UNIVERSITEIT VAN JOHANNESBURG — UNIVERSITY OF JOHANNESBURG

SEMESTER EXAMINATION
8 NOVEMBER 2014

COURSE:	MECHANICAL ENGINEERING	**TIME:**	180 mins
SUBJECT:	THEORY OF MACHINES MKE3B21	**MARKS:**	100

EXTERNAL EXAMINER: Mr JG Benade PrEng J.G. BENADÉ 05/11/2014.
INTERNAL EXAMINER: Dr CR Bester PrEng C.R. BESTER 2014/11/5

- Formula sheets attached
- No books, lecture notes, self-study-, homework- or practical material allowed
- No cellphone use or communication allowed
- Only UJ approved calculators allowed
- Do not write in pencil or red ink
- Answer all the questions in English
- Smoking is prohibited during the exam

Question 1 (20 marks)

Derive an equation for the power specific speed of a hydraulic turbine. Select the required parameters and then the primary parameters. Determine the dimensions of the parameters and apply Dimensional Analysis to obtain the power specific speed as a dimensionless ratio.

Question 2 (24 marks)

A circular cam as shown in Figure 1 has a radius of 35 mm and an eccentricity of 10 mm. The cam rotates at 6000 RPM and is in contact with a flat-ended follower.

Calculate the maxima of the follower displacement, -velocity and -acceleration during one revolution of the cam, from first principles.

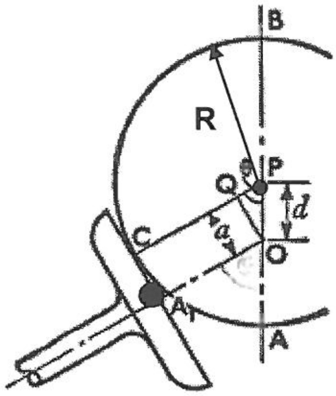

Figure 1: Circular cam with flat-ended follower

Question 3 **(25)**

The torque of an engine vs crank angle is given by:

$$T(\theta) = 2100\sin\theta + 900\sin 2\theta \quad \text{for} \quad 0 \le \theta \le \pi$$
$$T(\theta) = 375\sin\theta \quad \text{for} \quad \pi \le \theta \le 2\pi$$

This is repeated for every revolution of the engine.

The resisting torque is constant and the speed is 850 RPM. The total moment of inertia of the rotating parts of the engine and the driven member is 270 kgm^2.

Determine:

(i) The work done by the engine per revolution (5)
(ii) The engine power (3)
(iii) The resisting torque (3)
(iv) The maximum instantaneous angular acceleration of the engine and the value of θ where it occurs (14)

Question 4 **(20)**

Consider the epicyclic gear system shown in Figure 2. A is the annulus, L is the arm, S is the sun wheel and P are the planet wheels. The diameter of S is 300 mm and that of P is 100 mm. If S is the input and A is the output, calculate the overall ratio of the gearbox when L runs at 500 RPM and S runs at 800 RPM.

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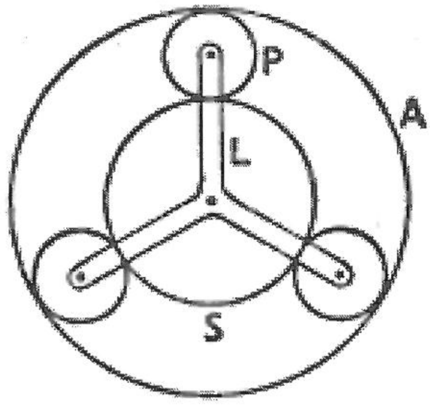

Figure 2: **Epicyclic gear system**

Question 5 **(11 marks)**

Derive an equation for the natural frequencies of a thin linear uniform homogeneous beam with both ends guided / sliding and express the first three natural frequencies in terms of the beam properties

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Various formulae

$$x = (R + r_0)\sec\theta - (R + r_0)$$
$$f = \ddot{x} = \omega^2 (R + r_0)(2\sec^3\theta - \sec\theta)$$
$$x = \{d\cos\phi + (r + r_0)\cos\gamma\} - (R + r_0)$$
$$\sin\gamma = (\sin\phi)/n$$
$$\cos\gamma = \sqrt{1 - \sin^2\gamma}$$
$$v = -\omega d\left\{\sin\phi + \frac{\sin 2\phi}{2\sqrt{n^2 - \sin^2\phi}}\right\}$$
$$f = -\omega^2 d\{1 + 1/n\}$$
$$v = -\omega d\sin\phi$$
$$f = -\omega^2 d$$
$$v = \omega d\sin\theta$$
$$P = mf$$
$$Fa = \{mf + S(x + y)\}(R + r_0)\sec\theta\tan\theta$$
$$Fa = F(\rho - R)\sin\theta$$
$$F_r = mr\Omega^2$$
$$F_{ix} = m_i r_i \Omega^2 \cos\theta_i$$
$$v = \dot{x} = \omega(R + r_0)\sec\theta\tan\theta$$
$$\tan\beta = (d\sin\alpha)/(R + r_0)$$
$$d\sin\phi = (r + r_0)\sin\gamma$$
$$n = (r + r_0)/d$$
$$x = d\left\{\cos\phi + \sqrt{n^2 - \sin^2\phi}\right\} - (R + r_0)$$
$$f = -\omega^2 d\left\{\cos\phi + \frac{\sin^4\phi + n^2\cos 2\phi}{(n^2 - \sin^2\phi)^{3/2}}\right\}$$
$$x = (d\cos\phi + r) - R$$
$$f = -\omega^2 d\cos\phi$$
$$x = d(1 - \cos\theta)$$
$$f = \omega^2 d\cos\theta$$
$$F\cos\theta = mf + S(x + y)$$
$$F = mf + S(x + y)$$
$$Fa = \{mf + S(x + y)\}(\rho - R)\sin\theta$$
$$\ddot{x} \approx a\Omega^2\left(\cos\theta + \frac{\cos 2\theta}{l/r}\right)$$
$$F_{iy} = m_i r_i \Omega^2 \sin\theta_i$$

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$$f = \frac{2gh_L}{V^2}\frac{D}{L}$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

$$Re = \rho DV/\mu$$

$$\ddot{y} \approx a\Omega^2\left(\cos\theta + \frac{\cos 2\theta}{R}\right)$$

$$f = f(Re, \quad \varepsilon/D)$$

$$R = r/a$$

$$\Omega = \dot{\theta}$$

$$\ddot{y}_p \approx a\Omega^2 \cos\theta$$

$$\ddot{y}_s \approx \left(a\Omega^2/R\right)\cos 2\theta$$

$$\theta_S = -\frac{T_A}{T_S}\theta_A$$

$$\theta_P = \frac{T_A}{T_P}\theta_A$$

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t)$$

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \frac{F}{m}$$

$$\omega_n = \sqrt{k/m}$$

$$\zeta = c/(2m\omega_n) = c/\left(2\sqrt{km}\right)$$

$$\tau = \mu\frac{\partial u}{\partial y}$$

$$\mu = \frac{\tau}{\partial u/\partial y}$$

$$f(t) = Fe^{j\omega t}$$

$$y(t) = Ye^{j\omega t}$$

$$\dot{y}(t) = j\omega Ye^{j\omega t}$$

$$\ddot{y}(t) = -\omega^2 Ye^{j\omega t}$$

$$\left|\frac{Y}{F/m\omega_n^2}\right| = \frac{1}{\sqrt{\left(1-r^2\right)^2 + (2\zeta r)^2}}$$

$$\therefore Q = Q(N, \quad D)$$

$$Q = CN^a D^b$$

$$Q = CND^3$$

$$Q/ND^3 = C$$

$$\therefore P = fcn(\rho, \quad N, \quad D)$$

$$\pi = \frac{P}{\rho N^2 D^2}$$

$$H = fcn(g, \quad N, \quad D)$$

$$P = \rho gH$$

$$\pi_1 = \frac{g}{N^2 D}$$

$$\pi_2 = \frac{H}{D}$$

$$\pi = \pi_1\pi_2 = \left(\frac{g}{N^2 D}\right)\left(\frac{H}{D}\right) = \frac{gH}{N^2 D^2}$$

$$\frac{\pi_1}{\pi} = \frac{P}{\rho N^2 D^2}\bigg/\frac{gH}{N^2 D^2} = \frac{P}{\rho gH}$$

$$\therefore \pi = \frac{gH}{N^2 D^2}$$

$$k = n - j$$

$$\pi_1 = \pi_Q = \frac{Q}{ND^3}$$

$$\pi_2 = \pi_P = \frac{P}{\rho N^2 D^2}$$

$$\pi_3 = \pi_{\dot{W}} = \frac{\dot{W}}{\rho N^3 D^5}$$

$$\pi_4 = \frac{\rho ND^2}{\mu}$$

$$\pi_5 = \frac{\varepsilon}{D}$$

$$Nu = Nu(Re, \quad Pr)$$

$$Nu = hx/k$$

$$Re = \rho xV/\mu$$

$$Pr = \mu C_p/k$$

$$Nu = CRe^m Pr^n$$

$$C = 0{,}332; \quad m = 1/2; \quad n = 1/3$$

$$N_s = \frac{(flow\ coefficient)^{1/2}}{(head\ coefficient)^{3/4}}$$

$$N_s = \frac{N\sqrt{Q}}{(gH)^{3/4}}$$

$$\eta = C_H C_Q / C_{\dot{W}}$$

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

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$$C_H = gH/N^2D^2$$

$$C_{\dot{W}} = \dot{W}/\rho N^3 D^5$$

$$\frac{\dot{W}}{\rho N^3 D^5} = f_5\left(\frac{Q}{ND^3}, \quad \frac{\rho ND^2}{\mu}, \quad \frac{\varepsilon}{D}\right)$$

$$C_{\dot{W}} = \frac{\dot{W}}{\rho N^2 D^2} = C_{\dot{W}}\left(C_Q\right)$$

$$\therefore \pi = N^{-c}D^{-c}V^c = \left(\frac{V}{ND}\right)^c$$

$$\frac{\dot{W}}{\rho N^3 D^5} \Big/ \frac{\tau}{\rho N^2 D^5} = \frac{\dot{W}}{\tau N}$$

$$\pi = \pi_F = \frac{F}{\rho N^2 D^4}$$

$$U_2 = r_2\Omega = \left(\frac{D_2}{2}\right)\left(\frac{2\pi N}{60}\right) = \frac{\pi N D_2}{60}$$

$$A_1 = \pi D_1 t, \quad A_2 = \pi D_2 t$$

$$\tau_1 = \underbrace{\underbrace{r_1 \frac{d}{dt}\Bigg(\underbrace{mc_{\theta 1}}_{momentum}\Bigg)}_{force}}_{moment} = r_1\Bigg(\dot{m}c_{\theta 1} + m\underbrace{\dot{c}_{\theta 1}}_{=0}\Bigg)$$

$$w_p = U_2 c_{\theta 2} - U_1 c_{\theta 1}$$

$$P_p = \rho\left(U_2 c_{\theta 2} - U_1 c_{\theta 1}\right)$$

$$S = \frac{N\sqrt{Q}}{\left(NPSH_{required}\right)^{3/4}}$$

$$\eta = \frac{Fluid \quad power}{Power \quad input} = \frac{\rho g Q H}{P_{in}} = \frac{\rho g Q H}{T_{in}\Omega}$$

$$H_p = aQ^2 + bQ + c$$

$$H_p = -2{,}55.10^6 Q^2 + 63{,}76 \; m \quad water$$

$$U_2 = \left(\frac{D_2}{2}\right)\left(\frac{2\pi N}{60}\right) = \frac{\pi N D_2}{60} = \frac{\pi N D_1}{60} = U_1$$

$$\pi_P = P_t/N^2D^2$$

$$\phi = Q\Big/\frac{\pi}{4}D^2U$$

$$\psi_{st} = P_{st}\Big/\frac{1}{2}\rho U^2$$

$$\eta_{st} = P_{st}Q/\dot{W}_{in}$$

$$N_s = \frac{N\sqrt{Q}}{P_{st}^{3/4}}$$

$$C_Q = Q/ND^3$$

$$\frac{gH}{N^2D^2} = f_1\left(\frac{Q}{ND^3}, \quad \frac{\rho ND^2}{\mu}, \quad \frac{\varepsilon}{D}\right)$$

$$C_H = \frac{gH}{N^2D^2} = C_H\left(C_Q\right)$$

$$\eta = \frac{C_H\left(C_Q\right)C_Q}{C_{\dot{W}}\left(C_Q\right)}$$

$$\pi = \pi_\tau = \frac{\tau}{\rho N^2 D^5}$$

$$\pi = \frac{\pi_\tau}{\pi_P} = \frac{\tau}{\rho N^2 D^5} \Big/ \frac{P}{\rho N^2 D^2} = \frac{\tau}{D^3 P}$$

$$U_1 = r_1\Omega = \left(\frac{D_1}{2}\right)\left(\frac{2\pi N}{60}\right) = \frac{\pi N D_1}{60}$$

$$\dot{m}_1 = \rho A_1 c_{r1} = \rho A_2 c_{r2} = \dot{m}_2$$

$$\rho \pi D_1 t c_{r1} = \rho \pi D_2 t c_{r2}$$

$$H_p = \dot{W}_p/\dot{m}g = \frac{U_2 c_{\theta 2} - U_1 c_{\theta 1}}{g}$$

$$\dot{W}_p = \tau_A \Omega = \dot{m}\left(U_2 c_{\theta 2} - U_1 c_{\theta 1}\right)$$

$$NPSH_{available} \geq NPSH_{required}$$

$$\frac{NPSH_{required\ 2}}{NPSH_{required\ 1}} = \left(\frac{N_2}{N_1}\right)^2$$

$$h_L = f\frac{L_{eq}}{D}\frac{V^2}{2g} = 0{,}02\frac{L_{eq}}{2gD}\left(\frac{Q}{A}\right)^2$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{Q}{A}\right)^2 = f\frac{L}{D}\frac{1}{2g}\frac{Q^2}{\left(\pi D^2/4\right)^2}$$

1 (US gal/min)$^{1/2}$ / (ft)$^{3/4}$ = 1,936.10^{-2} (m^3/s)$^{1/2}$ / (m)$^{3/4}$

$$\pi_Q = Q/ND^3$$

$$\pi_{\dot{W}} = \dot{W}/N^3D^5$$

$$\psi = P_t\Big/\frac{1}{2}\rho U^2$$

$$\eta = P_t Q/\dot{W}_{in}$$

$$\pi_{\dot{W}} = \phi\psi/\eta$$

$$\bar{c} = Q/A$$

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$$P_{dyn} = \frac{1}{2}\rho\left(\bar{c}\right)^2$$

$$P_t = P_{st} + P_{dyn}$$

$$\eta_t = \left(P_t Q/\dot{W}_{in}\right)$$

$$\eta_{st} = \left(P_{st} Q/\dot{W}_{in}\right)$$

$$\Omega_{sp} = \frac{\Omega\sqrt{\dot{W}/\rho}}{\left(gH_E\right)^{5/4}}$$

$$\pi_{sp} = \frac{(power\ coefficient)^{1/2}}{(head\ coefficient)^{5/4}}$$

$$\dot{W} = PQ$$

$$N_{sp} = \frac{N\sqrt{\dot{W}}}{H^{5/4}}$$

$$w_T = U_2 c_{\theta 2} - U_3 c_{\theta 3}$$

$$\dot{W}_T = \dot{m}\left(U_2 c_{\theta 2} - U_3 c_{\theta 3}\right) = \rho Q\left(U_2 c_{\theta 2} - U_3 c_{\theta 3}\right)$$

$$H_p = \dot{W}_T/\dot{m}g = \dot{W}_T/\rho Q g = \frac{U_2 c_{\theta 2} - U_3 c_{\theta 3}}{g}$$

$$P_T = \rho\left(U_2 c_{\theta 2} - U_3 c_{\theta 3}\right)$$

$$H_G = z_R - z_N$$

$$H_E = H_G - H_f = z_R - z_N - H_f$$

$$c_0 = \sqrt{2gH_E}$$

$$K_N = \frac{c_1}{c_0}$$

$$\eta_N = \frac{c_1^2}{2gH_E} = \frac{H_E - \Delta H_N}{H_E}$$

$$\eta_h = \eta_N \eta_R = \frac{c_1^2}{2gH_E}\frac{w_T}{c_1^2/2} = \frac{w_T}{gH_E}$$

$$\eta_o = \eta_N \eta_R \eta_m$$

$$A_j = \frac{Q}{2c_1}$$

$$A_j = \pi\frac{d_j^2}{4}$$

$$P\ \alpha\ \rho N^2 D^2; \quad Q\ \alpha\ ND^3$$

$$\frac{p_a}{\rho g} + H_E = \frac{p_2}{\rho g} + \frac{c_2^2}{2g} + z_2 + \Delta H_N$$

$$\frac{p_a}{\rho g} + H_E = \frac{p_3}{\rho g} + \frac{c_3^2}{2g} + z_3 + \Delta H_N + \Delta H_R + \frac{\dot{W}}{\rho Q g}$$

$$\frac{p_3}{\rho g} + \frac{c_3^2}{2g} + z_3 = \frac{p_a}{\rho g} + \frac{c_4^2}{2g} + \Delta H_{DT}$$

$$\frac{p_2}{\rho g} + \frac{c_2^2}{2g} + z_2 = \frac{p_3}{\rho g} + \frac{c_3^2}{2g} + z_3 + \Delta H_R + \frac{\dot{W}}{\rho Q g}$$

$$\frac{\dot{W}}{\rho Q g} = \frac{p_2 - p_3}{\rho g} + \frac{c_2^2 - c_3^2}{2g} + (z_2 - z_3) - \Delta H_R$$

$$\frac{\dot{W}}{\rho Q g} = \frac{p_{02} - p_{03}}{\rho g} + (z_2 - z_3) - \Delta H_R$$

$$\eta_H = \frac{turbine\ \ work}{g(available\ \ head)} = \frac{w_T}{gH_E}$$

$$\frac{p_3 - p_a}{\rho g} = \frac{c_4^2 - c_3^2}{2g} + \Delta H_{DT} - z_3$$

$$\pi_1 = \eta^{l_1} = \eta$$

$$\pi_2 = \gamma^{k_2} = \gamma$$

$$\pi_3 = P_{02}/P_{01}$$

$$\pi_4 = T_{02}/T_{01}$$

$$\pi_5 = \varepsilon/D$$

$$\pi_6 = ND^2/\nu$$

$$\pi_7 = ND/\sqrt{\gamma R T_{01}}$$

$$\pi_8 = \dot{m}\sqrt{\gamma R T_{01}}/P_{01}D^2$$

$$\eta_C\left(\frac{T_{02}}{T_{01}} - 1\right) = \left(\left(\frac{P_{02}}{P_{01}}\right)^{(\gamma-1)/\gamma} - 1\right)$$

$$\frac{\dot{m}\sqrt{\gamma R T_{01}}}{D^2 P_{01}} \rightarrow \frac{\dot{m}\sqrt{T_{01}}}{P_{01}}; \quad \frac{ND}{\sqrt{\gamma R T_{01}}} \rightarrow \frac{N}{\sqrt{T_{01}}}$$

$$\frac{P_{02}}{P_{01}} = f_1\left(\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}, \quad \frac{N}{\sqrt{T_{01}}}\right) = r_P$$

$$\eta_C = f_2\left(\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}, \quad \frac{N}{\sqrt{T_{01}}}\right)$$

$$\phi = \frac{c}{U} \quad \alpha \quad M_F/M_R$$

$$\psi = \frac{\Delta h_0}{U^2} = \frac{w}{U^2} = \frac{U\left(c_{\theta 2} - c_{\theta_1}\right)}{U^2} = \frac{\Delta c_0}{U}$$

$$R = \frac{h_2 - h_3}{h_{01} - h_{03}} = -\frac{w_{\theta 2} + w_{\theta 3}}{2U}$$

$$\phi = C_1\cos\beta x + C_2\sin\beta x + C_3\cosh\beta x + C_4\sinh\beta x$$

$$\omega_i = \beta_i^2\sqrt{EI/\rho A}$$

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6