UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

DEPARTMENT OF MATHEMATICS

MODULE	**ASMA2A1** SEQUENCES, SERIES AND VECTOR CALCULUS
CAMPUS	**APK**
EXAM	**SUPPLEMENTARY EXAM 2014** Main

EXAMINER(S)	MR F SCHULZ
INTERNAL MODERATOR	MRS C DUNCAN
DURATION	2.5 HOURS
MARKS	50

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

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NUMBER OF PAGES: 1 + 12

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

ASMA2A1 Supp Main

Question 1

Find all integers k for which the sequence

$$\left\{(-k)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n)^k \cdot (2n)!}\right\}_{n=1}^{\infty}$$

converges. [5]

Question 2

Prove the following theorem: If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. [4]

Question 3

Test the following series for convergence or divergence: [10]

(3.1) $\sum_{n=0}^{\infty} \ln\left(\frac{n^2+3}{3n^2+1}\right)$ (2)

(3.2) $\sum_{n=1}^{\infty} (-1)^n \frac{n \ln n}{(n+1)^3 \cdot 2^n}$ (4)

(3.3) $\sum_{n=1}^{\infty} \frac{2}{2n - n \cdot \cos^2 n}$ (4)

Question 4

Suppose that $\sum c_n (x-2)^n$ converges when $x = 6$ and diverges when $x = -8$. What can be said about the convergence or divergence of the following series? [3]

(a) $\sum_{n=0}^{\infty} c_n(-3)^n$ (b) $\sum_{n=0}^{\infty} c_n(11)^n$ (c) $\sum_{n=0}^{\infty} c_n(-4)^n$

Question 5

Find the Maclaurin series for f and its radius of convergence: [4]

$$f(x) = (1 - 3x)^{-5}.$$

Question 6

Use series to evaluate the limit: [3]

$$\lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{\arctan x - x}$$

Question 7

Find the curvature of the curve with parametric equations $x = 4\cos t$ and $y = 3\sin t$ at the points $(4, 0)$ and $(0, 3)$. [4]

Question 8

Find the velocity, acceleration and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t:[5]

$$\mathbf{r}(t) = 2e^{t}\,\mathbf{i} + \frac{1}{2}e^{-t}\,\mathbf{j} \;;\; t = \ln 3.$$

Question 9

A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\,\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\,\mathbf{i} + 12t^3\,\mathbf{j} - 6t\,\mathbf{k}$. Find its position function. [4]

Question 10

If v is the speed of a particle along a curve C, $\mathbf{T}$ and $\mathbf{N}$ the unit tangent and unit normal vectors respectively of the particle's position vector $\mathbf{r}$, and κ is the curvature of C, then show that the acceleration $\mathbf{a}$ of the particle is given by [4]

$$\mathbf{a} = v'\mathbf{T} + \kappa v^2\mathbf{N}.$$

Question 11

Find the vectors **T** and **N** at the given point: [4]

$$\mathbf{r}(t) = \langle \sin t, \cos t, \ln \sin t \rangle \quad ; \quad \langle 1, 0, 0 \rangle .$$