UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MODULE PHY002Y

CAMPUS APK

DATE: 4 June 2014	**SESSION 12:30**
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DURATION 2 HOURS	**MARKS 70**

NUMBER OF PAGES: 4 PAGES (including this page and the information sheet)

INSTRUCTIONS: **Answer all the questions.**

Question 1 [23]

(1a) Calculate the average volume per molecule for an ideal gas at room temperature and at atmospheric pressure. Use this to work out the average distance between molecules? [4]

(1b) For an ideal gas of one molecule in a smooth cylinder of volume V, with a piston at one end, show that:

1. the average pressure is given by: $\overline{P} = \frac{mv_x^2}{V}$, where v_x is the horizontal component of the velocity, i.e., in the direction towards the piston. [6]
2. From this derive an expression for the average translational kinetic energy of a large number of identical molecules. [4]

(1c) Give an example of a process in which no heat is added to a system, but its temperature increases. Then give an example of the opposite: a process in which heat is added to a system but its temperature does not change. [2]

(1d) How are the heat capacity at constant pressure and the heat capacity at constant volume related, for an ideal gas? Explain the meaning of all the terms. [3]

(1e) The specific heat capacity of Fattis and Monis pasta is approximately 1.8 J/g.°C. Suppose you toss 340 g of this pasta (at 25 °C) into 1.5 litres of boiling water. What effect does this have on the temperature of the water (before there is time for the stove to provide more heat)? The specific heat capacity of water is 4.186 J/g.°C. [4]

Question 2 [24]

(2a) Suppose that you flip 20 fair coins.

(i) How many possible outcomes (microstates) are there? [1]

(ii) What is the probability of getting the sequence HTHHTHHTTTHTHTTHTHHT (in exactly that order)? [1]

(iii) What is the probability of getting 12 heads and 8 tails (in any order)? [2]

(2c) When a hot object is placed in thermal contact with a cool object, heat flows from the hot object to the cool one, until the two objects are in thermal equilibrium. Explain why this is so. [3]

(2d) Derive a formula for the multiplicity of an Einstein solid containing a large number of oscillators and energy units, in the high-temperature limit. [8]

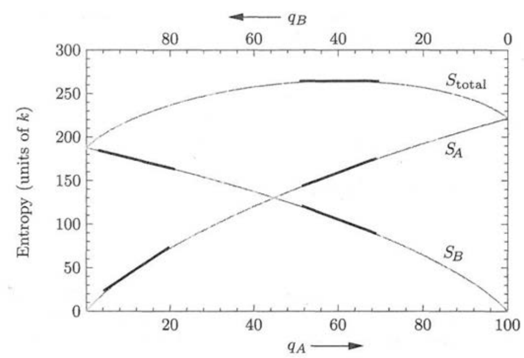
(2e) Show that the multiplicity of an ideal gas is given by

$$\Omega_N = \frac{1}{N!}\frac{V^N}{h^{3N}} \times (area\ of\ momentum\ hypersphere)$$

[9]

Question 3 [23]

(3a) Using the figure below (describing two weakly coupled Einstein oscillators A and B), give three arguments that lead to the definition of temperature as $T \equiv \left(\frac{\partial S}{\partial U}\right)^{-1}$ [6]



(3b) A bit of computer memory is some physical object that can be in two different states, often interpreted as 0 and 1. A byte is 8 bits, a kilobyte is 1024 (=2^{10}) bytes, a megabyte is 1024 kilobytes, and a gigabyte is 1024 megabytes. Suppose that your computer erases or overwrites one gigabyte of memory, keeping no record of the information that was stored. Explain why this process must create a certain amount of entropy, and calculate how much. [3]

(3c) Show that, for an ideal paramagnet, the energy of the system is given by

$$U = N\mu B \tanh\left(\frac{\mu B}{kT}\right),$$

where N is the number of dipoles, μ is their magnetic moment and B is the magnetic field. You should start by applying Stirling's approximation to the entropy. [6]

(3d)

(i) Use the thermodynamic identity to derive the heat capacity formula

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$ [3]

(ii) Now derive a similar formula for C_P, by first writing dH in terms of dS and dP. [5]

END of QUESTIONS

INFORMATION SHEET

$R = 8.31 \dfrac{J}{mol.K}$; $N_A = 6.022 \times 10^{23}$

Boltzmann's constant: $k = \dfrac{R}{N_A} = 1.381 \times 10^{-23} J/K$

Equipartition theorem: $U_{\text{per molecule}} = \dfrac{f}{2} kT$

$C_V = \left(\dfrac{\partial U}{\partial T}\right)_V$

$C_P = \left(\dfrac{\partial U}{\partial T}\right)_P + P\left(\dfrac{\partial V}{\partial T}\right)_P$

$H \equiv U + PV$

Adiabatic compression: $VT^{f/2} = \text{constant}$ and $V^\gamma P = \text{constant}$ where $\gamma = (f+2)/f$

Fourier heat conduction law: $\dfrac{Q}{\Delta t} = -k_t A \dfrac{dT}{dx}$

Two-state system multiplicity: $\Omega(N,n) = \dfrac{N!}{n! \cdot (N-n)!} = \dbinom{N}{n}$

Multiplicity of an Einstein solid: $\Omega(N,q) = \dfrac{(q+N-1)!}{q! \cdot (N-1)!} = \dbinom{q+N-1}{q}$

Stirling's approximation: $N! \approx N^N e^{-N} \sqrt{2\pi N}$ and $\ln N! \approx N \ln N - N$

Approximate form of the Heisenberg uncertainty principle: $(\Delta x)(\Delta p_x) \gtrapprox h$

Sackur-Tetrode equation: $S = Nk\left[\ln\left(\dfrac{V}{N}\left(\dfrac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \dfrac{5}{2}\right]$

$c_V(\text{water}) = 4186$ J/kg.K

$\dfrac{1}{T} \equiv \left(\dfrac{\partial S}{\partial U}\right)_{N,V}$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$; $\cosh x = \frac{1}{2}(e^x + e^{-x})$; $\tanh x = (\sinh x)/(\cosh x)$

Thermodynamic identity: $dU = TdS - PdV$