UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

MODULE **APM1B10**
INTRODUCTION TO DYNAMICS
CAMPUS **APK**

SUPPLEMENTARY EXAMINATION

DATE: 12/2014 **SESSION: 8:00 - 11:00**

ASSESSOR **DR JSC PRENTICE**
INTERNAL MODERATOR **PROF C M VILLET**

DURATION: **$2\frac{1}{2}$ HOURS** **MARKS: 50**

NUMBER OF PAGES: 4 PAGES

INSTRUCTIONS:

CALCULATORS MAY BE USED

ANSWER ALL THE QUESTIONS

SYMBOLS HAVE THEIR USUAL MEANING

PHYSICAL QUANTITIES ARE IN SI UNITS AND ANGLES ARE IN RADIANS

QUESTION 1

(a) Show that

$$\widehat{\theta} \cdot \widehat{n} = \frac{v_r a_n}{v_r a_\theta - v_\theta a_r}.$$

(b) The equation of the trajectory of a particle is given by

$$\mathbf{r}\,(t) = t^2 \widehat{x} - \cos{(4t)}\, \widehat{y} \equiv \left(t^2, -\cos{(4t)}\right).$$

Find, for $t = \frac{1}{2}$,

(i) $\mathbf{v}$ and $\mathbf{a}$,

(ii) $\widehat{\tau}$ and $\widehat{n}$,

(iii) the tangential and normal components of $\mathbf{a}$,

(iv) and the radius of curvature ρ.

[10]

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QUESTION 2

Calculate $\int_\Gamma \mathbf{a} \cdot d\mathbf{r}$ with $\mathbf{a} = \left(2xy, x^2 + xy\right)$, and where the path Γ is defined by

$$y = 2x^2 + 3.$$

In the integral, the lower limit is $(-2, 11)$, and the upper limit is $(3, 21)$. You should find that the integral is equal to 725.

[10]

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QUESTION 3

A tunnel is drilled from the surface of the Earth, through its centre, to the opposite surface. A particle a distance r from the centre of the Earth experiences a gravitational force proportional to only that mass of the Earth within a sphere of radius r. How long does it take for a particle to fall to the centre of the Earth?

[10]

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QUESTION 4

Calculate the directional derivative $\frac{\partial \psi}{\partial s}$ of

$$\psi = xz + y^2$$

in the direction

$$3\widehat{x} - 2\widehat{y} + \widehat{z}$$

at the point

$$(1, 0, -2)\,.$$

(a) Use the parameterization $\mathbf{r} = \mathbf{r}_0 + s\widehat{s}$.

(b) Use $\nabla\psi$.

[10]

QUESTION 5

A bead of mass m slides along a smooth circular ring from A to B (Figure 1). Calculate the speed at B if the initial speed was v_0. The angle θ is defined in the figure. Gravity acts vertically downwards.

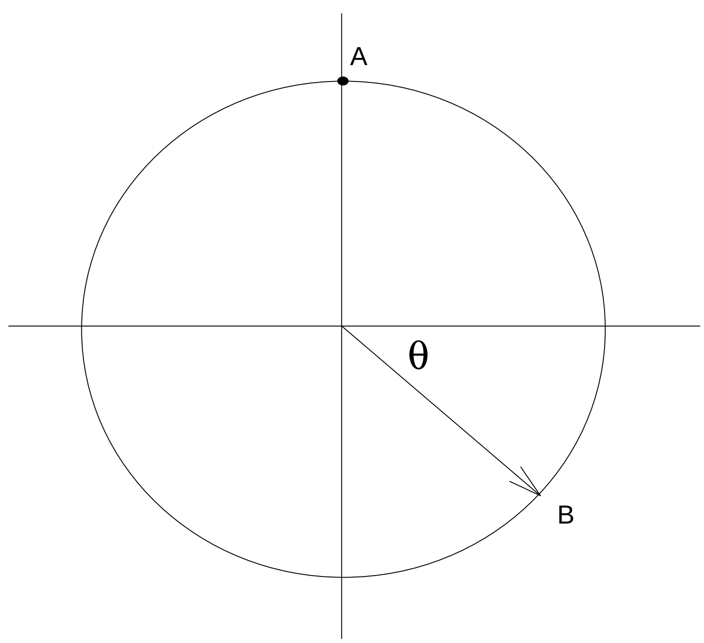

Figure 1

[10]

INFORMATION

$$
\begin{aligned}
\mathbf{r} &= (x, y) = (r\cos\theta, r\sin\theta) \\
\mathbf{v} &= v_r\widehat{r} + v_\theta\widehat{\theta} = v_\tau\widehat{\tau} + v_n\widehat{n} \\
&= \dot{r}\widehat{r} + r\dot{\theta}\widehat{\theta} = v\widehat{\tau} \\
\mathbf{a} &= a_r\widehat{r} + a_\theta\widehat{\theta} = a_\tau\widehat{\tau} + a_n\widehat{n} \\
&= \left(\ddot{r} - r\dot{\theta}^2\right)\widehat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\widehat{\theta} = \dot{v}\widehat{\tau} + \left(\frac{v^2}{\rho}\right)\widehat{n} \\
\widehat{r} &= (\cos\theta, \sin\theta) \qquad \widehat{\theta} = (-\sin\theta, \cos\theta) \\
\widehat{\tau} &= (\cos\psi, \sin\psi) \qquad \widehat{n} = (-\sin\psi, \cos\psi)
\end{aligned}
$$

$$\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F} dt = m\mathbf{v}_f - m\mathbf{v}_i$$

$$
\begin{aligned}
\frac{\partial\Psi}{\partial s} &= \nabla\Psi \cdot \widehat{s} \\
d\mathbf{r} &= (dx, dy) \text{ in two dimensions} \\
d\mathbf{r} &= (dx, dy, dz) \text{ in three dimensions}
\end{aligned}
$$

$$
\begin{aligned}
T &= \frac{m\left(\mathbf{v}\cdot\mathbf{v}\right)}{2} = \frac{mv^2}{2} \\
W &= \int_\Gamma \mathbf{F}\cdot d\mathbf{r} \\
\mathbf{F} &= -\frac{GMm}{r^2}\widehat{r}
\end{aligned}
$$