



FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: INTRODUCTORY MATHEMATICAL ANALYSIS A – MAA00A1
CAMPUS: APK
ASSESSMENT: EXAM

DATE: 31 MAY 2014

ASSESSORS: MR RJ MAARTENS
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DURATION: 2 HOURS

MARKS: 75

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

NUMBER OF PAGES: 5 (INCLUDING COVER PAGE)

INSTRUCTIONS: ANSWER ALL THE QUESTIONS IN PEN
ALL GRAPHS MUST BE DRAWN IN PEN
NO PENCIL ALLOWED
NO TIPEX ALLOWED
STATE ALL FORMULAS USED, MARKS ARE GIVEN TO FORMULAS
SHOW ALL THE NECESSARY CALCULATIONS AND STEPS
IF NECESSARY ROUND OFF TO FOUR DECIMAL PLACES WHEN
WORKING WITH INTEREST RATES, OTHERWISE TWO DECIMAL PLACES
SCIENTIFIC AND FINANCIAL CALCULATORS ARE ALLOWED

Question 1 [6]

Simplify the following expressions:

1.1 $\frac{2x-2}{x^2-2x-8} \div \frac{x^2-1}{x^2+5x+4}$ [3]

1.2 $\frac{x}{(xy^2)^2} \times \sqrt[3]{\frac{x^2y}{x^3}}$ [3]

Question 2 [11]

Solve for x in the following equations:

2.1 $21x^4 = 2 + x^2$ [2]

2.2 $\log_2(\log_3(x)) = 1$ [2]

2.3 $e^{2x-1} = 6$ [2]

2.4 $\log(x-4) + \log(x+5) = 2$ [5]

Question 3 [3]

Solve for x in the following inequality and graph your solution:

$$2|1 - 2x| - 1 \geq 9$$

Question 4 [3]

A student is setting up a sinking fund to buy himself a car worth R85,000 in 5 years. How much must he invest at the beginning of each month into this fund to meet his goal if the fund earns 7.5% interest, compounded monthly?

Question 5 [4]

Your company wants to take out a loan to expand one of its branches. Which one of the following three banks offers you the best interest rate?

- Bank X: 9.55% per year, compounded weekly
- Bank Y: 9.8% per year, compounded monthly
- Bank Z: 9.2% per year, compounded continuously

Question 6 [4]

You are only allowed to use a financial calculator for this question.

A borrower is repaying an R800,000 loan at 15% per year, compounded monthly, with monthly payments over 30 years.

- 6.1 Determine the monthly payment. [1]
- 6.2 Determine the balance outstanding after the 50th payment. [1]
- 6.3 Determine the interest contained in the 100th payment. [1]
- 6.4 Determine the amount that needs to be added or subtracted to the last payment to fully pay off the loan. [1]

Question 7 [8]

Determine the maximum and minimum profit for the following system of inequalities if the profit function is given by $P = 2x + 3y$:

$$\begin{aligned} 2x + y &\geq 120 & 0 \leq x &\leq 100 \\ x + 2y &\geq 120 & 0 \leq y &\leq 80 \\ & & 5y &\geq 2x \end{aligned}$$

Question 8 [4]

100 people were asked if they smoke and the results were recorded in the following table:

"Do you smoke?"	Yes	No	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

- 8.1 What is the probability that a randomly selected person is a smoker? [1]
- 8.2 What is the probability that a randomly selected female is a smoker? [1]
- 8.3 What is the probability that a randomly selected person is either male or a smoker? [2]

Question 9 [2]

- 9.1 The odds that ABC Suppliers declares a dividend this year is 2 to 5. What is the probability that ABC Suppliers will declare a dividend? [1]

- 9.2 The weather forecast predicts that it will rain tomorrow with probability 65%.
What is the odds that it will not rain tomorrow? [1]

Question 10 [2]

If $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cup F) = \frac{7}{10}$, find $P(F|E)$.

Question 11 [8]

Differentiate each of the following functions:

11.1 $f(x) = (1 - 4x + 7x^5)^{30}$ [2]

11.2 $g(x) = \sqrt[3]{8x^2 - 3} \cdot \ln(x^3 - 1)$ [3]

11.3 $h(x) = \frac{4x^3 - 7x}{5x^2 + 2}$ [3]

Question 12 [5]

If the total revenue and total cost is given by:

$$TR = 200Q - 4Q^2 \quad \text{and} \quad TC = \frac{Q^3}{3} - 12Q^2 + 164Q + 100$$

12.1 Find an expression for the marginal profit. [2]

12.2 Determine the maximum profit. [3]

Question 13 [3]

Given the consumption function:

$$C = 3 + \sqrt{I} + 2\sqrt[3]{I}$$

13.1 Find the marginal propensity to consume. [1]

13.2 Find the marginal propensity to consume if $I = 2$. [1]

13.3 Find the marginal propensity to save if $I = 2$. [1]

Question 14 [4]

Determine the equation of the tangent line at the point $x = 2$ on the curve:

$$f(x) = 3x^2 - 4x + \frac{2}{x}$$

Question 15**[8]**

Given the function

$$f(x) = \frac{x^3}{3} - 3x^2 + 8x$$

- 15.1 Determine the coordinates of the turning points. [2]
- 15.2 Determine whether the turning points are maximum or minimum. [2]
- 15.3 Determine the intervals along which the function is increasing and decreasing. [2]
- 15.4 Determine the coordinate of the point of inflection. [1]
- 15.5 Determine the intervals of curvature. [1]

End of assessment – Total 75 marks
