

FACULTY OF SCIENCE

	Examiner	Moderator
Paper 1 30 marks		
Paper 2 70 marks		
EM/100		

	Examiner	Moderator
SM		
EM		
FM		

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS NATIONAL DIPLOMA IN ENGINEERING:

Mechanical

MODULE CAMPUS

MAT3AW3 **ENGINEERING MATHEMATICS 3 (Paper 2)**

DFC

NOVEMBER EXAMINATION 2014

DATE:

ASSESSOR:

MODERATOR:

SESSION:

MRS E KIRCHNER

MRS Q VAN DER HOFF

DURATION: 3 HOURS

FULL MARKS: 100

SURNAME AND	
INITIALS	
STUDENT NUMBER	
CONTACT NUMBER	
LECTURER	

NUMBER OF PAGES: 16 PAGES

REQUIREMENTS:

MATHEMATICS INFORMATION BOOKLET

Instructions:

- Please fill in your particulars on the front page.
- Answer all the questions in the space provided.
- Do not write in pencil. Pencil will not be marked.
- You may use the back of each page (i.e. the left-hand side) for **rough** work OR to complete a question.
- Please indicate rough work as such.
- Rough work will not be marked.
- One non programmable calculator is permitted.
- Information booklets may be used.
- PLEASE CHECK THAT YOU HAVE RECEIVED 16 PAGES.

QUESTION 1

1.1 Determine the following: $L \left\{ e^{t-3}(t+3) H(t-3) \right\}$

(3)

1.2 Use the **Laplace transform** to solve the given differential equations, subject to the indicated initial conditions:

1.2.1	y'' - 2y' + 10y = 10	y(0) = 0; $y'(0) = 1$	(11)

1.2.2	$x'' + 9x = cos3t - cos3(t - \pi)H(t - \pi)$ x(0) = 0 and $x'(0) = 1$	(7)

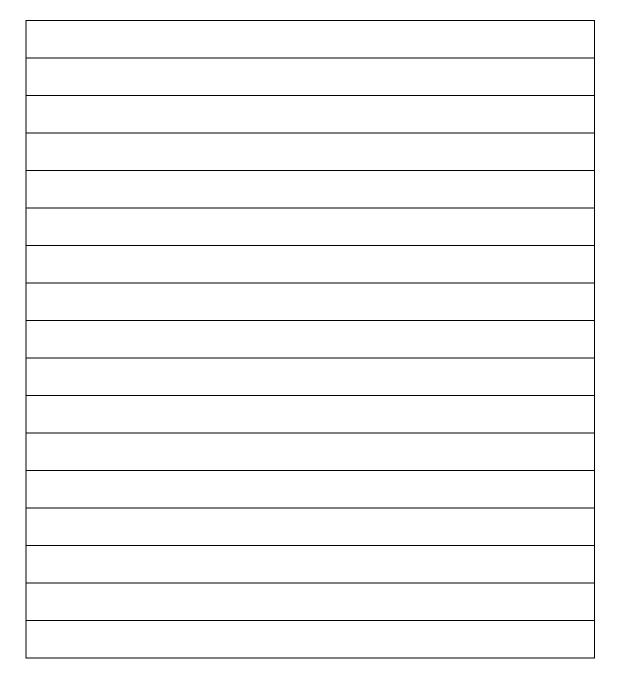
1.3	It is given that $y' + 0.2y = f(t)$, where $f(t) = \begin{cases} 8 & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$	
	1.3.1 Rewrite the function $f(t)$ above in terms of Heaviside functi	ons. (2)
	1.3.2 Use the Laplace transform to solve the given differential equation if $y(0) = 0$.	(2)

2.1 A particle is attached to a spring dashpot mechanism. At time t = 0, when the particle is at rest, an external force e^{-t} is applied to the system. At time t = 2, an additional force f(t) of very short duration is applied to the particle.

The model for this system is represented by the differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = e^{-t} + 3\delta(t-2)$$

Use the **Laplace transform** to find the position (y) of the particle at any time t if y(0)=0 and y'(0)=0. (6)



2.2 Given: f(t) = t - t H(t-2) + (t-3) H(t-3)

Sketch the graph of $f(t)$ for $t \ge 0$.	(3)

[9]

Find the general solution of the following differential equations, using **D-operator methods**.

3.1	$(D^2 + 2D + 1)y = 18 - 10 e^{-x}$	(7)

3.2	$y'' - 9y = 9t^2 + 50\cos 4t$	(7)

$$3.3 \quad \frac{d^{2}x}{dt^{2}} + \frac{1}{4}x = 2\sin\frac{1}{2}t \tag{4}$$

3.4	$(D^2 - 3D + 2)x = 24te^{-t}$	(7)

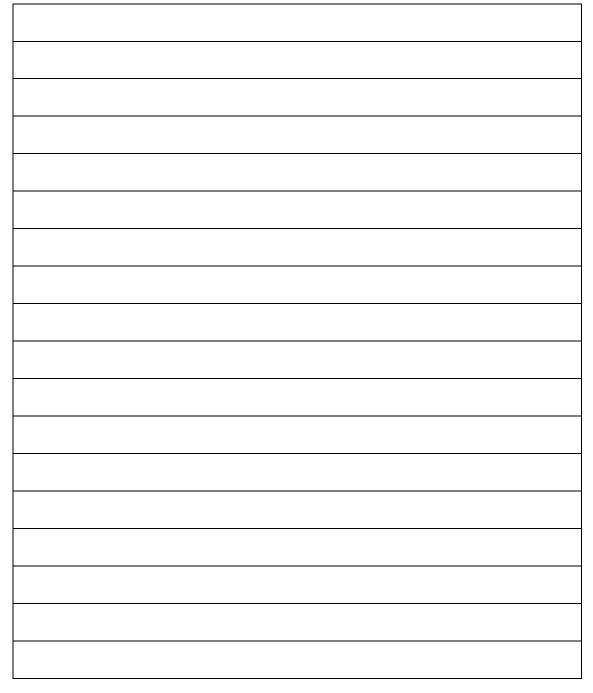
[25]

4.1 The motion of a certain forced spring-mass system is modelled by the following differential equation:

$$x'' + \frac{1}{8}x' + x = 8\cos 3t$$

- 4.1.1 Use **D** operator methods to determine the position (*x*) of the mass at any time *t*. (7)
- 4.1.2 Use your answer in 4.1.1 to discuss the progress of the motion when $t \rightarrow \infty$. (1)
- 4.1.3 Express the steady state of the solution in the form $x = R \sin(\omega t \pm \alpha)$.

(2)



4.2 Given the following system of simultaneous differential equations: $\frac{di_1}{dt} = 3i_1 - i_2 - 1$ $\frac{di_2}{dt} = i_2 + i_1 + 4e^t$ Use **D** – operator methods to solve for *i*₂ ONLY. (9)

5.1 Find the Fourier expansion of the ODD function f(x) given below:

$$f(x) = \begin{cases} x-3 & -2 \le x < 0 \\ x+3 & 0 \le x \le 2 \end{cases} \quad [f(x) = f(x+4)]$$
(8)

5.2 Determine a half range Fourier cosine series to represent the function $f(t) = 2 \sin t \cos t$ $(0 \le t \le \pi)$. It is given that $a_0 = 0$ and $f(t) = f(t + 2\pi)$. (8)

	(-)

[16]

TOTAL: 100