UNIVERSITY OF JOHANNESBURG

PROGRAMME : NATIONAL DIPLOMA
ENGINEERING : INDUSTRIAL

SUBJECT : **OPERATIONS RESEARCH III**

CODE : **BOA 321**

DATE : SUMMER EXAMINATION 2014
10 NOVEMBER 2014
:

DURATION : (SESSION 2) 12:30 - 15:30

TOTAL MARKS : **100**

EXAMINER : **PROF. H. M. CAMPBELL**

MODERATOR : **MR. T. NENZHELELE**

NUMBER OF PAGES : **21**

INSTRUCTIONS :

1. Do **ALL 3 QUESTIONS in PART I, which is COMPULSORY.**
2. **Then answer any other 3 QUESTIONS, from PART II.**
3. Total Marks : 100
4. Full Marks : 100

NOTE: FORMULA SHEET IS ATTACHED.
THIS PAPER CONSISTS OF: 21 TYPED PAGES (including the front page)

DO NOT TURN THE PAGE BEFORE PERMISSION IS GRANTED

(d) 195 to 215. *[2 marks]*

(e) 186.5 to 217. *[2 marks]*

QUESTION 3 **[Total = 16]**

Given the following data and seasonal index:

Month	Sales	
	Year 1	Year 2
Jan	8	8
Feb	7	9
Mar	5	6
Apr	10	11
May	9	12
June	12	16
July	15	20
Aug	20	25
Sept	4	4
Oct	3	2
Nov	8	7
Dec	9	9

(a) Compute the seasonal index using only Year 1 data. *[4 marks]*

(b) Determine the deseasonalized demand values using
Year 2 data, and Year 1's seasonal indices. *[4 marks]*

(c) Determine the trend line on year 2's deseasonalized data. *[4 marks]*

(d) Forecast the sales for the first 3 months of year 3, adjusting for seasonality. *[4 marks]*

PART 2 [Answer any 3 questions from PART II] [48]

QUESTION 4 **[Total = 16]**

Fresh First grocery store faces demand for freshly squeezed orange juice. The daily demand for freshly squeezed pomegranate juice ranges from 0 to 5 gallons. The grocery store offers the juice in a special 1 litre bottle that will be discarded after a day. Each gallon costs R2 to make and is sold for R5. The daily demand for pomegranate juice varies according to the following distribution:

Let X_1 = pounds using Ajax
X_2 = pounds using Bilco
X_3 = pounds using Hobo

QUESTION 8 MULTIPLE CHOICE [Total = 16]

Answer the Multiple Choice questions on the Sheet provided at back of your Examination Script.

1. Operations Research is known as

[1]

A) the science of numerical analysis.
B) the science of sensitivity analysis.
C) the science of better.
D) the science of modeling.
E) None of the above

2. When does $P(A|B) = P(A)$? [1]
A) when A and B are mutually exclusive
B) when A and B are statistically independent
C) when A and B are statistically dependent
D) when A and B are collectively exhaustive
E) when $P(B) = 0$

3. The following is an opportunity loss table. [1]

	States of Nature		
Alternatives	A	B	C
Alternative 1	0	90	85
Alternative 2	50	0	110
Alternative 3	75	80	0

What decision should be made based on the minimax regret criterion?
A) Alternative 1
B) Alternative 2
C) Alternative 3
D) State of Nature A
E) Does not matter

4. The coefficient of determination resulting from a particular regression analysis was 0.85. What was the slope of the regression line? [1]
A) 0.85
B) -0.85
C) 0.922

C) product demand
D) risk assessment
E) inventory costs

10. Which of the following is considered a decision variable in the production mix problem of maximizing profit? [1]
A) the amount of raw material to purchase for production
B) the number of product types to offer
C) the selling price of each product
D) the amount of each product to produce
E) None of the above

11. When using a general LP model for transportation problems, if there are 4 sources and 3 destinations, which of the following statements is true? [1]
A) There are typically 4 decision variables and 3 constraints.
B) There are typically 12 decision variables and 7 constraints.
C) There are typically 7 decision variables and 7 constraints.
D) There are typically 12 decision variables and 12 constraints.
E) There are typically 12 decision variables and 3 constraints.

12. If items being transported must go through an intermediate point before reaching a final destination, then this situation is known as a(n) [1]
A) transshipment problem.
B) assignment problem.
C) transportation problem.
D) intermediate point problem.
E) None of the above

13. In queuing theory, the calling population is another name for [1]
A) the queue size.
B) the servers.
C) the arrivals.
D) the service rate.
E) the market researchers.

14. A suburban specialty restaurant has developed a single drive-thru window. Customers order, pay, and pick up their food at the same window. Arrivals follow a Poisson distribution, while service times follow an exponential distribution.
If the average number of arrivals is 6 per hour and the service rate is 2 every 15 minutes, what is the average number of customers in the system? [1]
A) 0.50
B) 4.00
C) 2.25
D) 3.00
E) None of the above

TOTAL MARKS = [100]

FORMULA SHEET

1. Probability Concepts

Key Equations

(2-1) $0 \le P(\text{event}) \le 1$
A basic statement of probability.

(2-2) $P(A \text{ or } B) = P(A) + P(B)$
Law of addition for mutually exclusive events.

(2-3) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Law of addition for events that are not mutually exclusive.

(2-4) $P(AB) = P(A)P(B)$
Joint probability for independent events.

(2-5) $P(A|B) = \dfrac{P(AB)}{P(B)}$
Conditional probability.

(2-6) $P(AB) = P(A|B)P(B)$
Joint probability for dependent events.

(2-7) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$
Bayes' law in general form.

(2-8) $E(X) = \sum_{i=1}^{n} X_i P(X_i)$
An equation that computes the expected value (mean) of a discrete probability distribution.

(2-9) $\sigma^2 = \text{Variance} = \sum_{i=1}^{n} [X_i - E(X)]^2 P(X_i)$
An equation that computes the variance of a discrete probability distribution.

(2-10) $\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$
An equation that computes the standard deviation from the variance.

(2-11) $\text{Probability of } r \text{ successes in } n \text{ trials} = \dfrac{n!}{r!(n-r)!} p^r q^{n-r}$
A formula that computes probabilities for the binomial probability distribution.

(2-12) $\text{Expected value (mean)} = np$
The expected value of the binomial distribution.

(2-13) $\text{Variance} = np(1 - p)$
The variance of the binomial distribution.

(2-14) $f(X) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
The density function for the normal probabili tribution.

(2-15) $Z = \dfrac{X - \mu}{\sigma}$
An equation that computes the number of standar ations, Z, the point X is from the mean μ.

(2-16) $f(X) = \mu e^{-\mu x}$
The exponential distribution.

(2-17) $\text{Expected value} = \dfrac{1}{\mu}$
The expected value of an exponential distribution

(2-18) $\text{Variance} = \dfrac{1}{\mu^2}$
The variance of an exponential distribution.

(2-19) $P(X \le t) = 1 - e^{-\mu t}$
Formula to find the probability that an expo random variable (X) is less than or equal to time t

(2-20) $P(X) = \dfrac{\lambda^x e^{-\lambda}}{X!}$
The Poisson distribution.

(2-21) $\text{Expected value} = \lambda$
The mean of a Poisson distribution.

(2-22) $\text{Variance} = \lambda$
The variance of a Poisson distribution.

2. Decision Analysis

Key Equations

(5-1) $\text{MAD} = \dfrac{\sum|\text{forecast error}|}{n}$

A measure of overall forecast error called mean absolute deviation.

(5-2) $\text{MSE} = \dfrac{\sum(\text{error})^2}{n}$

A measure of forecast accuracy called mean squared error.

(5-3) $\text{MAPE} = \dfrac{\sum\left|\dfrac{\text{error}}{\text{actual}}\right|}{n}100\%$

A measure of forecast accuracy called mean absolute percent error.

(5-4) $\text{Moving average forecast} = \dfrac{\text{Sum of demands in previous } n \text{ periods}}{n}$

An equation for computing a moving average forecast.

(5-5) $F_{t+1} = \dfrac{Y_t + Y_{t-1} + \cdots + Y_{t-n+1}}{n}$

A mathematical expression for a moving average forecast.

(5-6) $F_{t+1} = \dfrac{\sum(\text{Weight in period } i)(\text{Actual value in period } i)}{\sum(\text{Weights})}$

An equation for computing a weighted moving average forecast.

(5-7) $F_{t+1} = \dfrac{w_1Y_t + w_2Y_{t-1} + \cdots + w_nY_{t-n+1}}{w_1 + w_2 + \cdots + w_n}$

A mathematical expression for a weighted moving average forecast.

(5-8) New forecast = Last period's forecast + α(Last period's actual demand − Last period's forecast)

An equation for computing an exponential smoothing forecast.

(5-9) $F_{t+1} = F_t + \alpha(Y_t - F_t)$

Equation 5-8 rewritten mathematically.

(5-10) $F_{t+1} = FIT_t + \alpha(Y_t - FIT_t)$

Equation to update the smoothed forecast (F_{t+1}) used in the trend adjusted exponential smoothing model.

(5-11) $T_{t+1} = T_t + \beta(F_{t+1} - FIT_t)$

Equation to update the smoothed trend value (T_{t+1}) used in the trend adjusted exponential smoothing model.

(5-12) $FIT_{t+1} = F_{t+1} + T_{t+1}$

Equation to develop forecast including trend (FIT) in the trend adjusted exponential smoothing model.

(5-13) $\text{Tracking signal} = \dfrac{\text{RSFE}}{\text{MAD}} = \dfrac{\sum(\text{forecast error})}{\text{MAD}}$

An equation for monitoring forecasts with a tracking signal.

5. Inventory Models

6. Waiting Lines and Queuing Theory Models

Key Equations

λ = mean number of arrivals per time period

μ = mean number of people or items served per time period

Equations 13-1 through 13-7 describe operating characteristics in the single-channel model that has Poisson arrival and exponential service rates.

(13-1) L = average number of units (customers) in the system

$$= \frac{\lambda}{\mu - \lambda}$$

(13-2) W = average time a unit spends in the system (Waiting time+Service time)

$$= \frac{1}{\mu - \lambda}$$

(13-3) L_q = average number of units in the queue

$$= \frac{\lambda^2}{\mu(\mu - \lambda)}$$

(13-4) W_q = average time a unit spends waiting in the queue

$$= \frac{\lambda}{\mu(\mu - \lambda)}$$

(13-5) ρ = utilization factor for the system $= \frac{\lambda}{\mu}$

(13-6) P_0 = probability of 0 units in the system (that is, the service unit is idle)

$$= 1 - \frac{\lambda}{\mu}$$

(13-7) $P_{n>k}$ = probability of more than k units in the system

$$= \left(\frac{\lambda}{\mu}\right)^{k+1}$$

Equations 13-8 through 13-12 are used for finding the costs of a queuing system.

(13-8) Total service cost $= mC_s$
where
m = number of channels
C_s = service cost (labor cost) of each channel

(13-9) Total waiting cost per time period $= (\lambda W)C_w$
C_w = cost of waiting
Waiting time cost based on time in the system.

(13-10) Total waiting cost per time period $= (\lambda W_q)C_w$
Waiting time cost based on time in the queue.

(13-11) Total cost $= mC_s + \lambda W C_w$
Waiting time cost based on time in the system.

(13-12) Total cost $= mC_s + \lambda W_q C_w$
Waiting time cost based on time in the queue.

Equations 13-13 through 13-18 describe operating characteristics in multichannel models that have Poisson arrival and exponential service rates, where m = the number of open channels.

(13-13) $$P_0 = \frac{1}{\left[\sum_{n=0}^{m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu - \lambda}}$$

for $m\mu > \lambda$

Probability that there are no people or units in the system.

(13-14) $$L = \frac{\lambda\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2}P_0 + \frac{\lambda}{\mu}$$

Average number of people or units in the system.

(13-15) $$W = \frac{\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2}P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

Average time a unit spends in the waiting line or being serviced (namely, in the system).

(13-16) $$L_q = L - \frac{\lambda}{\mu}$$

Average number of people or units in line waiting for service.

(13-17) $$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

Average time a person or unit spends in the queue waiting for service.

(13-18) $$\rho = \frac{\lambda}{m\mu}$$

Utilization rate.

Equations 13-19 through 13-22 describe operating characteristics in single-channel models that have Poisson arrivals and constant service rates.

(13-19) $$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

Average length of the queue.

(13-20) $$W_q = \frac{\lambda}{2\mu(\mu - \lambda)}$$

Average waiting time in the queue.

(13-21) $$L = L_q + \frac{\lambda}{\mu}$$

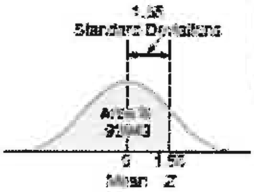
Average number of customers in the system.

(13-22) $$W = W_q + \frac{1}{\mu}$$

Average waiting time in the system.

Table A Normal Probabilities

Appendix A: Areas Under the Standard Normal Curve



Example: To find the area under the normal curve, you must know how many standard deviation that point is to the right of the mean. Then the area under the normal curve can be read directly from the normal table. For example, the total area under the normal curve for a point that is 1.55 standar deviations to the right of the mean is .93943.

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950

Table B BINOMIAL Probabilities

Appendix B: Binomial Probabilities

Probability of exactly r successes in n trials

		p									
n	r	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
1	0	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
	1	0.0500	0.1000	0.1500	0.2000	0.2500	0.3000	0.3500	0.4000	0.4500	0.5000
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.0950	0.1800	0.2550	0.3200	0.3750	0.4200	0.4550	0.4800	0.4950	0.5000
	2	0.0025	0.0100	0.0225	0.0400	0.0625	0.0900	0.1225	0.1600	0.2025	0.2500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.1354	0.2430	0.3251	0.3840	0.4219	0.4410	0.4436	0.4320	0.4084	0.3750
	2	0.0071	0.0270	0.0574	0.0960	0.1406	0.1890	0.2389	0.2880	0.3341	0.3750
	3	0.0001	0.0010	0.0034	0.0080	0.0156	0.0270	0.0429	0.0640	0.0911	0.1250
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
	1	0.1715	0.2916	0.3685	0.4096	0.4219	0.4116	0.3845	0.3456	0.2995	0.2500
	2	0.0135	0.0486	0.0975	0.1536	0.2109	0.2646	0.3105	0.3456	0.3675	0.3750
	3	0.0005	0.0036	0.0115	0.0256	0.0469	0.0756	0.1115	0.1536	0.2005	0.2500
	4	0.0000	0.0001	0.0005	0.0016	0.0039	0.0081	0.0150	0.0256	0.0410	0.0625
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313
	1	0.2036	0.3281	0.3915	0.4096	0.3955	0.3602	0.3124	0.2592	0.2059	0.1563
	2	0.0214	0.0729	0.1382	0.2048	0.2637	0.3087	0.3364	0.3456	0.3369	0.3125
	3	0.0011	0.0081	0.0244	0.0512	0.0879	0.1323	0.1811	0.2304	0.2757	0.3125
	4	0.0000	0.0005	0.0022	0.0064	0.0146	0.0284	0.0488	0.0768	0.1128	0.1563
	5	0.0000	0.0000	0.0001	0.0003	0.0010	0.0024	0.0053	0.0102	0.0185	0.0313
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
	1	0.2321	0.3543	0.3993	0.3932	0.3560	0.3025	0.2437	0.1866	0.1359	0.0938
	2	0.0305	0.0984	0.1762	0.2458	0.2966	0.3241	0.3280	0.3110	0.2780	0.2344
	3	0.0021	0.0146	0.0415	0.0819	0.1318	0.1852	0.2355	0.2765	0.3032	0.3125
	4	0.0001	0.0012	0.0055	0.0154	0.0330	0.0595	0.0951	0.1382	0.1861	0.2344
	5	0.0000	0.0001	0.0004	0.0015	0.0044	0.0102	0.0205	0.0369	0.0609	0.0938
	6	0.0000	0.0000	0.0000	0.0001	0.0002	0.0007	0.0018	0.0041	0.0083	0.0156
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.2573	0.3720	0.3960	0.3670	0.3115	0.2471	0.1848	0.1306	0.0872	0.0547
	2	0.0406	0.1240	0.2097	0.2753	0.3115	0.3177	0.2985	0.2613	0.2140	0.1641
	3	0.0036	0.0230	0.0617	0.1147	0.1730	0.2269	0.2679	0.2903	0.2918	0.2734
	4	0.0002	0.0026	0.0109	0.0287	0.0577	0.0972	0.1442	0.1935	0.2388	0.2734
	5	0.0000	0.0002	0.0012	0.0043	0.0115	0.0250	0.0466	0.0774	0.1172	0.1641
	6	0.0000	0.0000	0.0001	0.0004	0.0013	0.0036	0.0084	0.0172	0.0320	0.0547
	7	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0006	0.0016	0.0037	0.0078
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.2793	0.3826	0.3847	0.3355	0.2670	0.1977	0.1373	0.0896	0.0548	0.0313
	2	0.0515	0.1488	0.2376	0.2936	0.3115	0.2965	0.2587	0.2090	0.1569	0.1094

		p									
n	*r*	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
20	0	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000
	1	0.3774	0.2702	0.1368	0.0576	0.0211	0.0068	0.0020	0.0005	0.0001	0.0000
	2	0.1887	0.2852	0.2293	0.1369	0.0669	0.0278	0.0100	0.0031	0.0008	0.0002
	3	0.0596	0.1901	0.2428	0.2054	0.1339	0.0716	0.0323	0.0123	0.0040	0.0011
	4	0.0133	0.0898	0.1821	0.2182	0.1897	0.1304	0.0738	0.0350	0.0139	0.0046
	5	0.0022	0.0319	0.1028	0.1746	0.2023	0.1789	0.1272	0.0746	0.0365	0.0148
	6	0.0003	0.0089	0.0454	0.1091	0.1686	0.1916	0.1712	0.1244	0.0746	0.0370
	7	0.0000	0.0020	0.0160	0.0545	0.1124	0.1643	0.1844	0.1659	0.1221	0.0739
	8	0.0000	0.0004	0.0046	0.0222	0.0609	0.1144	0.1614	0.1797	0.1623	0.1201
	9	0.0000	0.0001	0.0011	0.0074	0.0271	0.0654	0.1158	0.1597	0.1771	0.1602
	10	0.0000	0.0000	0.0002	0.0020	0.0099	0.0308	0.0686	0.1171	0.1593	0.1762
	11	0.0000	0.0000	0.0000	0.0005	0.0030	0.0120	0.0336	0.0710	0.1185	0.1602
	12	0.0000	0.0000	0.0000	0.0001	0.0008	0.0039	0.0136	0.0355	0.0727	0.1201
	13	0.0000	0.0000	0.0000	0.0000	0.0002	0.0010	0.0045	0.0146	0.0366	0.0739
	14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0012	0.0049	0.0150	0.0370
	15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0013	0.0049	0.0148
	16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0013	0.0046
	17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0011
	18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
	19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

		p								
n	*r*	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
1	0	0.4500	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500
	1	0.5500	0.6000	0.6500	0.7000	0.7500	0.8000	0.8500	0.9000	0.9500
2	0	0.2025	0.1600	0.1225	0.0900	0.0625	0.0400	0.0225	0.0100	0.0025
	1	0.4950	0.4800	0.4550	0.4200	0.3750	0.3200	0.2550	0.1800	0.0950
	2	0.3025	0.3600	0.4225	0.4900	0.5625	0.6400	0.7225	0.8100	0.9025
3	0	0.0911	0.0640	0.0429	0.0270	0.0156	0.0080	0.0034	0.0010	0.0001
	1	0.3341	0.2880	0.2389	0.1890	0.1406	0.0960	0.0574	0.0270	0.0071
	2	0.4084	0.4320	0.4436	0.4410	0.4219	0.3840	0.3251	0.2430	0.1354
	3	0.1664	0.2160	0.2746	0.3430	0.4219	0.5120	0.6141	0.7290	0.8574
4	0	0.0410	0.0256	0.0150	0.0081	0.0039	0.0016	0.0005	0.0001	0.0000
	1	0.2005	0.1536	0.1115	0.0756	0.0469	0.0256	0.0115	0.0036	0.0005
	2	0.3675	0.3456	0.3105	0.2646	0.2109	0.1536	0.0975	0.0486	0.0135
	3	0.2995	0.3456	0.3845	0.4116	0.4219	0.4096	0.3685	0.2916	0.1715
	4	0.0915	0.1296	0.1785	0.2401	0.3164	0.4096	0.5220	0.6561	0.8145
5	0	0.0185	0.0102	0.0053	0.0024	0.0010	0.0003	0.0001	0.0000	0.0000
	1	0.1128	0.0768	0.0488	0.0283	0.0146	0.0064	0.0022	0.0004	0.0000
	2	0.2757	0.2304	0.1811	0.1323	0.0879	0.0512	0.0244	0.0081	0.0011
	3	0.3369	0.3456	0.3364	0.3087	0.2637	0.2048	0.1382	0.0729	0.0214
	4	0.2059	0.2592	0.3124	0.3602	0.3955	0.4096	0.3915	0.3280	0.2036
	5	0.0503	0.0778	0.1160	0.1681	0.2373	0.3277	0.4437	0.5905	0.7738

					p				
n	r	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
15	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0052	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0191	0.0074	0.0024	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	5	0.0515	0.0245	0.0096	0.0030	0.0007	0.0001	0.0000	0.0000	0.0000
	6	0.1048	0.0612	0.0298	0.0116	0.0034	0.0007	0.0001	0.0000	0.0000
	7	0.1647	0.1181	0.0710	0.0348	0.0131	0.0035	0.0005	0.0000	0.0000
	8	0.2013	0.1771	0.1319	0.0811	0.0393	0.0138	0.0030	0.0003	0.0000
	9	0.1914	0.2066	0.1906	0.1472	0.0917	0.0430	0.0132	0.0019	0.0000
	10	0.1404	0.1859	0.2123	0.2061	0.1651	0.1032	0.0449	0.0105	0.0006
	11	0.0780	0.1268	0.1792	0.2186	0.2252	0.1876	0.1156	0.0428	0.0049
	12	0.0318	0.0634	0.1110	0.1700	0.2252	0.2501	0.2184	0.1285	0.0307
	13	0.0090	0.0219	0.0476	0.0916	0.1559	0.2309	0.2856	0.2669	0.1348
	14	0.0016	0.0047	0.0126	0.0305	0.0668	0.1319	0.2312	0.3432	0.3658
	15	0.0001	0.0005	0.0016	0.0047	0.0134	0.0352	0.0874	0.2059	0.4633
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0150	0.0049	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
	7	0.0366	0.0146	0.0045	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000
	8	0.0727	0.0355	0.0136	0.0039	0.0008	0.0001	0.0000	0.0000	0.0000
	9	0.1185	0.0710	0.0336	0.0120	0.0030	0.0005	0.0000	0.0000	0.0000
	10	0.1593	0.1171	0.0686	0.0308	0.0099	0.0020	0.0002	0.0000	0.0000
	11	0.1771	0.1597	0.1158	0.0654	0.0271	0.0074	0.0011	0.0001	0.0000
	12	0.1623	0.1797	0.1614	0.1144	0.0609	0.0222	0.0046	0.0004	0.0000
	13	0.1221	0.1659	0.1844	0.1643	0.1124	0.0545	0.0160	0.0020	0.0000
	14	0.0746	0.1244	0.1712	0.1916	0.1686	0.1091	0.0454	0.0089	0.0003
	15	0.0365	0.0746	0.1272	0.1789	0.2023	0.1746	0.1028	0.0319	0.0022
	16	0.0139	0.0350	0.0738	0.1304	0.1897	0.2182	0.1821	0.0898	0.0133
	17	0.0040	0.0123	0.0323	0.0716	0.1339	0.2054	0.2428	0.1901	0.0596
	18	0.0008	0.0031	0.0100	0.0278	0.0669	0.1369	0.2293	0.2852	0.1887
	19	0.0001	0.0005	0.0020	0.0068	0.0211	0.0576	0.1368	0.2702	0.3774
	20	0.0000	0.0000	0.0002	0.0008	0.0032	0.0115	0.0388	0.1216	0.3585

--------------------------------**END**--