



FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: MATHEMATICAL ANALYSIS B – MAA00B1
CAMPUS: APK
ASSESSMENT: EXAM

DATE: 11 NOVEMBER 2014

ASSESSORS: MR RJ MAARTENS
MR W VAN REENEN

INTERNAL MODERATOR: MS S RICHARDSON

DURATION: 2 HOURS

MARKS: 90

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

NUMBER OF PAGES: 5 (INCLUDING COVER PAGE)

INSTRUCTIONS: ANSWER ALL THE QUESTIONS IN PEN
ALL GRAPHS MUST BE DRAWN IN PEN
NO PENCIL ALLOWED
NO TIPEX ALLOWED
STATE ALL FORMULAS USED, MARKS ARE GIVEN TO FORMULAS
SHOW ALL THE NECESSARY CALCULATIONS AND STEPS
ALL ANSWERS MUST BE SIMPLIFIED AS FAR AS POSSIBLE
IF NECESSARY ROUND OFF TO TWO DECIMAL PLACES
CALCULATORS ARE ALLOWED

Question 1**[5]**

Determine the following sums:

1.1 $4 + 9 + 14 + 19 + \dots + 254$ [3]

1.2 $\sum_{k=1}^{\infty} 5 \left(\frac{4}{3}\right)^{k-1}$ [2]

Question 2**[3]**

Graph the following plane:

$$4x - 5y - 3z = 60$$

Question 3**[4]**

Determine the inverse of the following matrix using reduction:

$$A = \begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix}$$

Question 4**[4]**

Determine whether the following functions are continuous or discontinuous. Motivate your answer.

4.1 $f(x) = \begin{cases} \sqrt{4x^2 + 20} & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ x^3 - 2 & \text{if } x > 2 \end{cases}$ [2]

4.2 $f(x) = \frac{16}{x}$ [2]

Question 5**[5]**

Determine the derivative of the following function using first principles:

$$f(x) = \frac{5}{1 - 3x}$$

Question 6**[5]**

Prove the following differentiation rule:

$$\text{If } y = \frac{f(x)}{g(x)} \text{ then } y' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}.$$

Question 7**[3]**

Determine the derivative of the following function:

$$f(x) = 3^x + \log_3(x^2 - 4) + e^2$$

Question 8**[8]**

Find an equation of the tangent line to

$$y = (x + 1)(x + 2)^2(x + 3)^2$$

at the point where $x = 0$.

Question 9**[28]**

Given the function

$$f(x) = \frac{6x^2}{(2x + 1)^2}$$

Determine

- 9.1 The intercepts that the graph of $f(x)$ will have with the axes. [1]
- 9.2 All the asymptotes that the graph of $f(x)$ will have. [3]
- 9.3 The domain and range of $f(x)$. [2]
- 9.4 If the graph of $f(x)$ has any symmetries. [2]
- 9.5 The coordinate(s) of the turning point(s) of $f(x)$. [5]
- 9.6 Whether the turning point(s), as in Question 13.5, are maximum or minimum? [4]
- 9.7 The intervals along which the function is increasing and decreasing. [3]
- 9.8 The coordinate(s) of the point(s) of inflection. [5]
- 9.9 The intervals of curvature. [3]

Question 10**[4]**

A company has fixed costs of R8,000 and a marginal cost function of

$$\frac{dc}{dq} = 0.08q^2 - 1.6q + 6.5.$$

Determine the total cost when $q = 25$.

Question 11**[10]**

Determine the following integrals:

11.1

[4]

$$\int \frac{x^2 + x + 1}{\sqrt[3]{x^3 + 1.5x^2 + 3x}} dx$$

11.2

[6]

$$\int_{-1}^2 (12x + 7)(6x^2 + 7x)^{20} dx$$

Question 12**[11]**

The demand equation for a product is

$$q = 400 - p^2$$

and the supply equation is

$$p = \frac{q}{60} + 5.$$

If equilibrium occurs when $q = 300$ and $p = 10$, determine the

12.1 Consumer surplus

[5]

12.1 Producer surplus

[5]

12.1 Total surplus

[1]**End of assessment – Total 90 marks**