STRENGTH OF MATERIALS 4A

SLR4A – EXAM

May 2014

COURSE : ENGINEERING
SUBJECT : STRENGTH OF MATERIALS 4A
Lecturer Prof R.F. Laubscher (UJ)
Moderator Prof C Polese (Wits)

TIME : 3 HOURS
FULL MARKS : 90

This paper consists of 6 pages

- *Equation sheet attached*
- *No books or notes are allowed*
- *No answers in pencil will be marked*
- *Answer all the questions*

Question 1 **[20]**

1.1 Write down the stress and strain matrixes ces for:

Plane strain [2]

Plane stress [2]

1.2 State the broad problem areas where the finite element method can be applied and give an example of each. [5]

1.3 Derive an expression for the maximum shear stress, for the plane stress case, if the basic transformation equations for plane stress are used as the starting point. [5]

1.4 For the simple load case ~~as~~ shown in Figure 1, determine the direction and ~~size~~ of the maximum expected shear stress. magnitude or value [6]

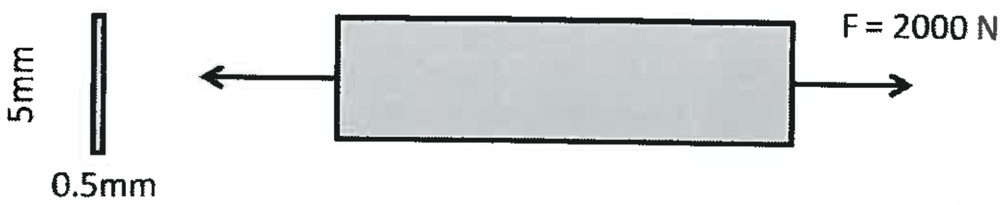

Figure 1

29/04/2014

R21

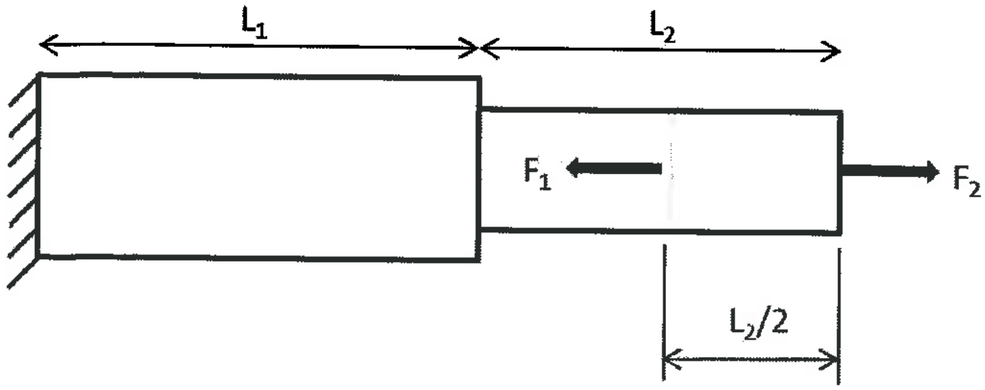

Figure 3 Bar system

Question 4 **[22]**

Consider the 2D plane stress element in Figure 4.

4.1 Calculate the shape functions associated with the three nodes i, j and k. [4]

4.2 Calculate the appropriate load vectors. [4]

4.3 Calculate the total force vector. [2]

4.4 Calculate the material property matrix. [2]

4.5 Calculate the matrix that relates the strains to the displacements. [2]

4.6 If the nodal displacements of the element are as follows:

$$\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 2 \\ 0 \\ 2 \\ 2 \end{Bmatrix} \times 10^{-3}\, mm$$

What are the displacements at the point (3,2)? [4]

4.7 Calculate the stress state of the element. [4]

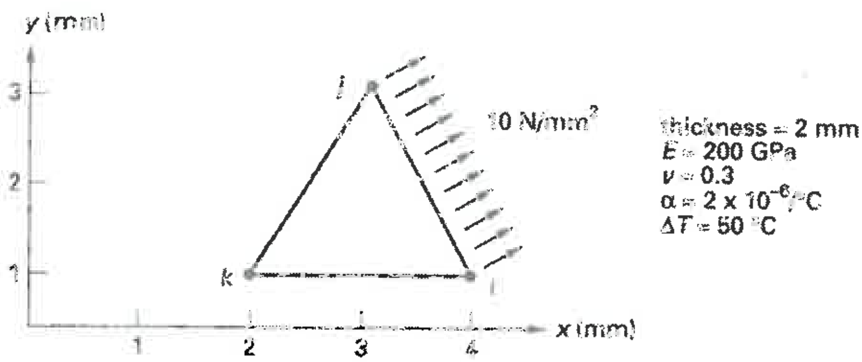

Figure 4 2D Plane stress element (dimensions in mm)

Total marks [90]

29/01/2014

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \varepsilon_{xy}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \varepsilon_{xy}\sin 2\theta$$

$$\varepsilon_{x'y'} = \frac{\varepsilon_y - \varepsilon_x}{2}\sin 2\theta + \varepsilon_{xy}\cos 2\theta$$

$$\left[[B]^T[D][B]tA\right]\begin{Bmatrix} u \\ v \end{Bmatrix} = \frac{\alpha Et(\Delta T)}{2(1-\nu)}\begin{Bmatrix} b_i \\ c_i \\ b_j \\ c_j \\ b_k \\ c_k \end{Bmatrix} + \frac{At}{3}\begin{Bmatrix} X \\ Y \\ X \\ Y \\ X \\ Y \end{Bmatrix} + \frac{t}{2}\left\{ H_{ij}\begin{Bmatrix} p_x \\ p_y \\ p_x \\ p_y \\ 0 \\ 0 \end{Bmatrix} + H_{jk}\begin{Bmatrix} 0 \\ 0 \\ p_x \\ p_y \\ p_x \\ p_y \end{Bmatrix} H_{ki}\begin{Bmatrix} p_x \\ p_y \\ 0 \\ 0 \\ p_x \\ p_y \end{Bmatrix}\right\} + \{P\}$$

$$\{\sigma\} = [D]\{\varepsilon\} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad \{\sigma\} = [D]\{\varepsilon\} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\varepsilon_z = -\frac{\upsilon}{E}(\sigma_x + \sigma_y) + \alpha\Delta T \qquad \sigma_z = \nu(\sigma_x + \sigma_y) - E\alpha\Delta T$$

$$\varepsilon_0 = \alpha\Delta T\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \qquad \varepsilon_0 = (1+\nu)\alpha\Delta T\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\{\varepsilon\} = [B]\{U\} \qquad \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{2A}\begin{bmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{bmatrix}\begin{Bmatrix} u_{2i-1} \\ u_{2i} \\ u_{2j-1} \\ u_{2j} \\ u_{2k-1} \\ u_{2k} \end{Bmatrix} = [B]\{U\}$$