FAKULTEIT NATUURWETENSKAPPE
FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

MODULE **APM1A1E**
VECTOR ALGEBRA AND GEOMETRY

CAMPUS **APK**

EXAM **11 NOVEMBER 2014**

DATE 11/11/2014 **SESSION** 08:30 - 11:30

ASSESSORS **DR M MOLELEKOA**
INTERNAL MODERATOR **PROF C M VILLET**

DURATION **2 $\frac{1}{2}$ HOURS** **MARKS** **50**

NUMBER OF PAGES: 3 PAGES

INSTRUCTIONS:
(1) **ANSWER ALL 5 QUESTIONS**
(2) **CALCULATORS MAY BE USED IN WHICH CASE WRITE ANSWERS** *CORRECT* **TO TWO DECIMAL PLACES**
(3) **SHOW ALL NECESSARY WORK**

QUESTION 1 *(Answer this question on a new page)*

(1.1) Sketch the solution space of the following system of inequalities, showing all the important numbers and labels on the coordinate axes:

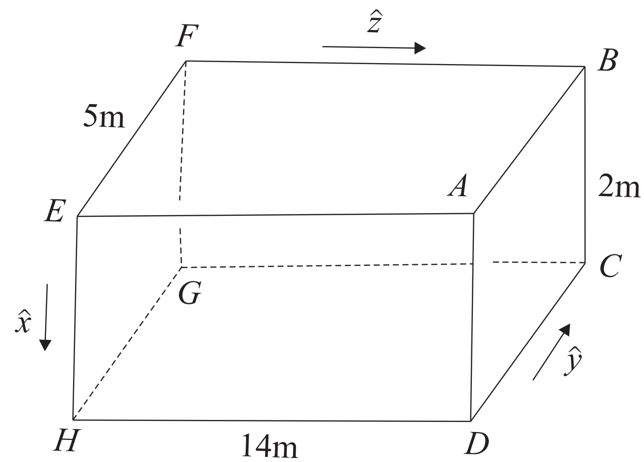
$3x + y < 3, -5x + 3y \leq 15$ and $2x - 3y > 6$. (5)

(1.2) A manufacturer makes two types of a certain product. Each working day he makes x units of type X and y units of type Y of the product. The time it takes him to make each product of type X is two-thirds of the time taken to make one of type Y. If only type Y of the product are made, the manufacturer can make up to thirty of them per day. Write down all the constraints that (x, y) must satisfy. (5)

[10]

QUESTION 2 *(Answer this question on a new page)*

(2.1) Consider the following rectangular box with given dimensions and directions of $\hat{x}, \hat{y}$ and $\hat{z}$:



(a) Determine the direction angles α, β and γ of $\overline{GA}$. (3)

(b) Calculate $\angle EAG$. (2)

(2.2) Consider a 3D XYZ-reference system in which the positive Z-axis is directed upwards. Suppose a point P is 6m above the XY-plane, at an elevation angle of 30° from the origin O. Find the x-coordinate of P if OP makes an angle of 122° with the positive X-axis. (5)

[**HINT:** First find how far P is from the origin.]

[10]

QUESTION 3 *(Answer this question on a new page)*

(3.1) Suppose $AB = BC$ in ΔABC. Use vector methods (only) to prove that $\angle CAB = \angle ACB$. (3)

[**HINT:** You may use suitable choices of $\bar{a}$ and $\bar{b}$.]

(3.2) Suppose $\bar{a} = 8\hat{x} - \hat{y} + 4\hat{z}$ and ΔABC has vertices $A(3, 1, 1)$, $B(-1, 0, 1)$ and $C(3, 2, 4)$. Calculate:

(a) the area of ΔABC and

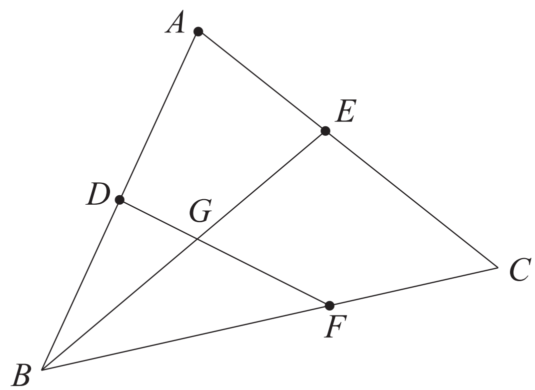
(b) the acute angle which $\bar{a}$ makes with the plane of ΔABC. (7)

[10]

QUESTION 4 *(Answer this question on a new page)*

(4.1) Solve for α if $\overline{PC} = \alpha\overline{PA} + (1-\alpha)\overline{PB}$ and B is a point of AC such that $AB : BC$ is $5 : 3$. (3)

(4.2) Consider the following figure in which $AD : DB = 2 : 3$, $EG : GB = 3 : 4$, $AE : EC = 1 : 3$, $\overline{BD} = 3\bar{a}$ and $\overline{BG} = 4\bar{b}$.



(a) Express $\overline{AE}, \overline{AC}$ and $\overline{BC}$ i.t.o. $\bar{a}$ and $\bar{b}$. (3)

(b) Hence (or otherwise) show that $BF = FC$. (4)

[**HINT:** Solve for $k = \dfrac{BF}{BC}$.]

[10]

QUESTION 5 *(Answer this question on a new page)*

(5.1) Let L be the line which passes through $P(4, 1, -3)$, and which is parallel to $3\hat{x} + 2\hat{y} + 4\hat{z}$.
Find (a) the shortest distance from P to the Y-axis. (2)

(b) the shortest distance between L and the Y-axis. (3)

(5.2) Consider the plane π whose equation is $4x - 7y + 4z = 1$.

(a) Show that $A(1, 1, 1)$ is a point of π. (1)

(b) Find how far the point $P(5, 2, 5)$ is from π. (4)

[10]

FIGURES FOR JUNE EXAMS APM1A2E 2014

Surname & Initials: ....

Student number: ....

Signature: ....

Figure (1.2)

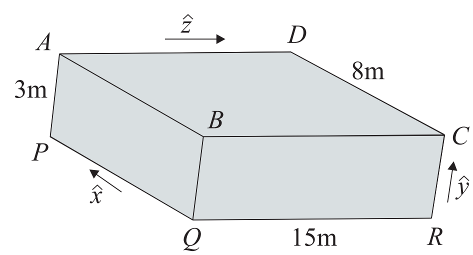


Figure (2.1)

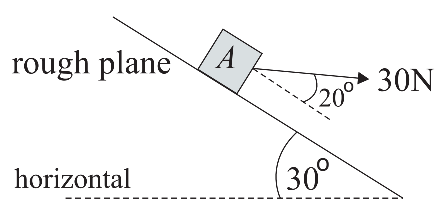


Figure (2.2)

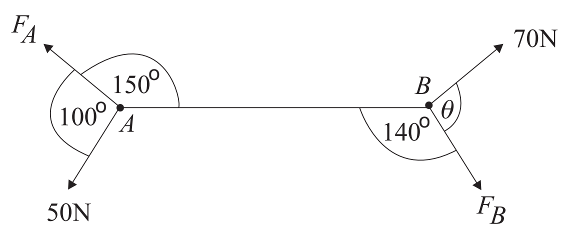


Figure (3.1)

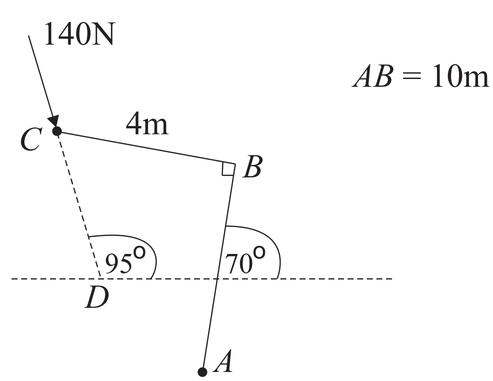


Figure (4.1)

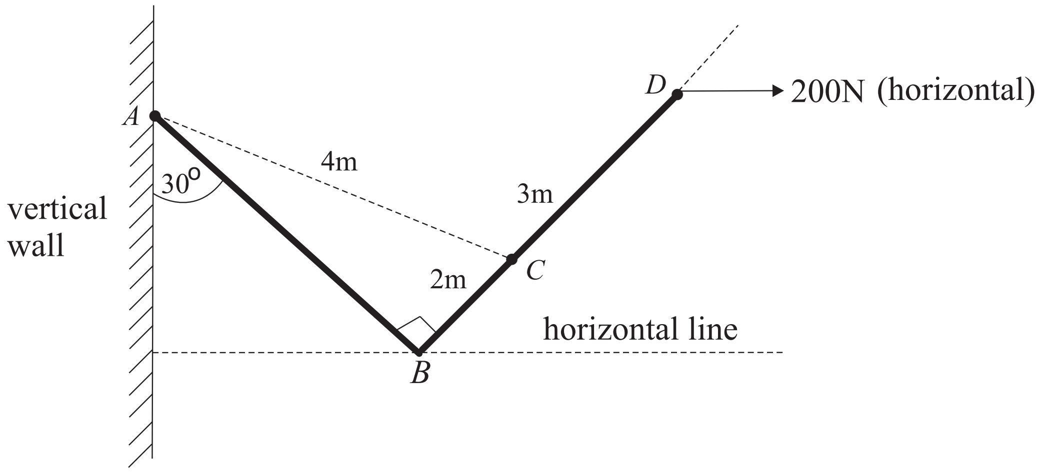


Figure (5.1)

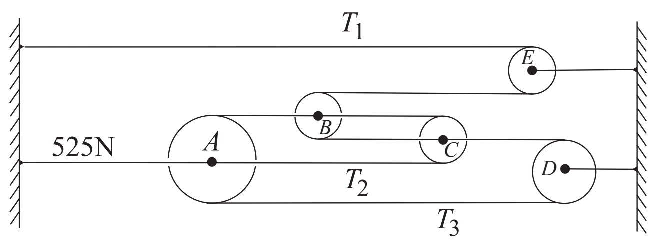


Figure (5.2)