SM	
EM	
FM	

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING:
ENGINEERING METALLURGY
EXTRACTION METALLURGY

MODULE MAT2AE2
ENGINEERING MATHEMATICS 2
CAMPUS **DFC**

NOVEMBER EXAMINATION

DATE 09/11/2014 **SESSION 08:30 – 11:30**

ASSESSORS **MR EZ MORAPELI**
MR IK LETLHAGE

INTERNAL MODERATOR **MR J BRUYNS**

DURATION 2 HOURS **MARKS 60**

SURNAME AND INITIALS: ______________________________

STUDENT NUMBER: ______________________________

COURSE: ______________________________

LECTURER: ______________________________

CONTACT NO: ______________________________

NUMBER OF PAGES: 15 (VERIFY THAT THE NUMBER OF PAGES IN YOUR SCRIPT IS CORRECT)
INSTRUCTIONS : ANSWER ALL THE QUESTIONS
USE THE BLANK PAGES AT THE BACK TO DO ROUGH WORK
NO PAGES SHOULD BE REMOVED FROM THIS PAPER.
USE ONLY BLUE OR BLACK INK TO WRITE. NO PENCIL.

REQUIREMENTS : INFORMATION BOOKLET
: NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES, WHERE APPLICABLE. USE THE BLANK PAGES FOR ROUGH WORK. USE PAGE 15 TO RE-DO ANY QUESTION YOU MAY HAVE CANCELLED OR IF YOU NEED MORE SPACE FOR WRITING. ANYTHING WRITTEN IN PENCIL WILL NOT BE MARKED.

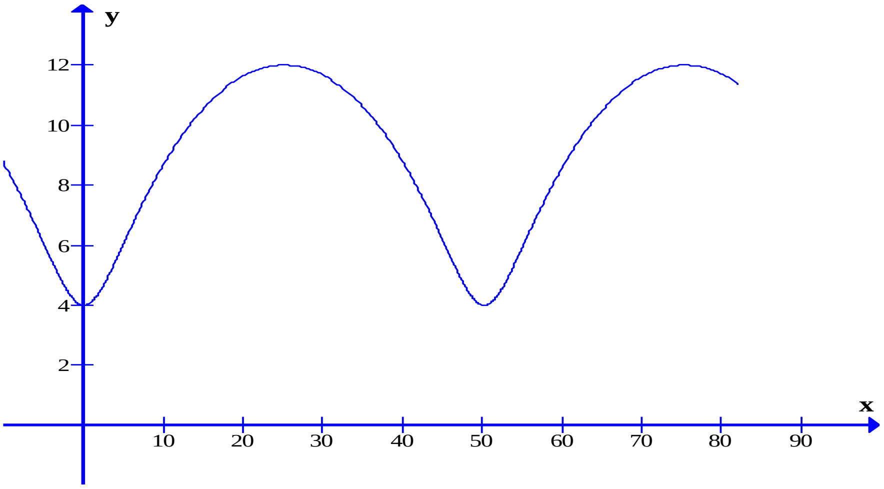
QUESTION 1

1.1 If $y = \sec^{-1}\sqrt{x^2 - 1}$, find $\dfrac{d^2y}{dx^2}$ in its simplest form. (3)

1.2 Given $y = \dfrac{\left(x^2 - 1\right)^{\tan x} \cdot \sqrt[3]{x^4 - 2x}}{e^{x^2}}$, find $\dfrac{dy}{dx}$ and write the answer in its simplest form. (5)

1.3 Use implicit differentiation to find $\frac{dy}{dx}$ in its simplest if $e^{x+y} = xy + x^2$ (3)

1.4 The curtate cycloid is defined by the following parametric equations:
$x = 8\theta - 4\sin\theta$, $y = 8 - 4\cos\theta$ and part of the graph is shown below.



Find $\dfrac{d^2y}{dx^2}$ in its simplest form. (5)

[16]

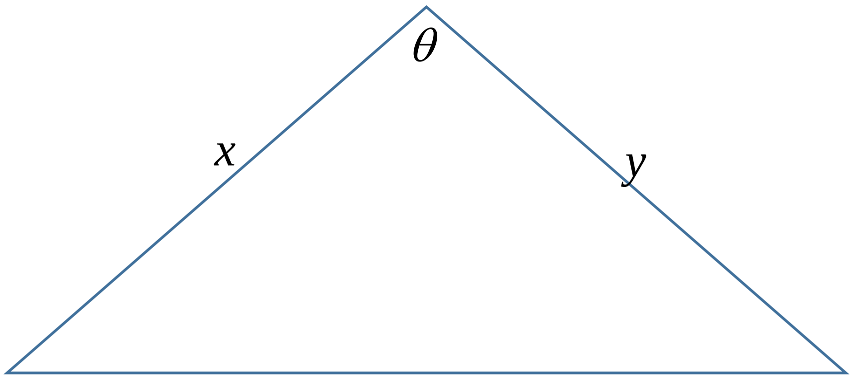
QUESTION 2

2.1 A function $z = f(x, y)$ is said to be **harmonic** if it satisfies **Laplace's equation:**

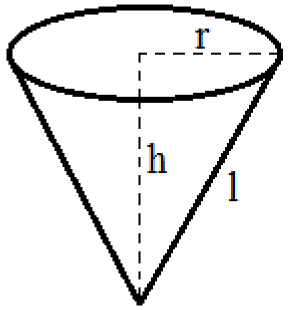
$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

Show that the function $z = \ln\sqrt{x^2 + y^2}$ is harmonic. (4)

2.2 A surveyor wants to calculate the area of a triangular field. She measures two adjacent sides and finds that the one side has length $x = 155m$ and the other side has length $y = 220m$. Each of these measurements has a possible error of $0.3m$. She measures the angle between the two sides and finds that it is $\theta = 30°$, with a possible error of $0.23°$. Find the maximum error in the calculation of the area, A, of the field. The area is given by $A = \frac{1}{2}xy\sin\theta$. (5)



2.3 A container in the shape of a right circular cone is filled with water. Let h denote the height and r the radius of the water level at a given instant. Suppose that the cone has a hole at the bottom and water is dripping through this hole. If the water is leaving the container at a rate of $0.1m^3/\min$ and the height of the water is decreasing at a rate of $0.3m/\min$ at the instant when $h=1m$ and $r=0.75m$, calculate the rate at which the radius is changing. Is this an increase or a decrease? Give reasons for your answer. (5)



The volume of a right circular cone is $V=\frac{1}{3}\pi r^2 h$.

[15]

QUESTION 3

Evaluate the following integrals. Show all the integration steps and, where applicable, give answers correct to 2 decimal places.

3.1 $\int \frac{1}{x+\sqrt[5]{x}}dt$ (2)

3.2 $\int_{0}^{\frac{\pi}{4}} \frac{1}{\cos^2 x\sqrt{9-4\tan^2 x}}dx$ (3)

3.3 $\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$ (6)

3.4 $\int \sin^2 x \cos^2 x dx$ (3)

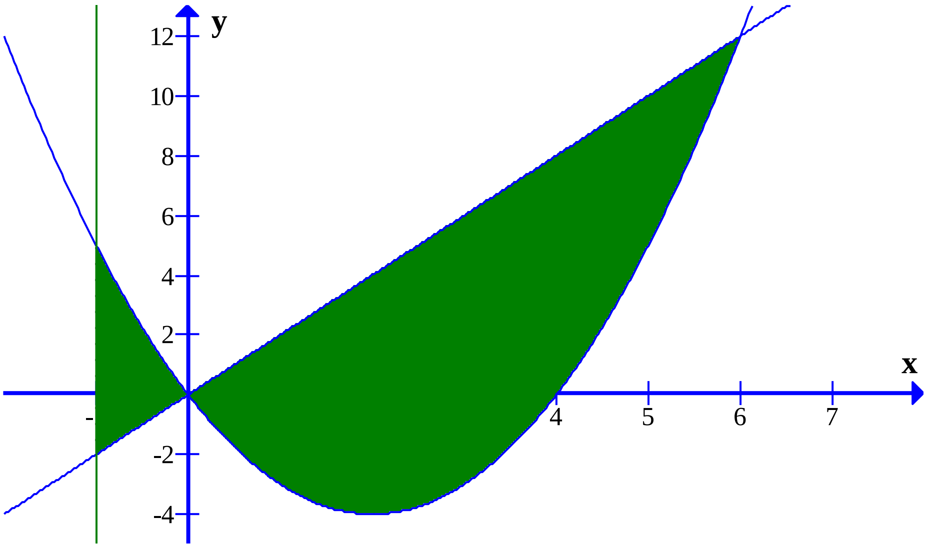
3.5 $\int \arctan\left(\frac{1}{x}\right) dx$ (5)

3.6 $\int \frac{(x+4)}{\sqrt{5+4x-x^2}}\,dx$ (5)

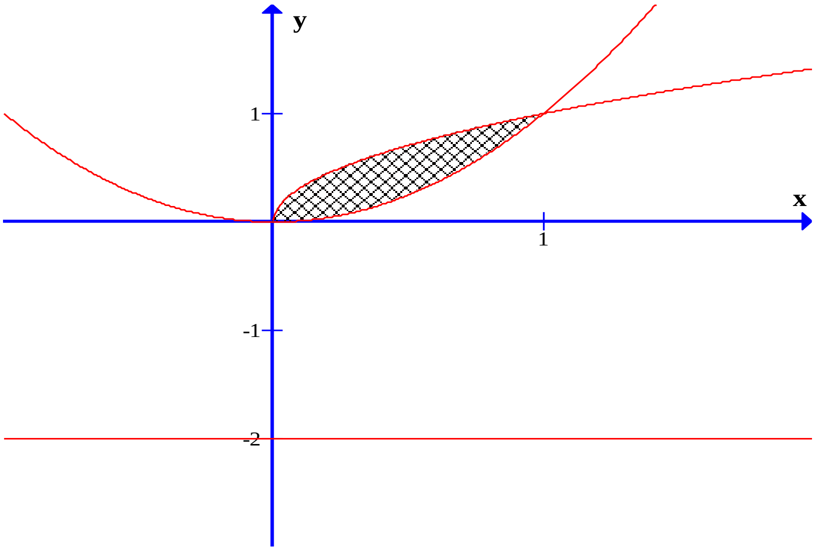
[24]

QUESTION 4

4.1 Calculate the area of the region bounded by the curves $y = 2x$, $y = x^2 - 4x$ and $x = -1$. (4)



4.2 The region bounded by the curves $y = x^2$ and $x = y^2$ is revolved about the line $y = -2$. Calculate the volume of the resulting solid. (4)



[8]

MARKS AVAILABLE : 63

TOTAL MARKS : 60

USE THIS SPACE TO RE-DO ANY QUESTION YOU MAY HAVE CANCELLED