UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE MAT1B01
APPLICATIONS OF CALCULUS

CAMPUS APK

EXAM DECEMBER SUPPLEMENTARY EXAM 2014

DATE 05/12/2014 **SESSION** 08:00 – 10:00

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DURATION 2 HOURS **MARKS** 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NR _____

NUMBER OF PAGES: 1 + 13 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE NEXT TO IT AND INDICATE THIS CLEARLY.

Question 1 [8 marks]

For questions 1.1 - 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	a	b	c	d	e
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1 The function $G(x) = \dfrac{(x+3)(x-2)}{x-1}$ does not satisfy the hypothesis of Rolle's Theorem on the interval $[-3, 2]$ because
(a) $G(x)$ is not continuous on $[-3, 2]$
(b) $G(x)$ is not differentiable on $[-3, 2]$
(c) $G(-3) = G(2) = 0$
(d) $G(0) \neq 0$
(e) None of the above

1.2 $\displaystyle\int_0^1 e^x \, dx =$
(a) e
(b) $\frac{1}{2}e^2 - \frac{1}{2}$
(c) divergent
(d) $e - 1$
(e) None of the above

1.3 $\dfrac{1}{x^3 + x^2}$ can be put in the form:
(a) $\dfrac{A}{x^3} + \dfrac{B}{x^2}$
(b) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}$
(c) $\dfrac{A}{x^2} + \dfrac{B}{x+1}$
(d) $\dfrac{Ax + B}{x^3 + x^2}$
(e) None of the above

1.4 An equation of a circle with centre $(3, -7)$ that goes through the point $(1, 1)$ is
(a) $(x+3)^2 + (y-7)^2 = 52$
(b) $(x-3)^2 + (y+7)^2 = 68$
(c) $(x-3)^2 + (y+7)^2 = 32$
(d) $(x+3)^2 + (y-7)^2 = 40$
(e) None of the above

1.5 Which of the following integrals gives the length of $y = \sqrt{x}$ on the interval [a,b] ?

(a) $\int_a^b \sqrt{x + x^2}\mathrm{d}x$

(b) $\int_a^b \sqrt{1 + \frac{1}{2x}}\mathrm{d}x$

(c) $\int_a^b \sqrt{1 - \frac{1}{2x}}\mathrm{d}x$

(d) $\int_a^b \sqrt{1 + \frac{1}{4x}}\mathrm{d}x$

(e) None of the above

1.6 The conic section defined by $4x^2 + 7y^2 + 32x - 56y + 148 = 0$ is a

(a) ellipse with centre $(4, -4)$ and foci at $(4 \pm \sqrt{3}, -4)$

(b) hyperbola with centre $(-4, 4)$ and foci at $(4, -4 \pm \sqrt{3})$

(c) ellipse with centre $(-4, 4)$ and foci at $(-4 \pm \sqrt{3}, 4)$

(d) hyperbola with centre $(4, -4)$ and foci at $(-4, 4 \pm \sqrt{3})$

(e) None of the above

1.7 Let f be a function of x such that, when $x = c$, f is increasing, concave down, and has an x-intercept at $x = c$. Which of the following is true?

(a) $f(c) < f'(c) < f''(c)$

(b) $f'(c) < f''(c) < f(c)$

(c) $f'(c) < f(c) < f''(c)$

(d) $f''(c) < f(c) < f'(c)$

(e) None of the above

1.8 If $f(x) = \int_2^{\sin x} \sqrt{1 + t^2}\mathrm{d}t$, then $f'(x) =$

(a) $(1 + x^2)^{\frac{3}{2}}$

(b) $(\cos x)\sqrt{1 + \sin^2 x}$

(c) $(\cos x)(1 + \sin^2 x)^{\frac{3}{2}}$

(d) $\sqrt{1 + \sin^2 x}$

(e) None of the above

Question 2 [17 marks]

Evaluate each of the following integrals if it exists, or otherwise show that it doesn't exist:

a) $\int \frac{x - 1}{\sqrt{x^2 - 2x}}\, dx$ [2]

b) $\int \frac{x^5}{\sqrt{1-x^2}}\,dx$ by using trig substitution [4]

c) $\int (\arcsin x)^2 \, dx$ [3]

d) $\int_0^2 \frac{dx}{2-x}$ [2]

e) Given: $f(x) = \dfrac{8}{(x+1)(x^2+3)}$

(i) Find A, B and C such that $f(x)$ can be expressed in the form $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+3}$. [3]

(ii) Hence find the value of a such that $\displaystyle\int_0^1 f(x)\,dx = \ln a + \frac{\pi}{\sqrt{3}a}$. [3]

Question 3 [4 marks]

Prove the Mean Value Theorem, i.e. prove that if f is a function such that
a) f is continuous on a closed interval $[a, b]$,
b) f is differentiable on an open interval (a, b),
then there is a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. [4]

Question 4 [3 marks]

Find the critical numbers, if any, of the function $f(t) = 4t - \sqrt{t^2+1}$. [3]

Question 5 [3 marks]

Sketch the graph of a function that satisfies the following conditions:

1. $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to 6} f(x) = -\infty$, $f(0) = 0$
2. $f'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$ and $(9, \infty)$
3. $f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$
4. $f'(-2) = f'(1) = f'(9) = 0$
5. $f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$
6. $f''(x) < 0$ on $(0, 6)$ and $(6, 12)$

[3]

Question 6 [2 marks]

Find the slant asymptote of the function $f(x) = \dfrac{x^3 + x}{x^2 + x + 1}$. [2]

Question 7 [4 marks]

Calculate the area of the surface obtained by rotating the curve about the x-axis : [4]

$$x = \frac{y^4}{16} + \frac{1}{2y^2} \qquad 1 \leqslant y \leqslant 2$$

Question 8 [6 marks]

Consider the region R in the first quadrant bound by $y = 9 - x^2$, $y = 0$, $x = 0$.

a) Find the area of the region R in terms of y. [3]

b) **Set up, but do not evaluate**, an integral for the volume of a solid rotating the region R about the line $x = 10$ using the **Cylindrical Shells Method**. [3]

Question 11 [2 marks]

Set up, but do not evaluate, an integral for the length of one arch of the cycloid : [2]

$$x = 2(t - \sin t), y = 2(1 - \cos t), \qquad 0 \leq t \leq 2\pi$$

Question 12 [3 marks]

Write down the first three terms in the binomial expansion of $\left(2 - \frac{1}{2x^2}\right)^5$. [3]

b) Identify the focus, vertex and directrix. Determine if the parabola opens to the right or left. [2]

Question 15 [2 marks]

Graph the hyperbola given by the equation $y^2 - 4x^2 - 16 = 0$. [2]

Question 16 [3 marks]

A cube is being compacted into a smaller cube. Given that the volume decreases at a rate of 2 cubic meters per minute, find the rate of change of an edge of the cube when the volume is exactly 27 cubic meters. [3]