UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

DEPARTMENT OF MATHEMATICS

MODULE	**MAT2B10** MULTIVARIABLE AND VECTOR CALCULUS Science Stream
CAMPUS	**APK**
EXAM	**NOVEMBER 2014**

EXAMINER(S)	MRS C DUNCAN
INTERNAL MODERATOR	MR F SCHULZ
DURATION	2 HOURS
MARKS	50

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 12

INSTRUCTIONS:

1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1 [3]

Describe the level surfaces of the function $f(x,y,z) = x^2 + y^2$. In particular, sketch the level surface corresponding to $k = 4$.

Question 2 [6]

(2.1) State the precise definition of the limit of a function of two variables. (2)

(2.1) Use the precise definition of the limit to show that

$$\lim_{(x,y)\to(0,0)} \frac{2x^3 + y^3}{x^2 + y^2} = 0$$

(4)

Question 3 [3]

If m and n are positive integers and $w = g(x^m y^n)$, then show that

$$nx\frac{\partial w}{\partial x} = my\frac{\partial w}{\partial y}$$

Question 4 [4]

Let $z = 2xy - 3x^2y$ and suppose x is increasing at 2cm/sec. Determine at what rate y must be changing in order for z to be neither increasing nor decreasing at the instant that $x = 3$cm and $y = 1$cm.

[4]

Question 5

The volume of an ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{4}{3}\pi abc$. For a fixed sum $a+b+c$, show that the ellipsoid of maximum volume is a sphere.

Question 6

[5]

By using an appropriate form of symmetry, determine the area of the region that lies within both graphs $r = 1$ and $r = 2\cos\theta$.

Question 7 [5]

Let E be the solid ellipsoid $x^2 + y^2 + 9z^2 = 16$ that lies in the first octant above the plane $z = 1$. Set up a triple integral to express the volume of E in spherical coordinates.

Question 8 [3]

Convert the following iterated integral to cylindrical coordinates:

$$\int_0^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_a^{a+\sqrt{a^2-x^2-y^2}} dz\,dy\,dx$$

[4]

Question 9

Let Ω be the region in the first quadrant bounded by the curves $xy = 1$, $xy = 4$, $y = x$ and $y = 4x$. By using the change of variables $x = \frac{u}{v}$ and $y = v$, determine the area of Ω.

Question 10

[4]

Let $\mathbf{F} = \nabla f$, where $f(x, y) = \sin(x - 2y)$. Find curves C_1 and C_2 that are not closed and satisfy the following:

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 0 \quad \text{and} \quad \int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 1$$

Question 11

[5]

Consider the following vector field:

$$\mathbf{F} = \langle y^3 + 1, 3xy^2 + 1 \rangle$$

(11.1) Is $\int_C \mathbf{F} \cdot d\mathbf{r}$ path-independent? Justify your answer clearly. (2)

(11.2) Show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 2$, where C is the semi-circular path in the first quadrant with starting point $(0, 0)$ and terminal point $(2, 0)$. (3)

[4]

Question 12

Suppose C is a positively oriented, piecewise-smooth, simple closed curve. Verify that the result of Green's Theorem holds for the vector field

$$\mathbf{F}(x, y) = (x - y)\,\mathbf{i} + x\,\mathbf{j}$$

and the region D bounded by the unit circle

$$C : \mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j}, \qquad 0 \leq t \leq 2\pi$$