UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

	Examiner	Moderator
Paper 1 30 marks		
Paper 2 70 marks		
EM/100		

	Examiner	Moderator
SM		
EM		
FM		

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS
NATIONAL DIPLOMA IN ENGINEERING:
Mechanical

MODULE **MAT3AW3** ENGINEERING MATHEMATICS 3 (Paper 2)
CAMPUS **DFC**

NOVEMBER EXAMINATION 2014

DATE: **SESSION:**

ASSESSOR: **MRS E KIRCHNER**

MODERATOR: **MRS Q VAN DER HOFF**

DURATION: **3 HOURS** **FULL MARKS: 100**

SURNAME AND INITIALS	
STUDENT NUMBER	
CONTACT NUMBER	
LECTURER	

NUMBER OF PAGES: **16 PAGES**

REQUIREMENTS: **MATHEMATICS INFORMATION BOOKLET**

Instructions:

- Please fill in your particulars on the front page.
- Answer all the questions in the space provided.
- Do not write in pencil. Pencil will not be marked.
- You may use the back of each page (i.e. the left-hand side) for **rough work OR to complete a question.**
- **Please indicate rough work as such.**
- Rough work will not be marked.
- One non programmable calculator is permitted.
- Information booklets may be used.
- **PLEASE CHECK THAT YOU HAVE RECEIVED 16 PAGES.**

QUESTION 1

1.1 Determine the following:

$L\left\{e^{t-3}(t+3)H(t-3)\right\}$ (3)

1.2 Use the **Laplace transform** to solve the given differential equations, subject to the indicated initial conditions:

1.2.1 $y'' - 2y' + 10y = 10$ $y(0) = 0 \quad ; y'(0) = 1$ (11)

1.2.2 $x'' + 9x = \cos 3t - \cos 3(t-\pi)H(t-\pi)$

$x(0) = 0$ and $x'(0) = 1$ (7)

1.3 It is given that $y' + 0{,}2y = f(t)$, where $f(t) = \begin{cases} 8 & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$

1.3.1 Rewrite the function *f(t)* above in terms of **Heaviside functions.** (2)

1.3.2 Use the **Laplace transform** to solve the given differential equation if $y(0) = 0$. (8)

[31]

QUESTION 2

2.1 A particle is attached to a spring dashpot mechanism. At time $t = 0$, when the particle is at rest, an external force e^{-t} is applied to the system. At time $t = 2$, an additional force $f(t)$ of very short duration is applied to the particle.
The model for this system is represented by the differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = e^{-t} + 3\delta(t-2)$$

Use the **Laplace transform** to find the position (y) of the particle at any time t if $y(0) = 0$ *and* $y'(0) = 0$. (6)

2.2 Given: $f(t) = t - t\,H(t-2) + (t-3)H(t-3)$

Sketch the graph of *f(t)* for $t \geq 0$. (3)

[9]

QUESTION 3

Find the general solution of the following differential equations, using **D-operator methods.**

3.1 $(D^2 + 2D + 1)y = 18 - 10\,e^{-x}$ (7)

3.2 $y'' - 9y = 9t^2 + 50\cos 4t$ (7)

3.3 $\frac{d^2x}{dt^2} + \frac{1}{4}x = 2\sin\frac{1}{2}t$ (4)

3.4 $(D^2 - 3D + 2)x = 24t\,e^{-t}$ (7)

[25]

QUESTION 4

4.1 The motion of a certain forced spring-mass system is modelled by the following differential equation:

$$x'' + \frac{1}{8}x' + x = 8\cos 3t$$

4.1.1 Use **D - operator methods** to determine the position (x) of the mass at any time t . (7)

4.1.2 Use your answer in 4.1.1 to discuss the progress of the motion when $t \to \infty$. (1)

4.1.3 Express *the steady state of the solution* in the form $x = R\sin(\omega t \pm \alpha)$. (2)

4.2 Given the following system of simultaneous differential equations:

$$\frac{di_1}{dt} = 3i_1 - i_2 - 1$$

$$\frac{di_2}{dt} = i_2 + i_1 + 4e^t$$

Use **D – operator methods** to solve for i_2 **ONLY**. (9)

[19]

QUESTION 5

5.1 Find the Fourier expansion of the ODD function *f(x)* given below:

$$f(x) = \begin{cases} x-3 & -2 \le x < 0 \\ x+3 & 0 \le x \le 2 \end{cases} \qquad [f(x) = f(x+4)] \qquad (8)$$

5.2 Determine a **half range Fourier cosine series** to represent the function $f(t) = 2\sin t \cos t \;\; (0 \le t \le \pi)$.
It is given that $a_0 = 0$ and $f(t) = f(t + 2\pi)$. (8)

[16]

TOTAL: 100