UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

DEPARTMENT OF MATHEMATICS

MODULE	**ASMA2A1** SEQUENCES, SERIES AND VECTOR CALCULUS
CAMPUS	**APK**
EXAM	**NOVEMBER EXAM 2014** PURE

EXAMINER(S)	MR F SCHULZ*
INTERNAL MODERATOR	MRS C DUNCAN
DURATION	2.5 HOURS
MARKS	50

SURNAME AND INITIALS ____________________

STUDENT NUMBER ____________________

CONTACT NUMBER ____________________

NUMBER OF PAGES: 1 + 12

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

ASMA 2A10 Exam Pure

Question 1

State the precise definition of a limit of a sequence. [3]

Question 2

Prove or disprove: If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent. [2]

Question 3

Find the sum of the following series: [4]

$$\sum_{n=0}^{\infty} \frac{n}{(n+1)!}$$

Question 4

Test the following series for convergence or divergence: [8]

(4.1) $\sum_{n=0}^{\infty} \frac{e^n}{n^2}$ (2)

(4.2) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)^n$ (3)

(4.3) $\sum_{n=1}^{\infty} \frac{\cos 2n}{1+n^2}$ (3)

Question 5

State and prove the Root Test. You **do not** have to show that it is inconclusive when $L = 1$. [6]

Question 6

Find the sum of the following series: [3]

$$-e + \frac{e^2}{2!} - \frac{e^3}{3!} + \frac{e^4}{4!} - \cdots.$$

Question 7

Let $\sum b_n$ be an absolutely convergent series and suppose that the sequence $\{a_n\}$ is bounded. Prove that $\sum a_n b_n$ converges. [3]

Question 8

Two particles travel along the space curves

$$\mathbf{r}_1 = \langle t, t^2, 4 \cdot 3^t \rangle$$

and

$$\mathbf{r}_2 = \langle t^2 - 12, 16, t \cdot 3^t \rangle.$$

Will these two particles collide? Show all working. [2]

Question 9

Determine $\mathbf{r}(t)$ if

$$\mathbf{r}'(t) = \left\langle \frac{t \ln(1+t^2)}{1+t^2}, e^{2t}, \sin 2t \right\rangle$$

and $\mathbf{r}(0) = \langle 1, 0, 0 \rangle$. [4]

Question 10

Reparametrize the following curve with respect to arc length measured from the point where $t = 0$ in the direction of increasing t: [4]

$$\mathbf{r}(t) = e^t \sin t \, \mathbf{i} + e^t \, \mathbf{j} + e^t \cos t \, \mathbf{k}.$$

Question 11

If $\mathbf{r}(t) \neq \mathbf{0}$, show that [3]

$$\frac{d}{dt}|\mathbf{r}(t)| = \frac{1}{|\mathbf{r}(t)|}\mathbf{r}(t) \cdot \mathbf{r}'(t).$$

Question 12

Prove that the curvature of a curve C with vector function $\mathbf{r}(t)$ is given by the following formula:[4]

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Question 13

Find the normal component of the acceleration vector if the position vector is given by [4]

$$\mathbf{r}(t) = t\,\mathbf{i} + t^2\mathbf{j} + 3t\,\mathbf{k}.$$