DEPARTMENT OF ECONOMICS AND ECONOMETRICS

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JUNE EXAM:QUANTITATIVE ECONOMICS QTE3AA3

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TIME: 3 Hours

TOTAL MARKS: 100

Instructions:

- Answer all the questions
- Write neatly and legibly
- Justify all your steps with mathematical theory
- Use pen, not pencil

Question 1

1. Suppose $F : U \to R^1$ is a real valued function of n variables, whose domain is a subset of R^n. $x^* \in U$ is a strict max if [2marks]

   (a) x^* is a max and $F(x^*) > F(x)$ for all $x^* \neq x$ in U

   (b) if $F(x*) = F(x)$ for all x not equals to x^* in U

   (c) if $F(x*) \leq F(x)$ for all $x \neq x^*$ in U

   (d) if $F(x*) \geq F(x)$ for all x in U

   (e) if $F(x*) \geq F(x)$ for all x not equals to x^* in U

2. A real valued function f defined on a convex subset U of R^n is concave if for all $x, y \in U$ and for all $0 \leq t \leq 1$ [2marks]

   (a) $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$

   (b) $f'(x)'(a)$

   (c) $f(tx + (1-t)y) = tf(x) + (1-t)f(y)$

   (d) $f(tx + (1-t)y) \geq tf(x) + (1-t)f(y)$

   (e) if f is defined on a non-covex set

3. Let A be a 5×5 matrix the 3rd order principal submatrix of A i.e $|A_3|$ is formed by deleting [2marks]

   (a) 2 rows and 3 colomns

   (b) 2 rows and 2 colomns

   (c) 1 row and 1 colomn

   (d) 3 rows and 1 colomn

   (e) 2 rows and 1 colomn

4. Let C be a subset of R^n. Let $u_1, ..., u_A$ be real valued functions on C. Then, $U = u_1, ..., u_A$ has a Pareto Optimum at $X^* \in C$ if there is no $X \in C$ such that [2marks]

   (a) $u_i(X) \leq u_i(X^*)$ for all i and $u_j(X) \leq u_j(X^*)$ for some j and $U(X) \geq U(X^*)$

   (b) $u_i(X) \leq u_i(X^*)$ for all i

   (c) $u_i(X) \geq u_i(X^*)$ for all i

   (d) $u_i(X) = u_i(X^*)$ for all i

   (e) $u_i(X) \geq u_i(X^*)$ for all i and $u_j(X) \geq u_j(X^*)$ for some j and $U(X) \neq U(X^*)$

5. Which of the following functions are monotonic transformation of R_+ [2marks]

   (a) (ii),(v),(iv) are correct

   (b) (i),(iv),(v) are correct

   (c) (i),(iii),(iv) are correct

   (d) (i),(ii),(v) are correct

   (e) (i),(v),(iii) are correct

   i $z^4 + z^2$

ii $z^4 - z^2$

iii $-z^2 + 4$

iv $\frac{z}{z+1}$

v $\sqrt{z^2 + 4}$

TOTAL MARKS FOR QUESTION 1 $\left[\text{ 10 marks}\right]$

QUESTION 2

1. Given the matrix $A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

   (a) Establish the definiteness of A. [3marks]

   (b) Find the corresponding $Q(X) = X^T AX$. [2marks]

2. Given the quadratic $Q(x_1, x_2, x_3) = x_1^2 - x_3^2 + 4x_1x_2 - 6x_2x_3$ subject to $x_1 + x_2 - x_3 = 0$

   (a) Form the bordered matrix H for the above quadratic and its constraints. [4marks]

3. Given, $f(x, y) = xy(9 - x - y)$ subject to the constraints $x + y \leq 5$, $x \geq 0$ and $y \geq 0$

   (a) Form the Kuhn-Tucker Lagrangian of the above equation. [3marks]

   (b) Compute the first order conditions in terms of Kuhn Tucker Lagrangian. [6marks]
   Hint: Kuhn Tucker first order conditions are $\frac{\partial L}{\partial X_i} \leq 0$, $\frac{\partial L}{\partial \lambda_i} \geq 0$, $\lambda_i \frac{\partial L}{\partial \lambda_i} = 0$, $X_i \frac{\partial L}{\partial X_i} = 0$

TOTAL MARKS FOR QUESTION 2 [20 marks]

QUESTION 3

1. Suppose we have a refinery that must ship finished goods to some storage tanks. Suppose further that there are two pipelines, A and B, to do the shipping. The cost of shipping x units on A is ax^2; the cost of shipping y units on B is by^2, where $a > 0$ and $b > 0$ are given. How can we ship Q units while minimizing cost? Hint: Minimize $ax^2 + by^2$ Subject to $x + y = Q$
   (a) Form the Lagrangian of (1) above. [3marks]
   (b) Solve for the optimal x^* and y^* as well as the multiplier. [4marks]
   (c) Establish that the cost is indeed minimised. [4marks]
   (d) What happens to the cost if Q increases by r percent. [1marks]
2. Given the function $f(x, y) = x^2 - 6xy + 2y^2 + 10x + 2y - 5$
   (a) Find the critical points of x and y. [4marks]
   (b) Find the Hessian of $f(x, y)$ above. [4marks]
   (c) Establish if the critical points are local max, local min or saddle points. [4marks]

TOTAL MARKS FOR QUESTION 3 [26 marks]

Question 4

1. Given the problem: Minimise $f(x, y) = x^2 - 2y$ subject to the constraints $x^2 + y^2 \leq 1$, $x \geq 0$,$y \geq 0$

   (a) Establish that the NDCQ holds, that is, find the rank of the Jacobian of the constraints. $\left[\text{2marks}\right]$

   (b) Form the Lagrangian of the above minimisation problem. $\left[\text{1mark}\right]$

   (c) State all the conditions that corresponds to all the existing multipliers. $\left[\text{4marks}\right]$

   (d) Solve for x, y and multipliers such that the conditions stated above are satisfied.Hint: Also solve for values that lead to contradiction, and clearly state that there is a contradiction if that is the case.
   $\left[\text{4marks}\right]$

2. Suppose the demand function is given by $P = a - bQ$ where P is the price and Q is the quantity. Assume that the parameters $a, b, c \geq 0$ and $c < a$, where c is the cost. The monopolist wants to maximise the profit function $\pi(Q, a, b, c) = (P - c)Q$

   (a) Write the expression for the profit function. Hint:$P = a - bQ$. $\left[\text{2marks}\right]$

   (b) Solve for $Q^*(a, b, c)$. $\left[\text{2marks}\right]$

   (c) Use the Envelope Theorem to come up with an expression $\frac{d\pi}{da}, \frac{d\pi}{db}\ \frac{d\pi}{dc}$. $\left[\text{6marks}\right]$

   (d) Determine the sign of $\frac{d\pi}{dc}$, and its interpretation. $\left[\text{2marks}\right]$

   (e) Plug $Q^*(a, b, c)$ into $\pi(Q, a, b, c)$ to solve for the value function $\pi(a, b, c)$. $\left[\text{2marks}\right]$

TOTAL MARKS FOR QUESTION 4 $\left[\text{ 24 marks}\right]$

Question 5

1. Prove that the product of a homogeneous functions is homogeneous. [4marks]
2. Suppose the utility function for goods x and y is given by $U(X,Y) = \alpha lnx + \beta lny$ subject to $xP_x + yP_y = I$
   (a) Calculate the uncompensated (Marshallian) demand functions for x and y . [4marks]
   (b) How does the demand for x changes when I changes. [2marks]
   (c) How does the demand for y changes when I changes. [2marks]
   (d) Compute own price elasticity, cross price elasticity and income elasticity of Good x and Good y.[6marks]
3. Consider the Cobb-Douglas function $f(x; y) = cx^a yb$ with $a; b; c > 0$ in the first orthant $x > 0; y > 0$. Establish the concavity of $f(x; y)$. [4marks]

TOTAL MARKS FOR QUESTION 2 [22 marks]