

$$y_{BC} = \frac{Fa}{6EI}(3x^2 + a^2 - 3lx)$$

$$y_{\max} = \frac{Fa}{24EI}(4a^2 - 3l^2)$$

10 Simple supports—overhanging load




$$R_1 = \frac{Fa}{l} \qquad R_2 = \frac{F}{l}(l + a)$$

$$V_{AB} = -\frac{Fa}{l} \qquad V_{BC} = F$$

$$M_{AB} = -\frac{Fax}{l} \qquad M_{BC} = F(x - l - a)$$

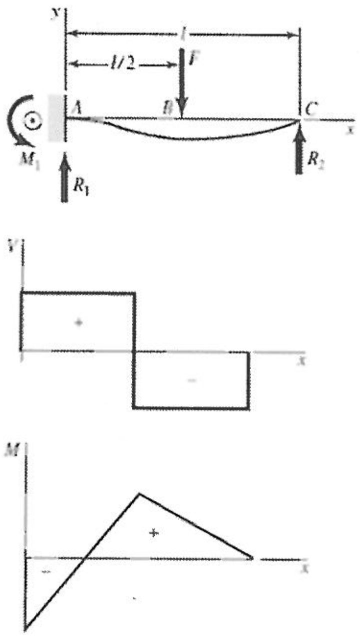
$$y_{AB} = \frac{Fax}{6EIl}(l^2 - x^2)$$

$$y_{BC} = \frac{F(x - l)}{6EI}[(x - l)^2 - a(3x - l)]$$

$$y_c = -\frac{Fa^2}{3EI}(l + a)$$

(continued)

11 One fixed and one simple support—center load




$$R_1 = \frac{11F}{16} \qquad R_2 = \frac{5F}{16} \qquad M_1 = \frac{3Fl}{16}$$

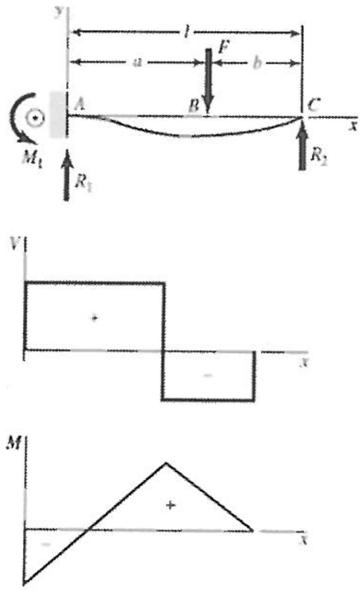
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{F}{16}(11x - 3l) \qquad M_{BC} = \frac{5F}{16}(l - x)$$

$$y_{AB} = \frac{Fx^2}{96EI}(11x - 9l)$$

$$y_{BC} = \frac{F(l - x)}{96EI}(5x^2 + 2l^2 - 10lx)$$

12 One fixed and one simple support—intermediate load




$$R_1 = \frac{Fb}{2l^3}(3l^2 - b^2) \qquad R_2 = \frac{Fa^2}{2l^3}(3l - a)$$

$$M_1 = \frac{Fb}{2l^2}(l^2 - b^2)$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

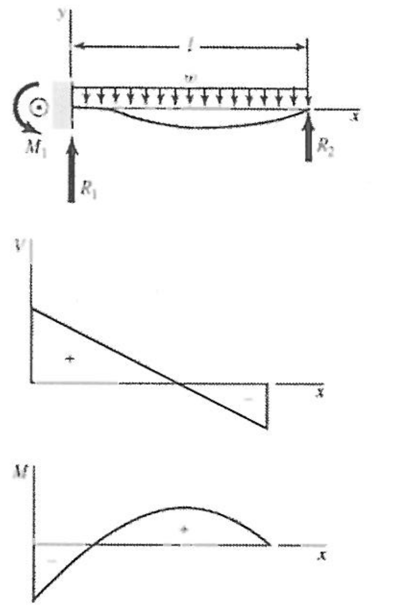
$$M_{AB} = \frac{Fb}{2l^3}[b^2 l - l^3 + x(3l^2 - b^2)]$$

$$M_{BC} = \frac{Fa^2}{2l^3}(3l^2 - 3lx - al + ax)$$

$$y_{AB} = \frac{Fbx^2}{12EIl^3}[3l(b^2 - l^2) + x(3l^2 - b^2)]$$

$$y_{BC} = y_{AB} - \frac{F(x - a)^3}{6EI}$$

### 13 One fixed and one simple support—uniform load



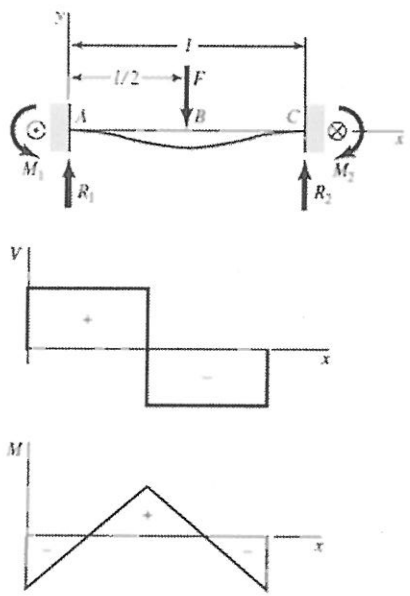
$$R_1 = \frac{5wl}{8} \qquad R_2 = \frac{3wl}{8} \qquad M_1 = \frac{wl^2}{8}$$

$$V = \frac{5wl}{8} - wx$$

$$M = -\frac{w}{8}(4x^2 - 5lx + l^2)$$

$$y = \frac{wx^2}{48EI}(l - x)(2x - 3l)$$

### 14 Fixed supports—center load



$$R_1 = R_2 = \frac{F}{2} \qquad M_1 = M_2 = \frac{Fl}{8}$$

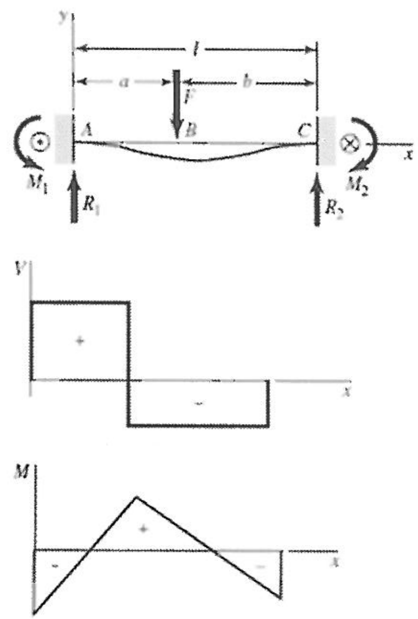
$$V_{AB} = -V_{BC} = \frac{F}{2}$$

$$M_{AB} = \frac{F}{8}(4x - l) \qquad M_{BC} = \frac{F}{8}(3l - 4x)$$

$$y_{AB} = \frac{Fx^2}{48EI}(4x - 3l)$$

$$y_{max} = -\frac{Fl^3}{192EI}$$

### 15 Fixed supports—intermediate load



$$R_1 = \frac{Fb^2}{l^3}(3a + b) \qquad R_2 = \frac{Fa^2}{l^3}(3b + a)$$

$$M_1 = \frac{Fab^2}{l^2} \qquad M_2 = \frac{Fa^2b}{l^2}$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

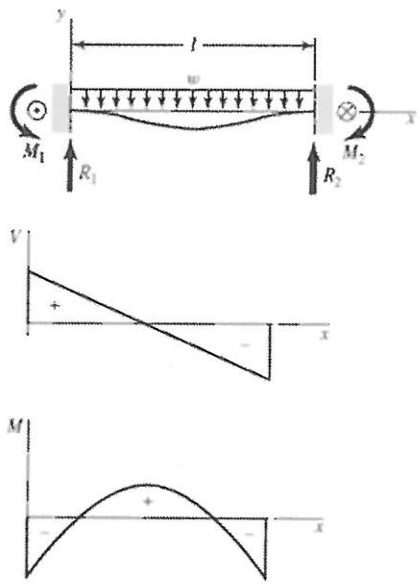
$$M_{AB} = \frac{Fb^2}{l^3}[x(3a + b) - al]$$

$$M_{BC} = M_{AB} - F(x - a)$$

$$y_{AB} = \frac{Fb^2x^2}{6EIl^3}[x(3a + b) - 3al]$$

$$y_{BC} = \frac{Fa^2(l - x)^2}{6EIl^3}[(l - x)(3b + a) - 3bl]$$

### 16 Fixed supports—uniform load



$$R_1 = R_2 = \frac{wl}{2} \qquad M_1 = M_2 = \frac{wl^2}{12}$$

$$V = \frac{w}{2}(l - 2x)$$

$$M = \frac{w}{12}(6lx - 6x^2 - l^2)$$

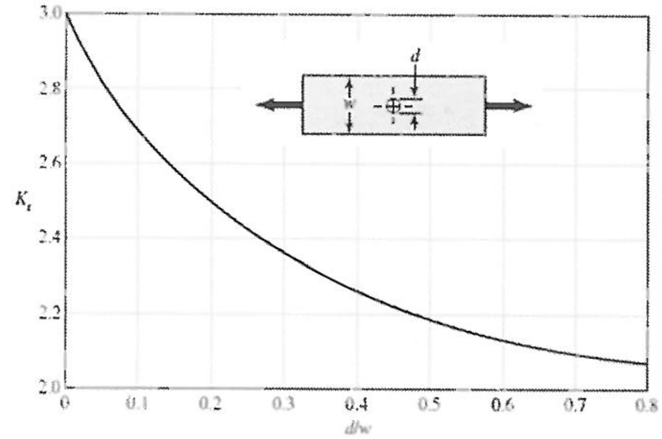
$$y = -\frac{wx^2}{24EI}(l - x)^2$$

$$y_{max} = -\frac{wl^4}{384EI}$$

(continued)

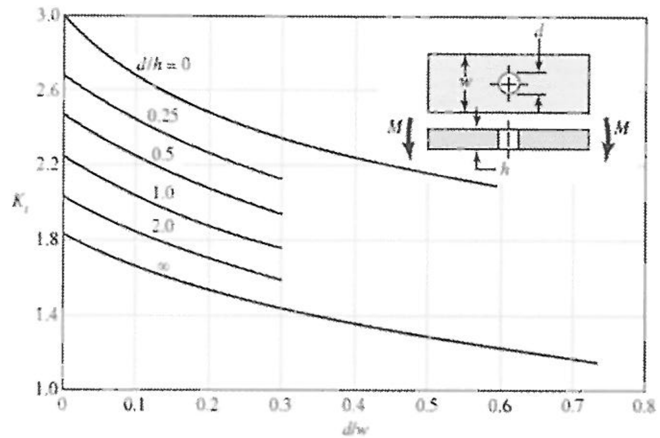
## Figure A-15-1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and $t$ is the thickness.



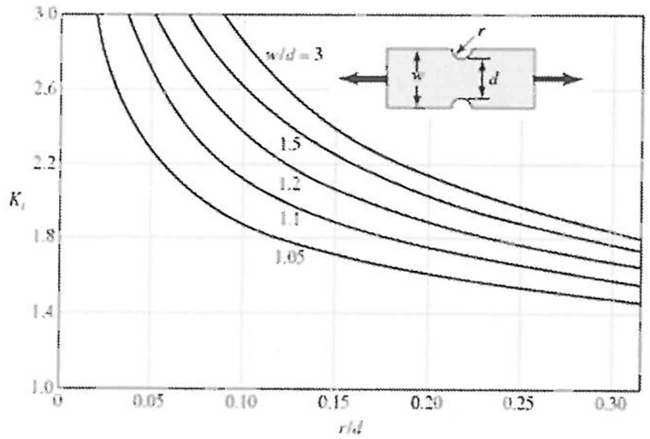

## Figure A-15-2

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.



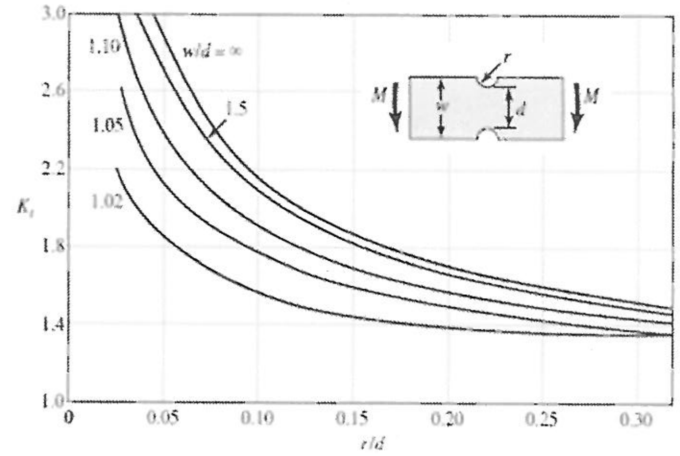

## Figure A-15-3

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and $t$ is the thickness.



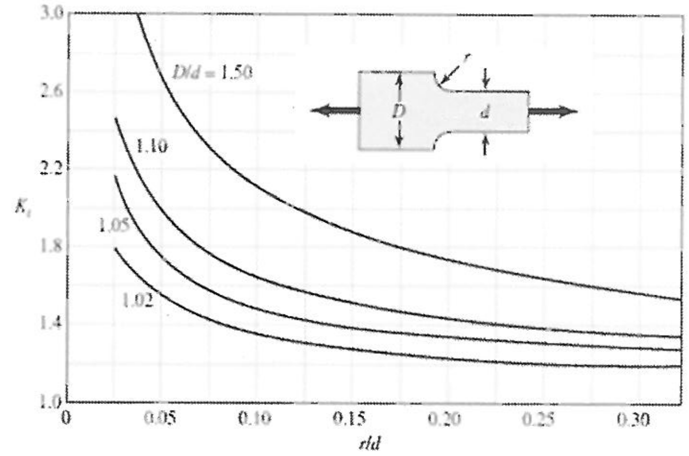

## Figure A-15-4

Notched rectangular bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, and $t$ is the thickness.



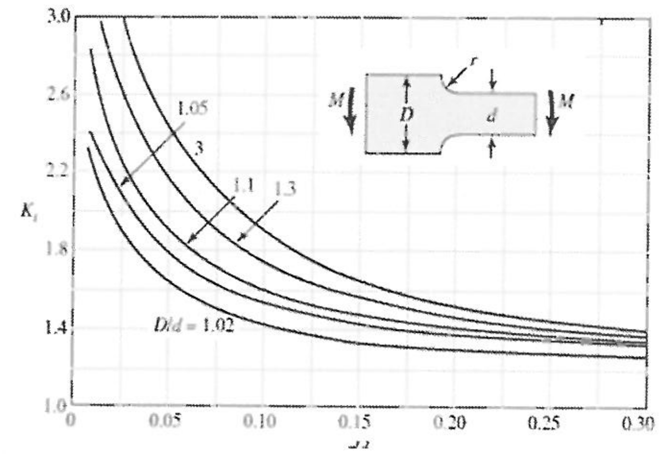

## Figure A-15-5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and $t$ is the thickness.



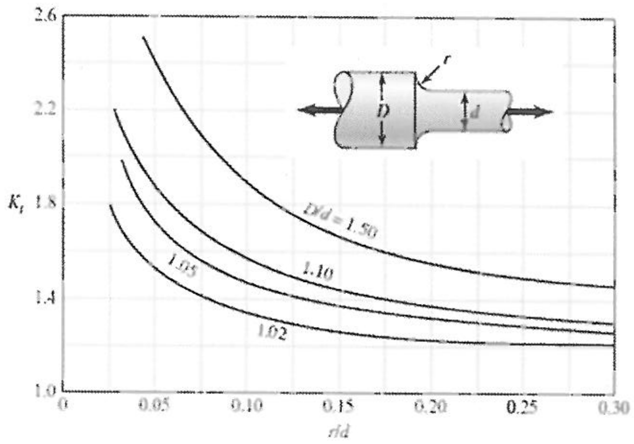

## Figure A-15-6

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, $t$ is the thickness.



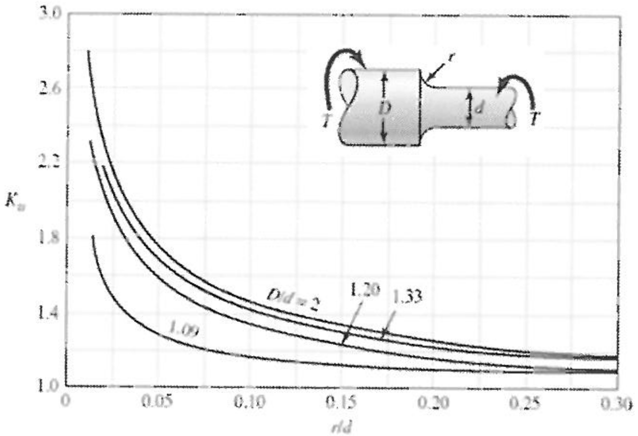
## Figure A–15–7

Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.



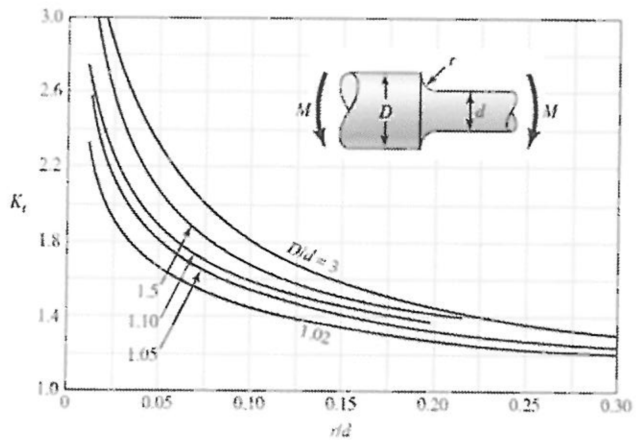

## Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



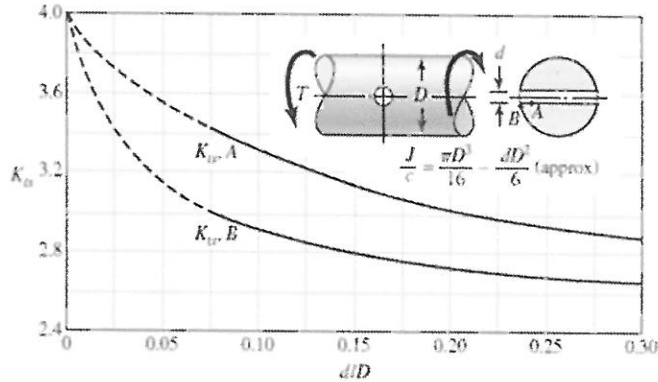

## Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



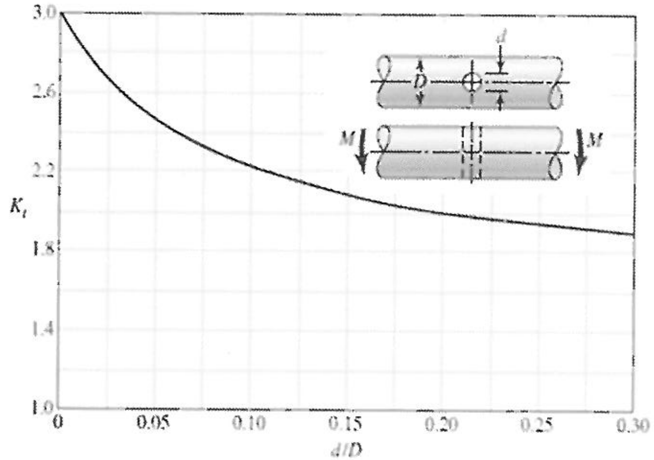

## Figure A–15–10

Round shaft in torsion with transverse hole.




## Figure A–15–11

Round shaft in bending with a transverse hole. $\sigma_0 = M/[(\pi D^3/32) - (dD^2/6)]$, approximately.




## Figure A–15–12

Plate loaded in tension by a pin through a hole. $\sigma_0 = F/A$, where $A = (w - d)t$. When clearance exists, increase $K_t$ 35 to 50 percent. *(M. M. Frocht and H. N. Hill, "Stress Concentration Factors around a Central Circular Hole in a Plate Loaded through a Pin in Hole," J. Appl. Mechanics, vol. 7, no. 1, March 1940, p. A-5.)*

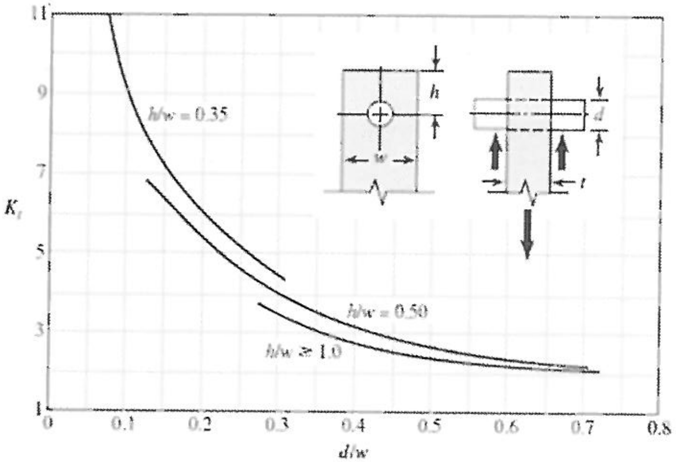


**Figure A–15–13**

Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

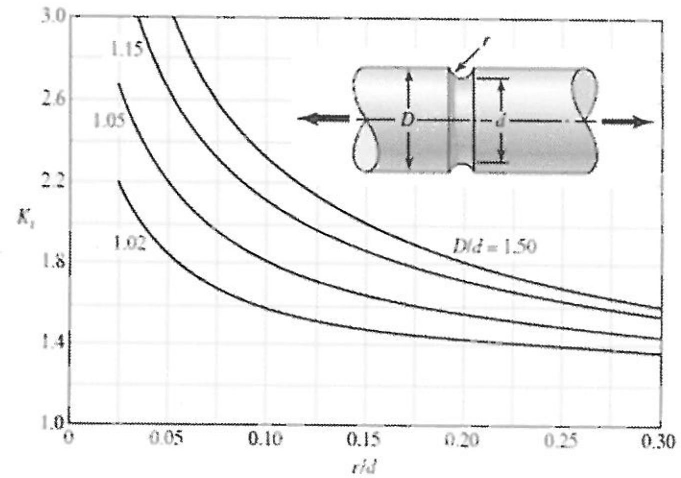



**Figure A–15–14**

Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

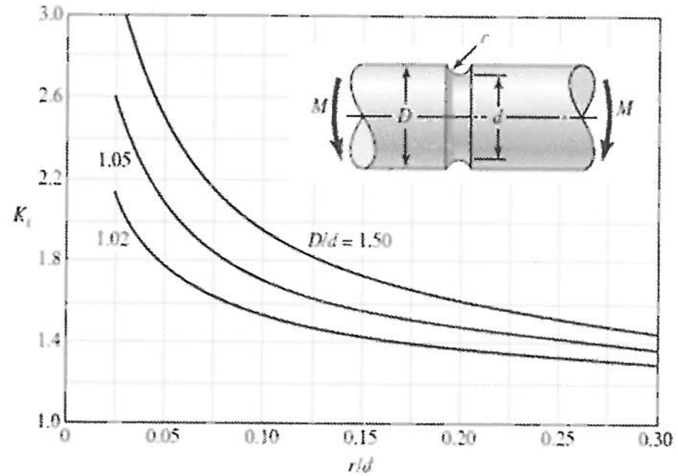
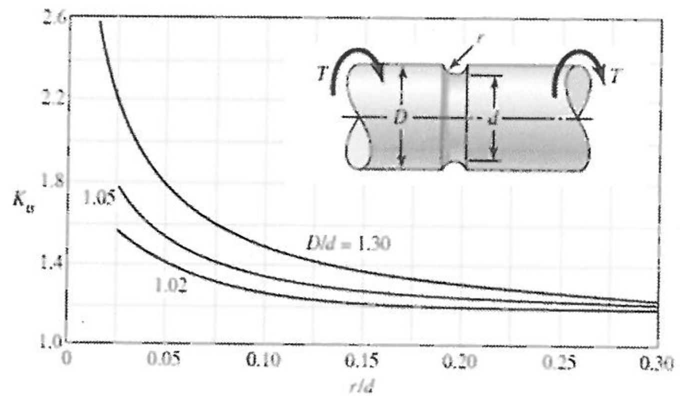



**Figure A–15–15**

Grooved round bar in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



## Mechanisms

### DOF

**Computing DOF - Spatial:** $M = 6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5$

M – Mobility/DOF

L – No. of links

$J_1$ – No. of joints capturing 5DOF

$J_2$ – No. of joints capturing 4DOF

$J_3$ – No. of joints capturing 3DOF

$J_4$ – No. of joints capturing 2DOF

$J_5$ – No. of joints capturing 1DOF

**Computing DOF - Planar:**

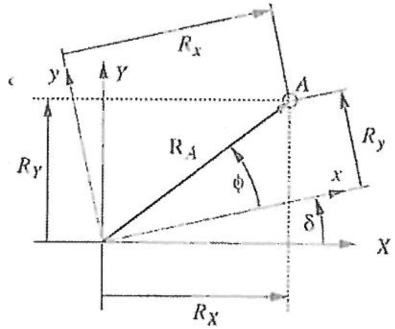
$$M = 3L - 2J - 3G = 3(L-1) - 2J = 3(L-1) - 2J_1 - J_2$$

M – DOF/Mobility, G –Ground link, L – No. of links,

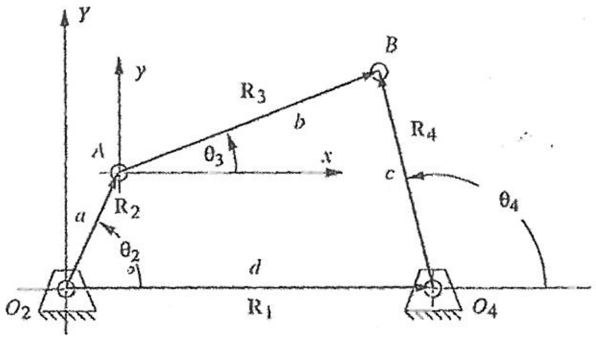
$J_1$ – No. of full joints, $J_2$ – No. of half joints

### Coordinate Transformation

$$\begin{bmatrix} R_X \\ R_Y \end{bmatrix} = \begin{bmatrix} Cos\delta & -Sin\delta \\ Sin\delta & Cos\delta \end{bmatrix} \begin{bmatrix} R_x \\ R_y \end{bmatrix}$$




### Four Bar Linkage




**Position Analysis:**

$$\vec{R_2} + \vec{R_3} - \vec{R_4} - \vec{R_1} = 0 \quad , \quad ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j0} = 0$$

$$k_1 = \frac{d}{a} \quad , \quad k_2 = \frac{d}{c} \quad , \quad k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} \quad , k_4 = \frac{d}{b} \quad , \quad k_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab}$$

$$A = Cos\theta_2 - k_1 + k_3 - k_2 Cos\theta_2 \quad , \quad B = -2Sin\theta_2 \quad , \quad C = k_1 - k_2 Cos\theta_2 + k_3 - Cos\theta_2$$

$$D = Cos\theta_2 - k_1 + k_5 + k_4 Cos\theta_2 \quad , \quad E = -2Sin\theta_2 \quad , \quad F = k_1 + k_4 Cos\theta_2 + k_5 - Cos\theta_2$$

$$Tan\frac{\theta_4}{2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \therefore \quad \theta_4 = 2Tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$$

$$Tan\frac{\theta_3}{2} = \frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \quad \therefore \quad \theta_3 = 2Tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right]$$

**Velocity Analysis:**

$$\vec{V}_A + \vec{V}_{BA} - \vec{V}_B = 0 \qquad a\,j\omega_2 e^{j\theta_2} + b\,j\omega_3 e^{j\theta_3} - c\,j\omega_4 e^{j\theta_4} = 0$$

$$\omega_3 = \frac{a\omega_2}{b}\frac{Sin(\theta_4 - \theta_2)}{Sin(\theta_3 - \theta_4)} \qquad \omega_4 = \frac{a\omega_2}{c}\frac{Sin(\theta_2 - \theta_3)}{Sin(\theta_4 - \theta_3)}$$

$$\vec{V}_A = a\omega_2(-Sin\theta_2 + jCos\theta_2) \; , \; \vec{V}_B = c\,\omega_4(-Sin\theta_4 + jCos\theta_4) \; , \; \vec{V}_{BA} = b\omega_3(-Sin\theta_3 + jCos\theta_3)$$

**Acceleration Analysis:**

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) + \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \left(j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4}\right) = 0$$

$A = c\,Sin\theta_4 \qquad B = b\,Sin\theta_3$

$C = a\alpha_2 Sin\theta_2 + a\omega_2^2 Cos\theta_2 + b\omega_3^2 Cos\theta_3 - c\omega_4^2 Cos\theta_4$

$D = c\,Cos\theta_4$

$E = b\,Cos\theta_3$

$F = a\alpha_2 Cos\theta_2 - a\omega_2^2 Sin\theta_2 - b\omega_3^2 Sin\theta_3 + c\omega_4^2 Sin\theta_4$

$$\alpha_3 = \frac{C.D - A.F}{A.E - B.D} \qquad \alpha_4 = \frac{C.E - B.F}{A.E - B.D}$$

### Mechanical Advantage & Efficiency

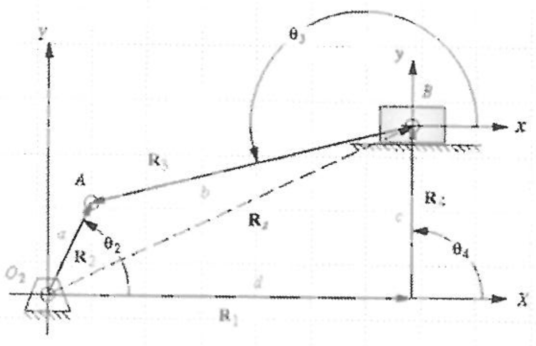
Mechanical Advantage:

$$m_A = \frac{F_{out}}{F_{in}} = \frac{\omega_{in}}{\omega_{out}}\frac{r_{in}}{r_{out}}$$

For 100% efficiency: $P_{out} = P_{in} \Rightarrow \frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}}$

Efficiency: $\varepsilon = \frac{P_{out}}{P_{in}}$

## Slider Crank



**Position Analysis:**

$$\overrightarrow{R_2} - \overrightarrow{R_3} - \overrightarrow{R_4} - \overrightarrow{R_1} = 0$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j0} = 0$$

$$\theta_{31} = \arcsin\left(\frac{a\sin(\theta_2) - c}{b}\right)$$

$$\theta_{32} = \arcsin\left(-\frac{a\sin(\theta_2) - c}{b}\right) + \pi \ , \qquad \theta_{33} = \arcsin\left(-\frac{a\sin(\theta_2) + c}{b}\right) + \pi|$$

$$d = a\cos(\theta_2) - b\cos(\theta_3)$$

**Velocity Analysis:**

$$\omega_3 = \frac{a\cos(\theta_2)}{b\cos(\theta_3)}\omega_2 \quad , \quad \dot{d} = -a\omega_2\sin(\theta_2) + b\omega_3\sin(\theta_3)$$

$$\vec{V}_A = a\omega_2(-sin\theta_2 + jcos\theta_2) \ , \ \vec{V}_{AB} = b\omega_3(-sin\theta_3 + jcos\theta_3) \ , \ \vec{V_B} = \dot{d} = -a\omega_2\, sin\theta_2 + b\omega_3 \sin\theta_3$$

**Acceleration Analysis:**

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) - \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \ddot{d} = 0$$

$$\alpha_3 = \frac{a\alpha_2\cos(\theta_2) - a\omega_2^2\sin(\theta_2) + b\omega_3^2\sin(\theta_3)}{b\cos(\theta_3)}$$

$$\ddot{d} = -a\alpha_2\sin(\theta_2) - a\omega_2^2\cos(\theta_2) + b\alpha_3\sin(\theta_3) + b\omega_3^2\cos(\theta_3)$$

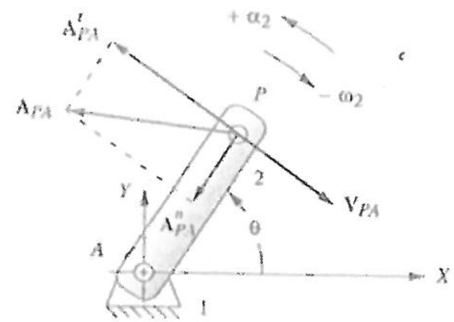
## General Formulae

Angular velocity & acceleration/velocity & acceleration at a point:

$$\omega = \frac{d\theta}{dt} \quad , \quad \alpha = \frac{d\omega}{dt} \quad , \quad V = \frac{dR}{dt} \quad , \quad A = \frac{dV}{dt} \quad , \quad v = r\omega$$

*A* is acceleration vector at a point of interest, *V* is velocity vector at the point of interest and R is the position vector of the point of interest. α is the angular acceleration of a link, *r* is the radius of the link, *ω* is the angular velocity of the rigid link and *v* is the magnitude of velocity.

**Expressions for 1-DOF link:**



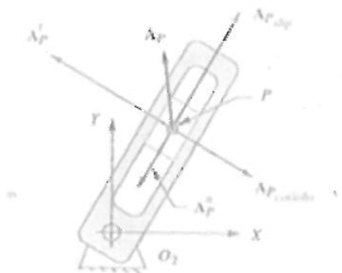
$$R_{PA} = pe^{j\theta} \qquad V_{PA} = \frac{dR_{PA}}{dt}$$

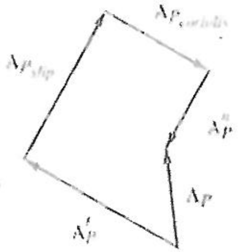
$$= pje^{j\theta}\frac{d\theta}{dt} = p\omega je^{j\theta}$$

$$A_{PA} = \frac{dV_{PA}}{dt} = pj\frac{d\omega}{dt}e^{j\theta} + pj\omega j\frac{d\theta}{dt}e^{j\theta}$$

$$= pj\alpha e^{j\theta} - p\omega^2 e^{j\theta}$$

**Coriolis Acceleration:**





$$R_P = pe^{j\theta_2} \quad , \quad V_P = pj\omega_2 e^{j\theta_2} + \dot{p}e^{j\theta_2} \quad , \quad V_P = V_{P_{trans}} + V_{P_{slip}}$$

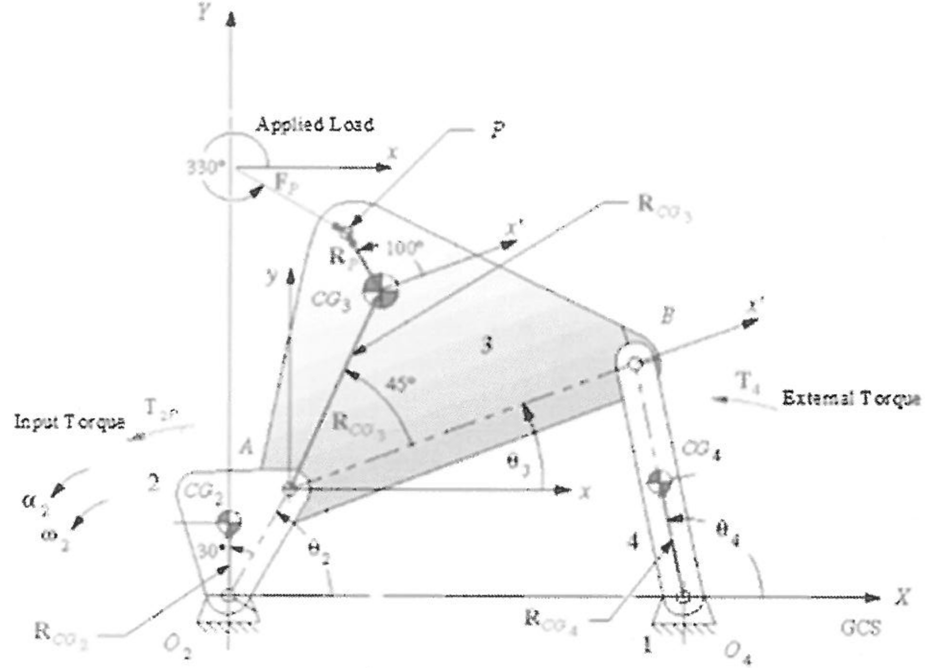
$$A_P = p\alpha_2 je^{j\theta_2} - p\omega_2^2 e^{j\theta_2} + 2\dot{p}j\omega_2 e^{j\theta_2} + \ddot{p}e^{j\theta_2} = A_{P_{tangential}} + A_{P_{normal}} + A_{P_{coriolis}} + A_{P_{slip}}$$

# Dynamics

## EOM for a Planar System:

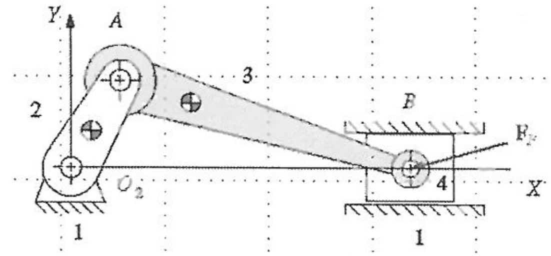
$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum T = I_G \alpha$$

## Four-bar linkage




$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{34y} & -R_{34x} & -R_{14y} & R_{14x} & 0 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ F_{14x} \\ F_{14y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2x} \\ m_2 a_{G2y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3x} - F_{Px} \\ m_3 a_{G3y} - F_{Py} \\ I_{G3}\alpha_3 - R_{Px}F_{Py} + R_{Py}F_{Px} \\ m_4 a_{G4x} \\ m_4 a_{G4y} \\ I_{G4}\alpha_4 - T_4 \end{bmatrix}$$

## Slider Crank



$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \pm\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ F_{14y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2x} \\ m_2 a_{G2y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3x} \\ m_3 a_{G3y} \\ I_{G3}\alpha_3 \\ m_4 a_{G4x} - F_{P_x} \\ -F_{P_y} \end{bmatrix}$$

| | | |
|---|---|---|
| **PROGRAM** | : | BACCALAUREUS INGENERIAE<br>*MECHANICAL ENGINEERING* |
| **SUBJECT** | : | **Design (Mechanical) 2A** |
| **CODE** | : | **OWM2A11** |
| **DATE** | : | WINTER SUPPLEMENTARY EXAMINATION<br>June 2016 |
| **DURATION** | : | 3 hours (1-PAPER) |
| **WEIGHT** | : | 50 : 50 |
| **TOTAL MARKS** | : | 80 |

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| | | |
|---|---|---|
| **EXAMINER** | : | Dr BW Botha |
| **MODERATOR** | : | Dr A Maneschijn |
| **NUMBER OF PAGES** | : | 3 PAGES AND 1 ANNEXURE |

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| | | |
|---|---|---|
| **INSTRUCTIONS** | : | QUESTION PAPERS MUST BE HANDED IN. |
| **REQUIREMENTS** | : | ANSWER BOOKLET. |

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2/...

**INSTRUCTIONS TO CANDIDATES:**

PLEASE ANSWER ALL THE QUESTIONS.

## QUESTION 1

**25 Marks**

A four bar linkage has the data and layout as given in the table and figure below. Perform a dynamic analysis. Draw the free body diagrams, write the equations of motion and compile the equations of motion in a matrix form. Fill the matrices with all the known variables and clearly identify the unknown variables.

Part 1 Lengths in mm, angles in degrees, angular acceleration in rad/sec$^2$

| Row | link 2 | link 3 | link 4 | link 1 | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a. | 101.6 | 304.8 | 203.2 | 381.0 | 45 | 24.97 | 99.30 | 20 | 75.29 | 244.43 |

Part 2 Angular velocity in rad/sec, mass in kg, mass moment of inertia in kg-m$^2$, torque in N-m

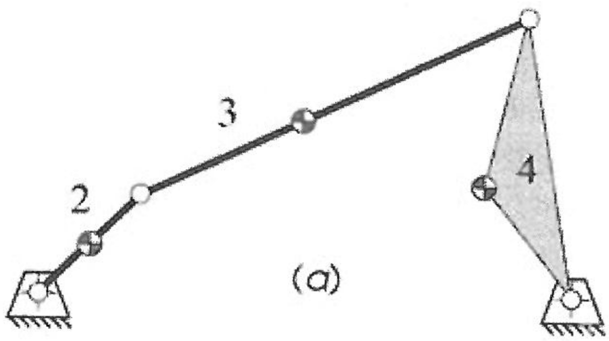
| Row | $\omega_2$ | $\omega_3$ | $\omega_4$ | $m_2$ | $m_3$ | $m_4$ | $I_2$ | $I_3$ | $I_4$ | $T_3$ | $T_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| a. | 20 | -5.62 | 3.56 | 0.35 | 3.50 | 17.51 | 0.011 | 0.023 | 0.056 | -1.69 | 2.82 |

Part 3 Lengths in mm angles in degrees, linear accelerations in m/sec$^2$

| Row | $R_{g_2}$ mag | $R_{g_2}$ ang | $R_{g_3}$ mag | $R_{g_3}$ ang | $R_{g_4}$ mag | $R_{g_4}$ ang | $a_{g_2}$ mag | $a_{g_2}$ ang | $a_{g_3}$ mag | $a_{g_3}$ ang |
|---|---|---|---|---|---|---|---|---|---|---|
| a. | 50.8 | 0 | 127.0 | 0 | 101.6 | 30 | 20.35 | 222.14 | 42.96 | 208.24 |

Part 4 Linear accelerations in m/sec$^2$, forces in N, lengths in mm, angles in degrees

| Row | $a_{g_4}$ mag | $a_{g_4}$ ang | $F_{P_3}$ mag | $\delta F_{P_3}$ ang | $R_{P_3}$ mag | $\delta R_{P_3}$ ang | $F_{P_4}$ mag | $\delta F_{P_4}$ ang | $R_{P_4}$ mag | $\delta R_{P_4}$ ang |
|---|---|---|---|---|---|---|---|---|---|---|
| a. | 24.87 | 222.27 | 0.00 | 0 | 0.00 | 0 | 177.93 | -30 | 203.2 | 0 |

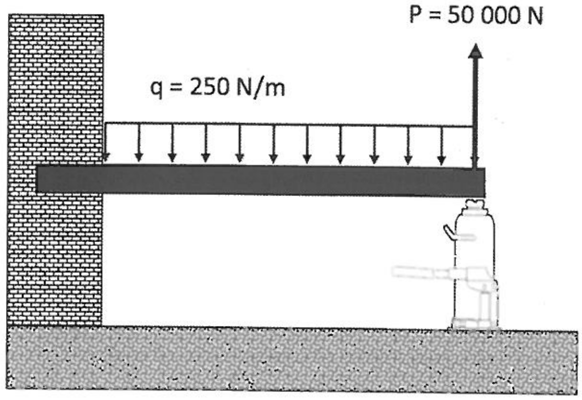


(a)

## QUESTION 2

**15 Marks**

In order to calibrate the digital readout of a hydraulic cylinder a cantilever beam with a width of 75mm, a height of 150mm and a weight of 850 N/m is fitted with strain gauges to determine the actual force exerted. The cylinder is placed at a distance of 5 m from the supporting structure. The steel used has a yield strength of 350 MPa and a modulus of elasticity of 200 GPa. Using the principle of super positioning calculate the deflection and end slope that can be expected if the cylinder exerts a maximum force of 10 000 N at the end of the beam.

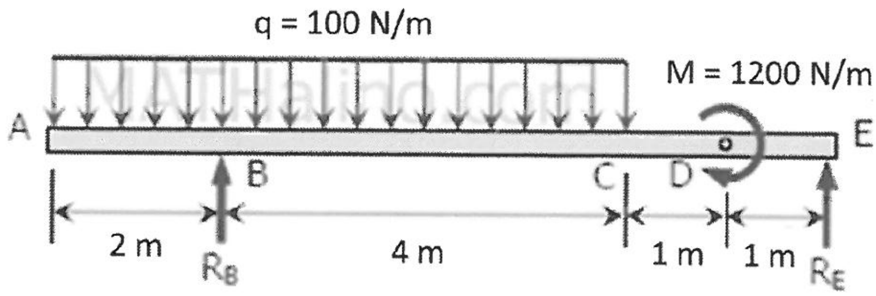



## QUESTION 3

**25 Marks**

A beam is loaded and supported as shown in the figure. For this beam

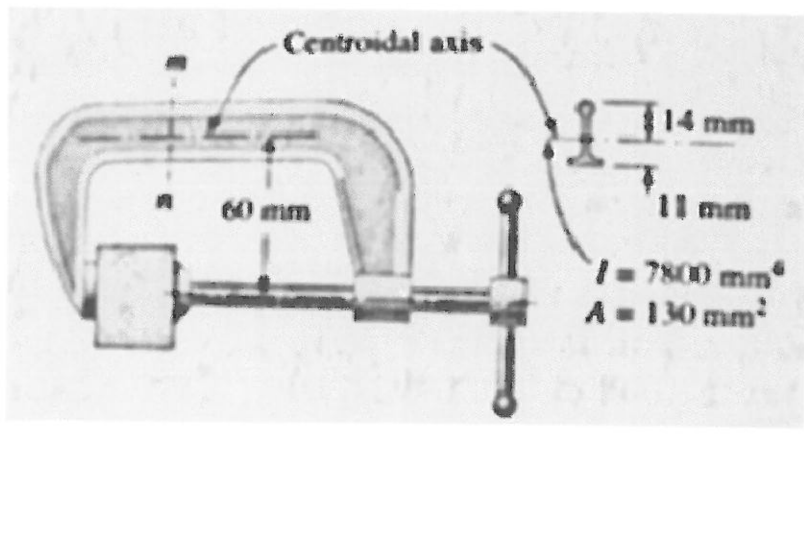
(a) Calculate the reaction forces
(b) Determine the equations for the shear force and the bending moment as functions of *x*.
(c) Draw complete shear force and bending moment diagrams.




## QUESTION 4

**15 Marks**

Find the maximum clamping force that may be exerted by the cast-iron C-clamp if the allowable normal stresses on Section m-n are 30 MPa in compression and 15 MPa in tension and a factor of safety of 1.2 is required.



# ADDITIONAL INFORMATION
# OWM2A

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi$$

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\nu\frac{\sigma_x}{E}$$

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$1 \; [ \quad \sigma_x + \sigma_y)]$$

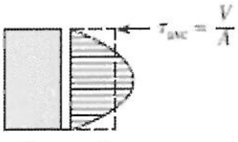
$$E = 2G(1 + \nu)$$

$$\sigma_x = -\frac{My}{I} \qquad I = \int y^2 dA$$

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

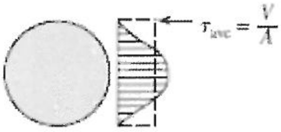
$$Q = \int_{y_1}^{c} y\,dA = \bar{y}' A'$$

$$\tau = \frac{VQ}{Ib}$$



Rectangular

$$\tau_{max} = \frac{3V}{2A}$$




$$\tau_{max} = \frac{4V}{3A}$$

$$\theta = \frac{Tl}{GJ} \qquad \tau_{max} = \frac{Tr}{J}$$

$$J = \frac{\pi d^4}{32} \qquad \theta = \frac{Tl}{\beta bc^3 G}$$

$$\tau_{max} = \frac{T}{\alpha bc^2} \doteq \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right)$$

$$T = \int \tau t r\,ds = (\tau t)\int r\,ds = \tau t(2A_m) = 2A_m t\tau$$

$$\tau = \frac{T}{2A_m t} \qquad \theta_1 = \frac{TL_m}{4GA_m^2 t}$$

$$\tau = G\theta_1 c = \frac{3T}{Lc^2}$$

$$K_t = \frac{\sigma_{max}}{} \qquad K_{ts} = \frac{\tau_{max}}{}$$

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2(p_o - p_i)/r^2}{r_o^2 - r_i^2} \qquad \sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2(p_o - p_i)/r^2}{r_o^2 - r^2} \qquad \sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

$$(\sigma_t)_{av} = \frac{pd_i}{2t} \qquad (\sigma_t)_{max} = \frac{p(d_i + t)}{2t} \qquad \sigma_l = \frac{pd_i}{4t}$$

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right)$$

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]}$$

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right]$$

$$\sigma = -\epsilon E = -\alpha(\Delta T)E \qquad \sigma = -\frac{\alpha(\Delta T)E}{1 - \nu}$$

| $b/c$ | 1.00 | 1.50 | 1.75 | 2.00 | 2.50 | 3.00 | 4.00 | 6.00 | 8.00 | 10 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.208 | 0.231 | 0.239 | 0.246 | 0.258 | 0.267 | 0.282 | 0.299 | 0.307 | 0.313 | 0.333 |
| $\beta$ | 0.141 | 0.196 | 0.214 | 0.228 | 0.249 | 0.263 | 0.281 | 0.299 | 0.307 | 0.313 | 0.333 |

## ADDITIONAL INFORMATION (OWM2A) (cntd.)

$k = \dfrac{F}{y}$ $\qquad \dfrac{1}{\rho} = \dfrac{M}{EI}$ $\qquad U = \dfrac{F^2 l}{2AE}$ $\qquad U = \dfrac{F^2 l}{2AG}$ $\qquad U = \dfrac{CV^2 l}{2AG}$

$\delta = \dfrac{Fl}{AE}$ $\qquad \dfrac{q}{EI} = \dfrac{d^4 y}{dx^4}$ $\qquad U = \displaystyle\int \dfrac{F^2}{2AE}\,dx$ $\qquad U = \displaystyle\int \dfrac{F^2}{2AG}\,dx$ $\qquad U = \displaystyle\int \dfrac{CV^2}{2AG}\,dx$

$k = \dfrac{AE}{l}$ $\qquad \dfrac{V}{EI} = \dfrac{d^3 y}{dx^3}$ $\qquad U = \dfrac{T^2 l}{2GJ}$ $\qquad U = \dfrac{M^2 l}{2EI}$

$\theta = \dfrac{Tl}{GJ}$ $\qquad \dfrac{M}{EI} = \dfrac{d^2 y}{dx^2}$ $\qquad U = \displaystyle\int \dfrac{T^2}{2GJ}\,dx$ $\qquad U = \displaystyle\int \dfrac{M^2}{2EI}\,dx$

$k = \dfrac{T}{\theta} = \dfrac{GJ}{l}$ $\qquad \theta = \dfrac{dy}{dx}$ $\qquad P_{cr} = \dfrac{\pi^2 EI}{l^2}$ $\qquad P_{cr} = \dfrac{C\pi^2 EI}{l^2}$ $\qquad \dfrac{P_{cr}}{A} = \dfrac{C\pi^2 E}{(l/k)^2}$

$y = f(x)$ $\qquad \sigma_c = \dfrac{P}{A} + \dfrac{Mc}{I} = \dfrac{P}{A} + \dfrac{PecA}{IA} = \dfrac{P}{A}\left(1 + \dfrac{ec}{k^2}\right)$

$\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2} \geq \dfrac{S_y}{2}$ $\quad$ or $\quad \sigma_1 - \sigma_3 \geq S_y$

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2}$$

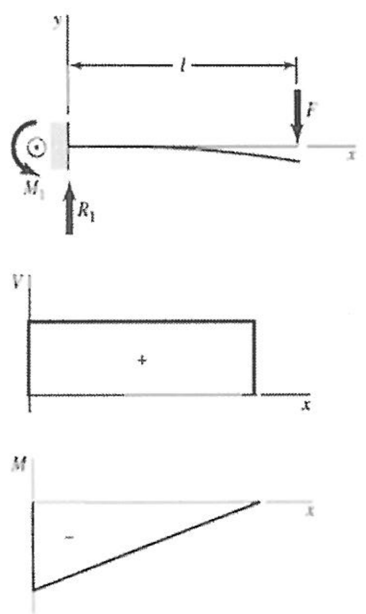
$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2}$$

$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$ $\qquad \sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$\sigma_x = \dfrac{K_I}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)$ $\qquad \tau_{xy} = \dfrac{K_I}{\sqrt{2\pi r}}\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\cos\dfrac{3\theta}{2}$ $\qquad K_I = \beta\sigma\sqrt{\pi a}$

$\sigma_y = \dfrac{K_I}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1 + \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)$ $\qquad \sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases}$

1 Cantilever—end load



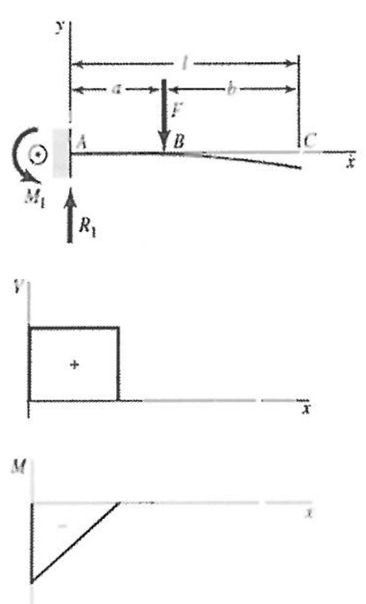

$R_1 = V = F \qquad M_1 = Fl$

$M = F(x - l)$

$$y = \frac{Fx^2}{6EI}(x - 3l)$$

$$y_{max} = -\frac{Fl^3}{3EI}$$

2 Cantilever—intermediate load




$R_1 = V = F \qquad M_1 = Fa$

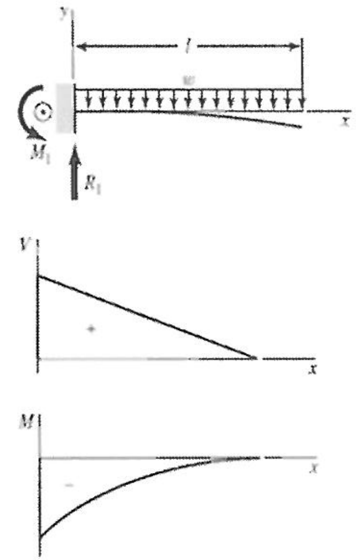
$M_{AB} = F(x - a) \qquad M_{BC} = 0$

$$y_{AB} = \frac{Fx^2}{6EI}(x - 3a)$$

$$y_{BC} = \frac{Fa^2}{6EI}(a - 3x)$$

$$y_{max} = \frac{Fa^2}{6EI}(a - 3l)$$

3 Cantilever—uniform load



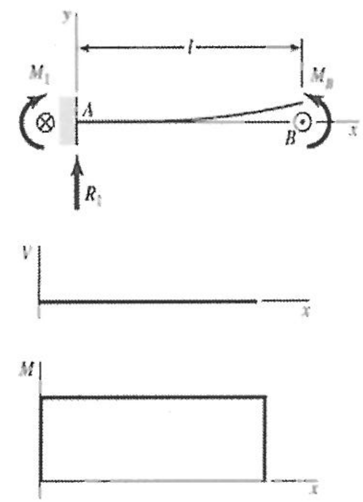

$R_1 = wl \qquad M_1 = \frac{wl^2}{2}$

$V = w(l - x) \qquad M = -\frac{w}{2}(l - x)^2$

$$y = \frac{wx^2}{24EI}(4lx - x^2 - 6l^2)$$

$$y_{max} = -\frac{wl^4}{8EI}$$

4 Cantilever—moment load

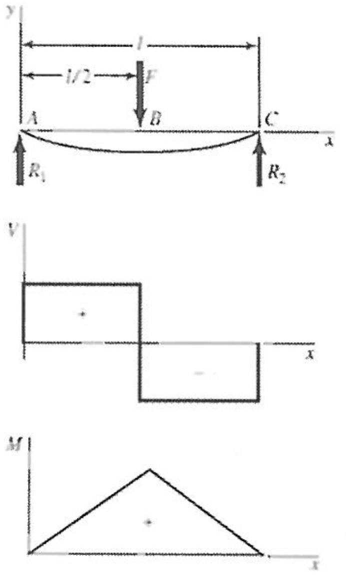



$R_1 = V = 0 \qquad M_1 = M = M_B$

$$y = \frac{M_B x^2}{2EI} \qquad y_{max} = \frac{M_B l^2}{2EI}$$

(continued)

5 Simple supports—center load




$$R_1 = R_2 = \frac{F}{2}$$

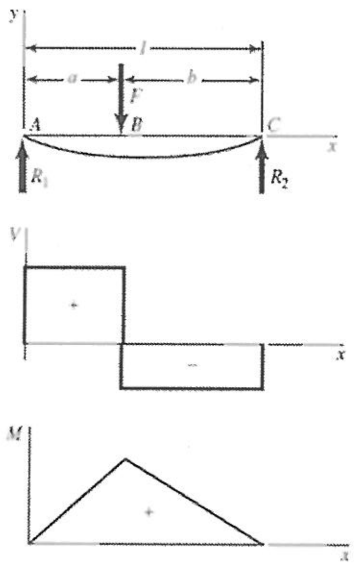
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fx}{2} \qquad M_{BC} = \frac{F}{2}(l - x)$$

$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$y_{max} = -\frac{Fl^3}{48EI}$$

6 Simple supports—intermediate load




$$R_1 = \frac{Fb}{l} \qquad R_2 = \frac{Fa}{l}$$

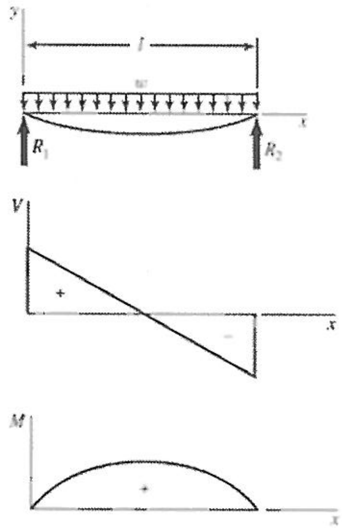
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fbx}{l} \qquad M_{BC} = \frac{Fa}{l}(l - x)$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$y_{BC} = \frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

7 Simple supports—uniform load



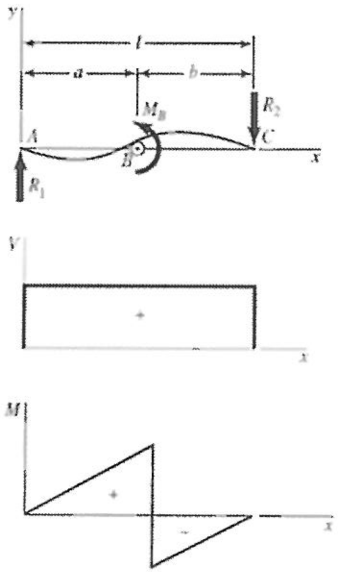

$$R_1 = R_2 = \frac{wl}{2} \qquad V = \frac{wl}{2} - wx$$

$$M = \frac{wx}{2}(l - x)$$

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3)$$

$$y_{max} = -\frac{5wl^4}{384EI}$$

8 Simple supports—moment load




$$R_1 = R_2 = \frac{M_B}{l} \qquad V = \frac{M_B}{l}$$

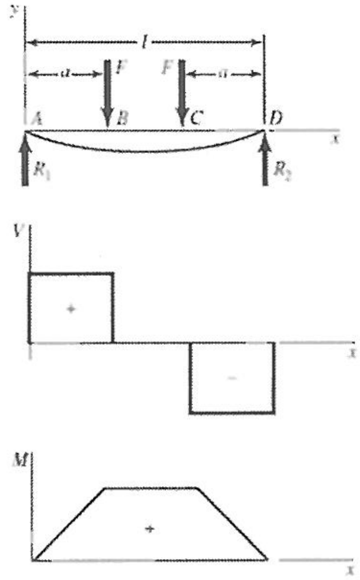
$$M_{AB} = \frac{M_B x}{l} \qquad M_{BC} = \frac{M_B}{l}(x - l)$$

$$y_{AB} = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$y_{BC} = \frac{M_B}{6EIl}[x^3 - 3lx^2 + x(2l^2 + 3a^2) - 3a^2 l]$$

(continued)

9 Simple supports—twin loads



$$R_1 = R_2 = F \qquad V_{AB} = F \qquad V_{BC} = 0$$

$$V_{CD} = -F$$

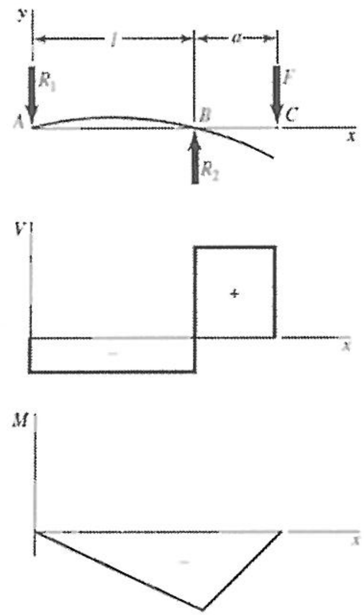
$$M_{AB} = Fx \qquad M_{BC} = Fa \qquad M_{CD} = F(l - x)$$

$$y_{AB} = \frac{Fx}{6EI}(x^2 + 3a^2 - 3la)$$

$$y_{BC} = \frac{Fa}{6EI}(3x^2 + a^2 - 3lx)$$

$$y_{max} = \frac{Fa}{24EI}(4a^2 - 3l^2)$$

10 Simple supports—overhanging load



$$R_1 = \frac{Fa}{l} \qquad R_2 = \frac{F}{l}(l + a)$$

$$V_{AB} = -\frac{Fa}{l} \qquad V_{BC} = F$$

$$M_{AB} = -\frac{Fax}{l} \qquad M_{BC} = F(x - l - a)$$

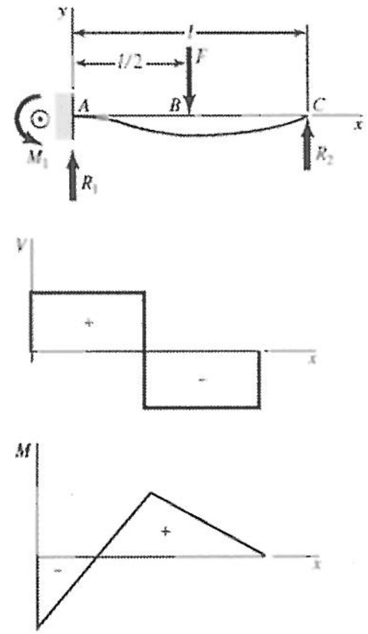
$$y_{AB} = \frac{Fax}{6EIl}(l^2 - x^2)$$

$$y_{BC} = \frac{F(x - l)}{6EI}[(x - l)^2 - a(3x - l)]$$

$$y_c = -\frac{Fa^2}{3EI}(l + a)$$

(continued)

11 One fixed and one simple support—center load



$$R_1 = \frac{11F}{16} \qquad R_2 = \frac{5F}{16} \qquad M_1 = \frac{3Fl}{16}$$

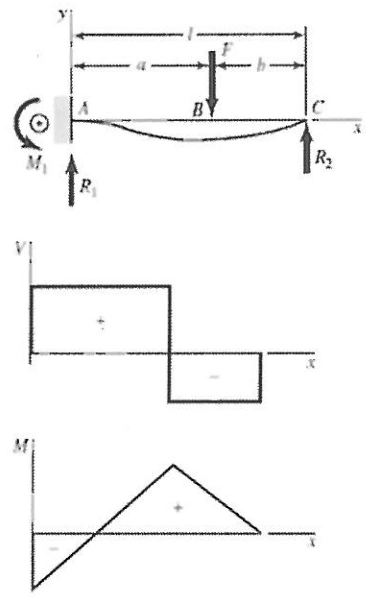
$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{F}{16}(11x - 3l) \qquad M_{BC} = \frac{5F}{16}(l - x)$$

$$y_{AB} = \frac{Fx^2}{96EI}(11x - 9l)$$

$$y_{BC} = \frac{F(l - x)}{96EI}(5x^2 + 2l^2 - 10lx)$$

12 One fixed and one simple support—intermediate load



$$R_1 = \frac{Fb}{2l^3}(3l^2 - b^2) \qquad R_2 = \frac{Fa^2}{2l^3}(3l - a)$$

$$M_1 = \frac{Fb}{2l^2}(l^2 - b^2)$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

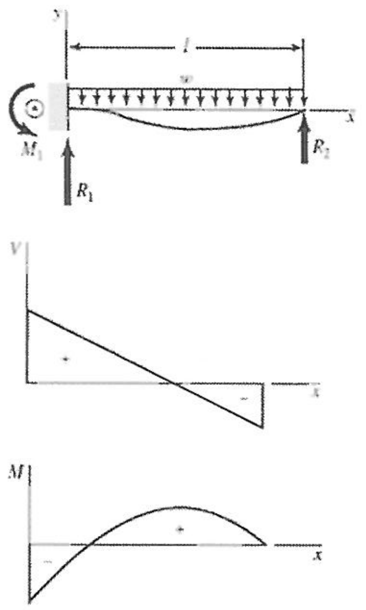
$$M_{AB} = \frac{Fb}{2l^3}[b^2l - l^3 + x(3l^2 - b^2)]$$

$$M_{BC} = \frac{Fa^2}{2l^3}(3l^2 - 3lx - al + ax)$$

$$y_{AB} = \frac{Fbx^2}{12EIl^3}[3l(b^2 - l^2) + x(3l^2 - b^2)]$$

$$y_{BC} = y_{AB} - \frac{F(x - a)^3}{6EI}$$

13 One fixed and one simple support—uniform load



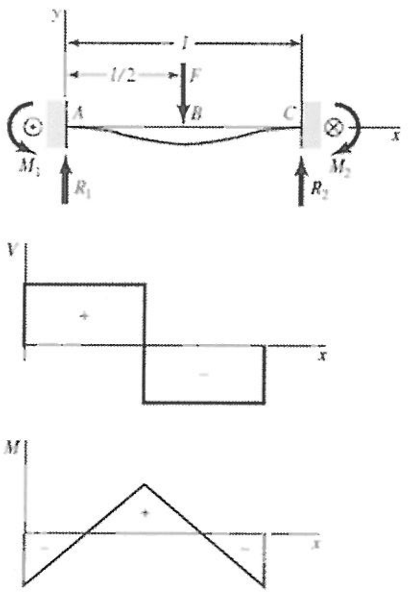
$$R_1 = \frac{5wl}{8} \qquad R_2 = \frac{3wl}{8} \qquad M_1 = \frac{wl^2}{8}$$

$$V = \frac{5wl}{8} - wx$$

$$M = -\frac{w}{8}(4x^2 - 5lx + l^2)$$

$$y = \frac{wx^2}{48EI}(l - x)(2x - 3l)$$

14 Fixed supports—center load



$$R_1 = R_2 = \frac{F}{2} \qquad M_1 = M_2 = \frac{Fl}{8}$$

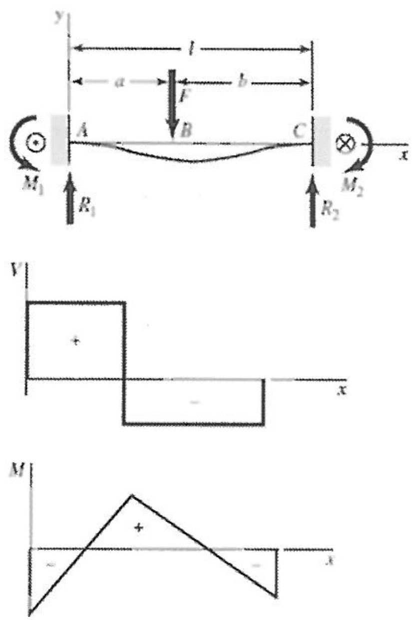
$$V_{AB} = -V_{BC} = \frac{F}{2}$$

$$M_{AB} = \frac{F}{8}(4x - l) \qquad M_{BC} = \frac{F}{8}(3l - 4x)$$

$$y_{AB} = \frac{Fx^2}{48EI}(4x - 3l)$$

$$y_{max} = -\frac{Fl^3}{192EI}$$

15 Fixed supports—intermediate load



$$R_1 = \frac{Fb^2}{l^3}(3a + b) \qquad R_2 = \frac{Fa^2}{l^3}(3b + a)$$

$$M_1 = \frac{Fab^2}{l^2} \qquad M_2 = \frac{Fa^2b}{l^2}$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

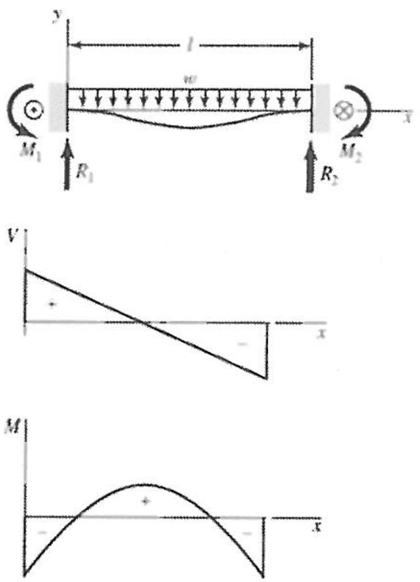
$$M_{AB} = \frac{Fb^2}{l^3}[x(3a + b) - al]$$

$$M_{BC} = M_{AB} - F(x - a)$$

$$y_{AB} = \frac{Fb^2x^2}{6EIl^3}[x(3a + b) - 3al]$$

$$y_{BC} = \frac{Fa^2(l - x)^2}{6EIl^3}[(l - x)(3b + a) - 3bl]$$

16 Fixed supports—uniform load



$$R_1 = R_2 = \frac{wl}{2} \qquad M_1 = M_2 = \frac{wl^2}{12}$$

$$V = \frac{w}{2}(l - 2x)$$

$$M = \frac{w}{12}(6lx - 6x^2 - l^2)$$

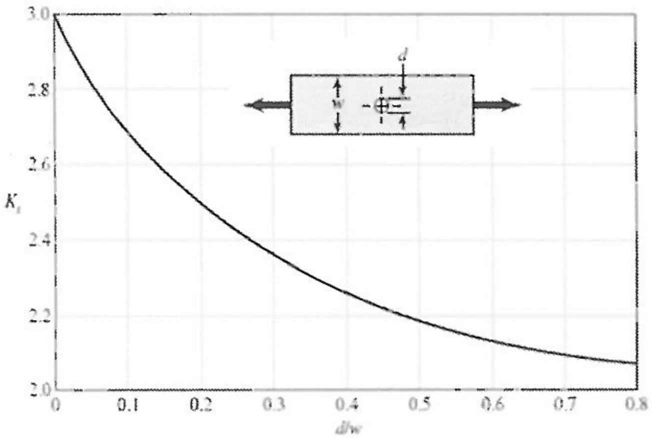
$$y = -\frac{wx^2}{24EI}(l - x)^2$$

$$y_{max} = -\frac{wl^4}{384EI}$$

*(continued)*

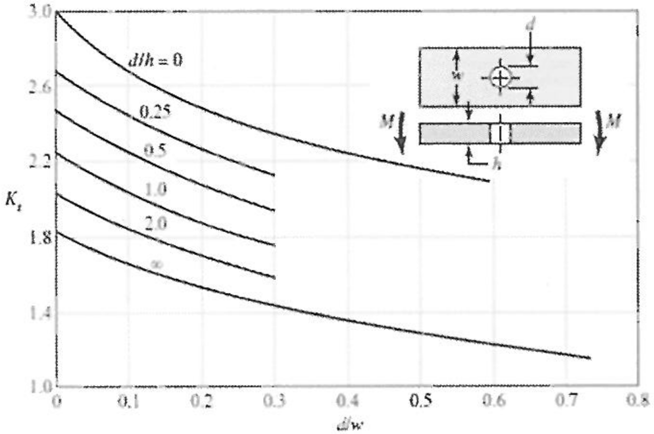
## Figure A-15-1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and $t$ is the thickness.



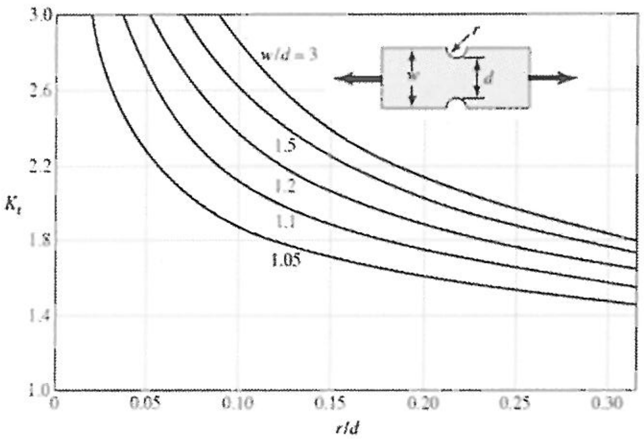

## Figure A-15-2

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.



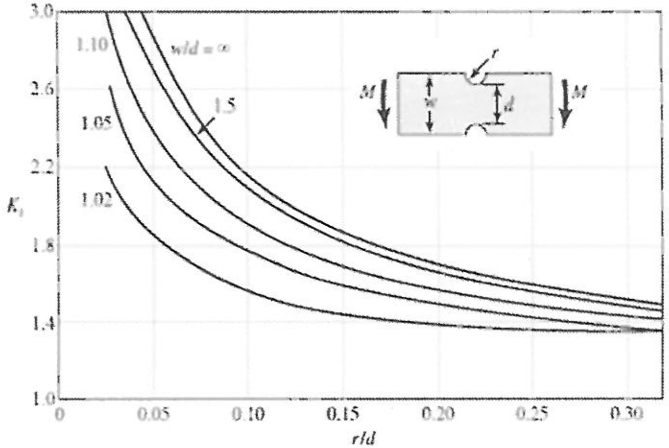

## Figure A-15-3

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and $t$ is the thickness.



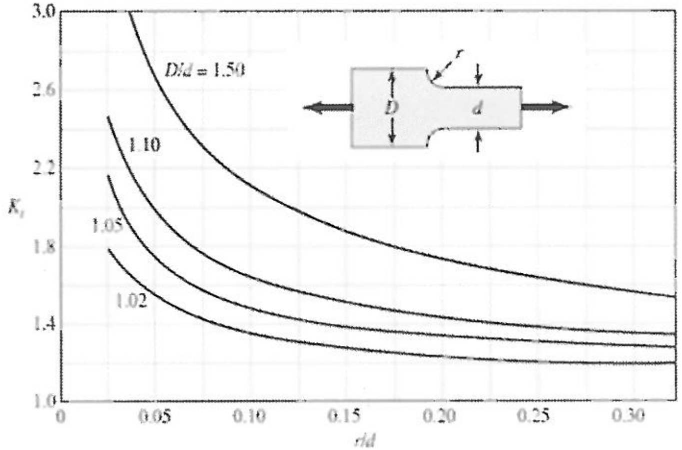

## Figure A-15-4

Notched rectangular bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, and $t$ is the thickness.




## Figure A-15-5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and $t$ is the thickness.




## Figure A-15-6

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, $t$ is the thickness.

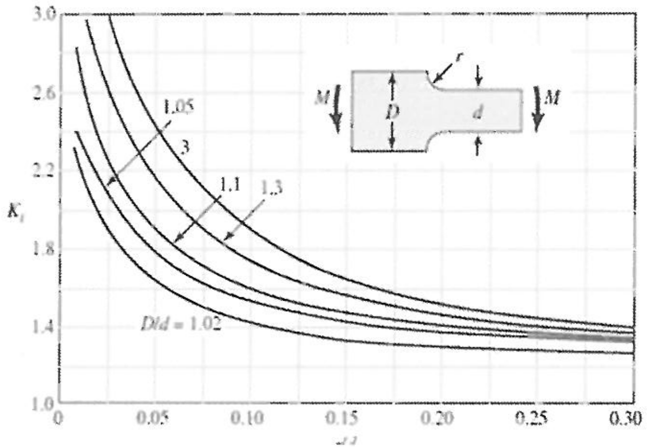


**Figure A–15–7**

Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

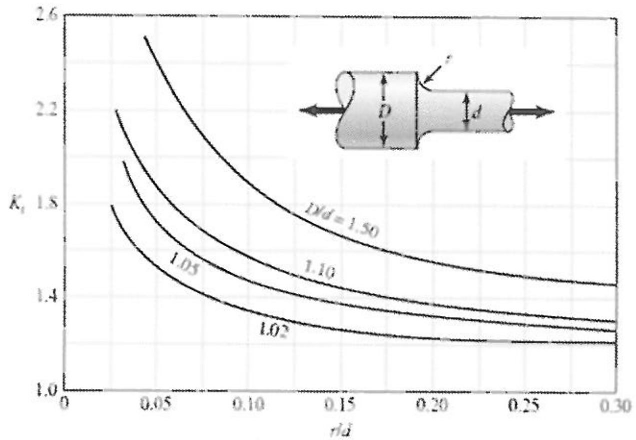



**Figure A–15–8**

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

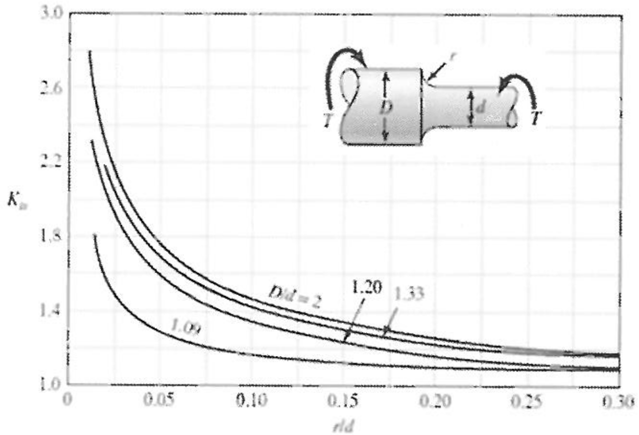



**Figure A–15–9**

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

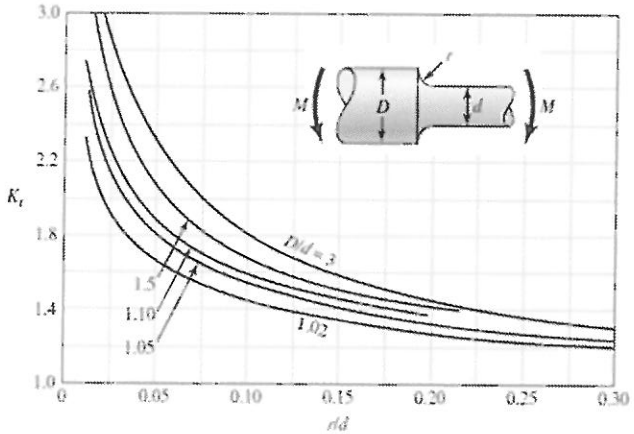



**Figure A–15–10**

Round shaft in torsion with transverse hole.

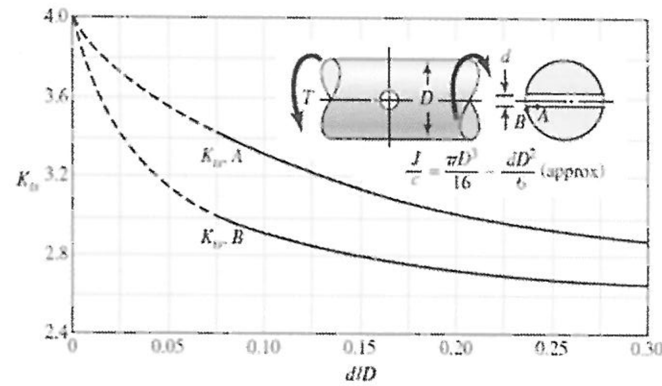



**Figure A–15–11**

Round shaft in bending with a transverse hole. $\sigma_0 = M/[(\pi D^3/32) - (dD^2/6)]$, approximately.

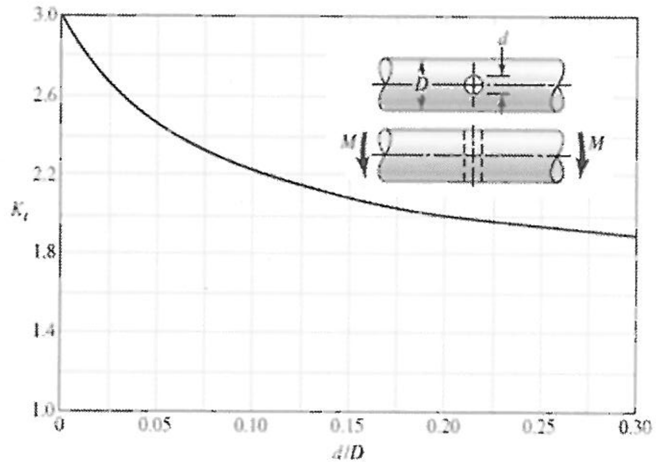



**Figure A–15–12**

Plate loaded in tension by a pin through a hole. $\sigma_0 = F/A$, where $A = (w - d)t$. When clearance exists, increase $K_t$ 35 to 50 percent. (M. M. Frocht and H. N. Hill, "Stress Concentration Factors around a Central Circular Hole in a Plate Loaded through a Pin in Hole," *J. Appl. Mechanics*, vol. 7, no. 1, March 1940, p. A-5.)

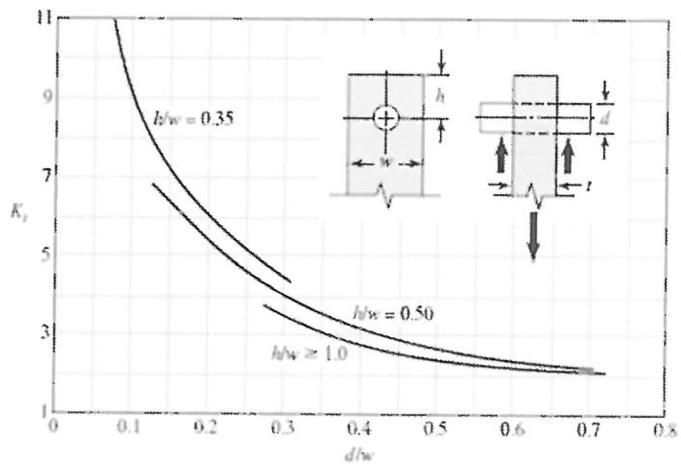


**Figure A–15–13**

Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

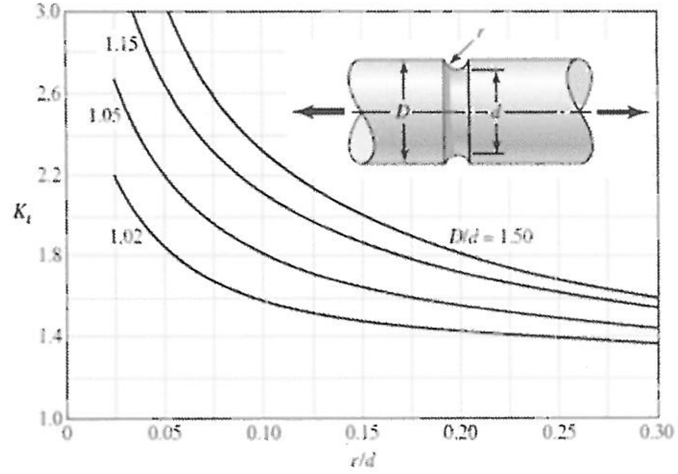



**Figure A–15–14**

Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

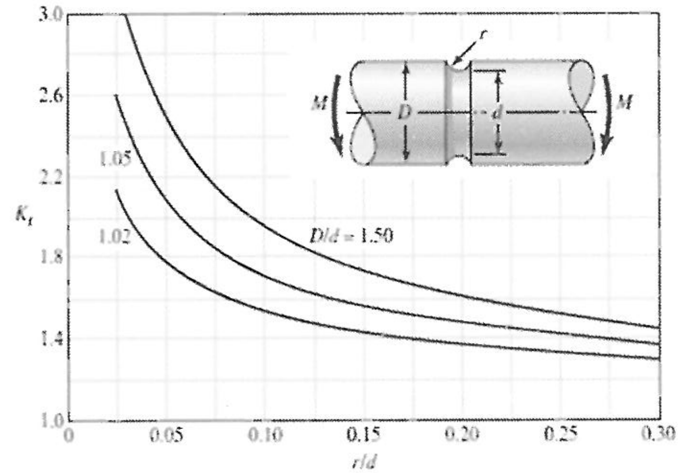
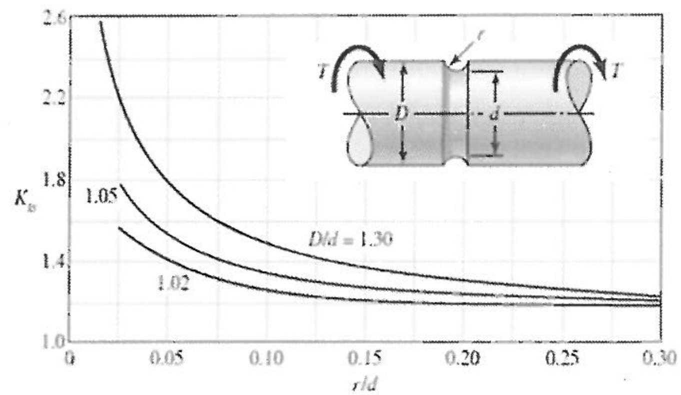



**Figure A–15–15**

Grooved round bar in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



## Mechanisms

### DOF

**Computing DOF - Spatial:** $M=6(L-1)-5J_1-4J_2-3J_3-2J_4-J_5$

M – Mobility/DOF
L – No. of links
$J_1$ – No. of joints capturing 5DOF
$J_2$ – No. of joints capturing 4DOF
$J_3$ – No. of joints capturing 3DOF
$J_4$ – No. of joints capturing 2DOF
$J_5$ – No. of joints capturing 1DOF

**Computing DOF - Planar:**

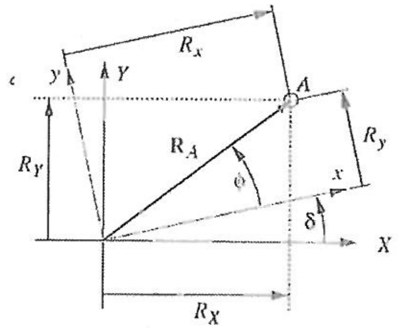
$$M=3L-2J-3G=3(L-1)-2J=3(L-1)-2J_1-J_2$$

M – DOF/Mobility, G –Ground link, L – No. of links,

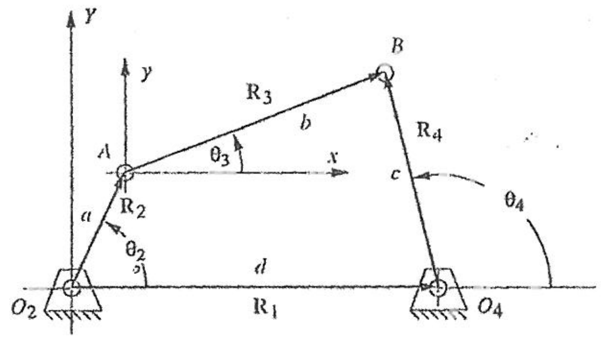
$J_1$ – No. of full joints, $J_2$ – No. of half joints

## Coordinate Transformation

$$\begin{bmatrix} R_X \\ R_Y \end{bmatrix}=\begin{bmatrix} Cos\delta & -Sin\delta \\ Sin\delta & Cos\delta \end{bmatrix}\begin{bmatrix} R_x \\ R_y \end{bmatrix}$$




## Four Bar Linkage




**Position Analysis:**

$$\vec{R_2}+\vec{R_3}-\vec{R_4}-\vec{R_1}=0 \quad , \quad ae^{j\theta_2}+be^{j\theta_3}-ce^{j\theta_4}-de^{j0}=0$$

$$k_1=\frac{d}{a} \quad , \quad k_2=\frac{d}{c} \quad , \quad k_3=\frac{a^2-b^2+c^2+d^2}{2ac} \quad , k_4=\frac{d}{b} \quad , \quad k_5=\frac{c^2-d^2-a^2-b^2}{2ab}$$

$$A=Cos\theta_2-k_1+k_3-k_2Cos\theta_2 \quad , \quad B=-2Sin\theta_2 \quad , \quad C=k_1-k_2Cos\theta_2+k_3-Cos\theta_2$$

$$D=Cos\theta_2-k_1+k_5+k_4Cos\theta_2 \quad , \quad E=-2Sin\theta_2 \quad , \quad F=k_1+k_4Cos\theta_2+k_5-Cos\theta_2$$

$$Tan\frac{\theta_4}{2}=\frac{-B\pm\sqrt{B^2-4AC}}{2A} \quad \therefore \quad \theta_4=2Tan^{-1}\left[\frac{-B\pm\sqrt{B^2-4AC}}{2A}\right]$$

$$Tan\frac{\theta_3}{2}=\frac{-E\pm\sqrt{E^2-4DF}}{2D} \quad \therefore \quad \theta_3=2Tan^{-1}\left[\frac{-E\pm\sqrt{E^2-4DF}}{2D}\right]$$

**Velocity Analysis:**

$$\vec{V}_A+\vec{V}_{BA}-\vec{V}_B=0 \qquad a\,j\,\omega_2e^{j\theta_2}+b\,j\,\omega_3e^{j\theta_3}-c\,j\,\omega_4e^{j\theta_4}=0$$

$$\omega_3=\frac{a\omega_2}{b}\frac{Sin(\theta_4-\theta_2)}{Sin(\theta_3-\theta_4)} \qquad \omega_4=\frac{a\omega_2}{c}\frac{Sin(\theta_2-\theta_3)}{Sin(\theta_4-\theta_3)}$$

$$\vec{V}_A=a\omega_2(-Sin\theta_2+jCos\theta_2) \ , \ \vec{V}_B=c\,\omega_4(-Sin\theta_4+jCos\theta_4) \ , \ \vec{V}_{BA}=b\,\omega_3(-Sin\theta_3+jCos\theta_3)$$

**Acceleration Analysis:**

$$\left(j^2a\omega_2^2e^{j\theta_2}+ja\alpha_2e^{j\theta_2}\right)+\left(j^2b\omega_3^2e^{j\theta_3}+jb\alpha_3e^{j\theta_3}\right)-\left(j^2c\,\omega_4^2e^{j\theta_4}+jc\alpha_4e^{j\theta_4}\right)=0$$

$$A=c\,Sin\theta_4 \qquad B=b\,Sin\theta_3$$

$$C=a\alpha_2Sin\theta_2+a\omega_2^2Cos\theta_2+b\omega_3^2Cos\theta_3-c\omega_4^2Cos\theta_4$$

$$D=c\,Cos\theta_4$$

$$E=b\,Cos\theta_3$$

$$F=a\alpha_2Cos\theta_2-a\omega_2^2Sin\theta_2-b\omega_3^2Sin\theta_3+c\,\omega_4^2Sin\theta_4$$

$$\alpha_3=\frac{C.D-A.F}{A.E-B.D} \qquad \alpha_4=\frac{C.E-B.F}{A.E-B.D}$$

## Mechanical Advantage & Efficiency

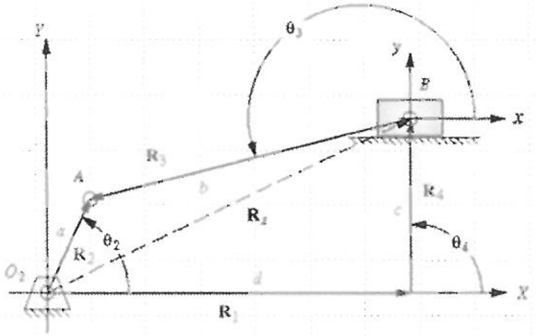
Mechanical Advantage:

$$m_A=\frac{F_{out}}{F_{in}}=\frac{\omega_{in}}{\omega_{out}}\frac{r_{in}}{r_{out}}$$

For 100% efficiency: $P_{out}=P_{in}\Rightarrow\frac{T_{out}}{T_{in}}=\frac{\omega_{in}}{\omega_{out}}$

Efficiency: $\varepsilon=\frac{P_{out}}{P_{in}}$

## Slider Crank



**Position Analysis:**

$$\overrightarrow{R_2} - \overrightarrow{R_3} - \overrightarrow{R_4} - \overrightarrow{R_1} = 0$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j0} = 0$$

$$\theta_{31} = \arcsin\left(\frac{a\sin(\theta_2)-c}{b}\right)$$

$$\theta_{32} = \arcsin\left(-\frac{a\sin(\theta_2)-c}{b}\right)+\pi \quad , \quad \theta_{33} = \arcsin\left(-\frac{a\sin(\theta_2)+c}{b}\right)+\pi|$$

$$d = a\cos(\theta_2) - b\cos(\theta_3)$$

**Velocity Analysis:**

$$\omega_3 = \frac{a}{b}\frac{\cos(\theta_2)}{\cos(\theta_3)}\omega_2 \quad , \quad \dot{d} = -a\omega_2\sin(\theta_2) + b\omega_3\sin(\theta_3)$$

$$\vec{V}_A = a\omega_2(-sin\theta_2 + jcos\theta_2) \ , \ \vec{V}_{AB} = b\omega_3(-sin\theta_3 + jcos\theta_3) \ , \ \overrightarrow{V_B} = \dot{d} = -a\omega_2 sin\theta_2 + b\omega_3 \sin\theta_3$$

**Acceleration Analysis:**

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) - \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \ddot{d} = 0$$

$$\alpha_3 = \frac{a\alpha_2\cos(\theta_2) - a\omega_2^2\sin(\theta_2) + b\omega_3^2\sin(\theta_3)}{b\cos(\theta_3)}$$

$$\ddot{d} = -a\alpha_2\sin(\theta_2) - a\omega_2^2\cos(\theta_2) + b\alpha_3\sin(\theta_3) + b\omega_3^2\cos(\theta_3)$$

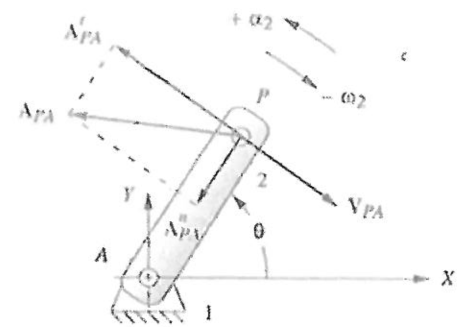
## General Formulae

Angular velocity & acceleration/velocity & acceleration at a point:

$$\omega = \frac{d\theta}{dt} \quad , \quad \alpha = \frac{d\omega}{dt} \quad , \quad V = \frac{dR}{dt} \quad , \quad A = \frac{dV}{dt} \quad , \quad v = r\omega$$

*A* is acceleration vector at a point of interest, *V* is velocity vector at the point of interest and R is the position vector of the point of interest. α is the angular acceleration of a link, *r* is the radius of the link, *ω* is the angular velocity of the rigid link and *v* is the magnitude of velocity.

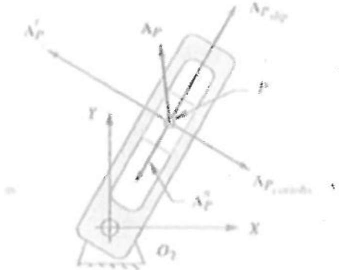
**Expressions for 1-DOF link:**

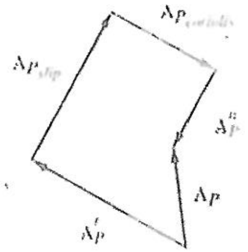


$$R_{PA} = pe^{j\theta} \qquad V_{PA} = \frac{dR_{PA}}{dt} = pje^{j\theta}\frac{d\theta}{dt} = p\omega je^{j\theta}$$

$$A_{PA} = \frac{dV_{PA}}{dt} = pj\frac{d\omega}{dt}e^{j\theta} + pj\omega j\frac{d\theta}{dt}e^{j\theta} = pj\alpha e^{j\theta} - p\omega^2 e^{j\theta}$$

**Coriolis Acceleration:**





$$R_P = pe^{j\theta_2} \quad , \quad V_P = pj\omega_2 e^{j\theta_2} + \dot{p}e^{j\theta_2} \quad , \quad V_P = V_{P_{trans}} + V_{P_{slip}}$$

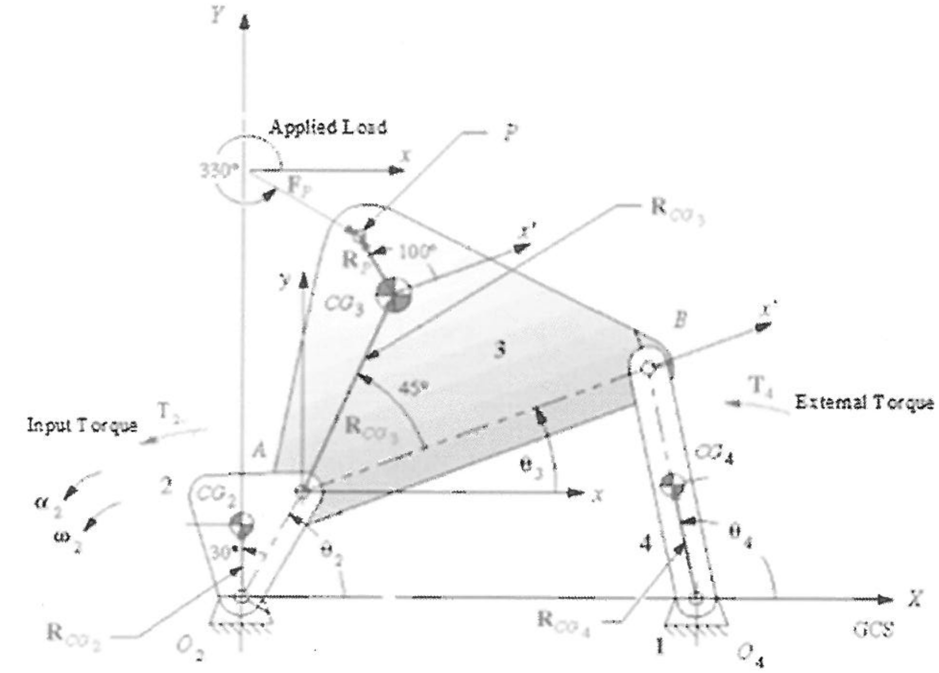
$$A_P = p\alpha_2 je^{j\theta_2} - p\omega_2^2 e^{j\theta_2} + 2\dot{p}j\omega_2 e^{j\theta_2} + \ddot{p}e^{j\theta_2} = A_{P_{tangential}} + A_{P_{normal}} + A_{P_{coriolis}} + A_{P_{slip}}$$

# Dynamics

## EOM for a Planar System:

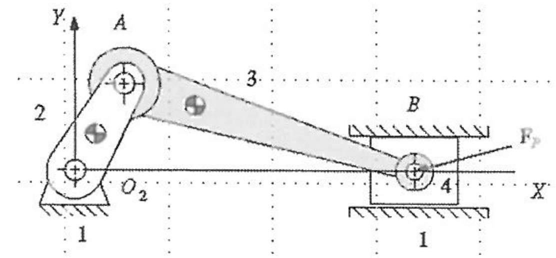
$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum T = I_G \alpha$

## Four-bar linkage




$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{34y} & -R_{34x} & -R_{14y} & R_{14x} & 0 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ F_{14x} \\ F_{14y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2x} \\ m_2 a_{G2y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3x} - F_{Px} \\ m_3 a_{G3y} - F_{Py} \\ I_{G3}\alpha_3 - R_{Px}F_{Py} + R_{Py}F_{Px} \\ m_4 a_{G4x} \\ m_4 a_{G4y} \\ I_{G4}\alpha_4 - T_4 \end{bmatrix}$$

## Slider Crank




$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \pm\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ F_{14y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2x} \\ m_2 a_{G2y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3x} \\ m_3 a_{G3y} \\ I_{G3}\alpha_3 \\ m_4 a_{G4x} - F_{P_x} \\ -F_{P_y} \end{bmatrix}$$