



## **FACULTY OF SCIENCES**

### **DEPARTMENT OF PURE AND APPLIED MATHEMATICS**

**MODULE: INTRODUCTORY MATHEMATICAL ANALYSIS A – MAA00A1**

**CAMPUS: APK**

**ASSESSMENT: EXAM**

**DATE: 31 MAY 2014**

**ASSESSORS: MR RJ MAARTENS  
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**INTERNAL MODERATOR: MS S RICHARDSON**

**DURATION: 2 HOURS**

**MARKS: 75**

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**INITIALS AND SURNAME:** \_\_\_\_\_

**STUDENT NUMBER:** \_\_\_\_\_

**CONTACT NUMBER:** \_\_\_\_\_

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**NUMBER OF PAGES: 5 (INCLUDING COVER PAGE)**

**INSTRUCTIONS:**

- ANSWER ALL THE QUESTIONS IN PEN**
- ALL GRAPHS MUST BE DRAWN IN PEN**
- NO PENCIL ALLOWED**
- NO TIPEX ALLOWED**
- STATE ALL FORMULAS USED, MARKS ARE GIVEN TO FORMULAS**
- SHOW ALL THE NECCESARY CALCULATIONS AND STEPS**
- IF NECESSARY ROUND OFF TO FOUR DECIMAL PLACES WHEN**
- WORKING WITH INTEREST RATES, OTHERWISE TWO DECIMAL PLACES**
- SCIENTIFIC AND FINANCIAL CALCULATORS ARE ALLOWED**

**Question 1****[6]**

Simplify the following expressions:

1.1  $\frac{2x-2}{x^2-2x-8} \div \frac{x^2-1}{x^2+5x+4}$  [3]

1.2  $\frac{x}{(xy^2)^2} \times \sqrt[3]{\frac{x^2y}{x^3}}$  [3]

**Question 2****[11]**

Solve for  $x$  in the following equations:

2.1  $21x^4 = 2 + x^2$  [2]

2.2  $\log_2(\log_3(x)) = 1$  [2]

2.3  $e^{2x-1} = 6$  [2]

2.4  $\log(x-4) + \log(x+5) = 2$  [5]

**Question 3****[3]**

Solve for  $x$  in the following inequality and graph your solution:

$$2|1 - 2x| - 1 \geq 9$$

**Question 4****[3]**

A student is setting up a sinking fund to buy himself a car worth R85,000 in 5 years. How much must he invest at the beginning of each month into this fund to meet his goal if the fund earns 7.5% interest, compounded monthly?

**Question 5****[4]**

Your company wants to take out a loan to expand one of its branches. Which one of the following three banks offers you the best interest rate?

- |         |  |
|---------|--|
| Bank X: | 9.55% per year, compounded weekly      |
| Bank Y: | 9.8% per year, compounded monthly      |
| Bank Z: | 9.2% per year, compounded continuously |

**Question 6****[4]**

*You are only allowed to use a financial calculator for this question.*

A borrower is repaying an R800,000 loan at 15% per year, compounded monthly, with monthly payments over 30 years.

6.1 Determine the monthly payment. [1]

6.2 Determine the balance outstanding after the 50<sup>th</sup> payment. [1]

6.3 Determine the interest contained in the 100<sup>th</sup> payment. [1]

6.4 Determine the amount that needs to be added or subtracted to the last payment to fully pay off the loan. [1]

**Question 7****[8]**

Determine the maximum and minimum profit for the following system of inequalities if the profit function is given by  $P = 2x + 3y$ :

$$\begin{aligned} 2x + y &\geq 120 & 0 \leq x &\leq 100 \\ x + 2y &\geq 120 & 0 \leq y &\leq 80 \\ 5y &\geq 2x \end{aligned}$$

**Question 8****[4]**

100 people were asked if they smoke and the results were recorded in the following table:

"Do you smoke?"	Yes	No	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

8.1 What is the probability that a randomly selected person is a smoker? [1]

8.2 What is the probability that a randomly selected female is a smoker? [1]

8.3 What is the probability that a randomly selected person is either male or a smoker? [2]

**Question 9****[2]**

9.1 The odds that ABC Suppliers declares a dividend this year is 2 to 5. What is the probability that ABC Suppliers will declare a dividend? [1]

- 9.2 The weather forecast predicts that it will rain tomorrow with probability 65%.  
What is the odds that it will not rain tomorrow? [1]

**Question 10** [2]

If  $P(E) = \frac{4}{5}$ ,  $P(F) = \frac{3}{10}$  and  $P(E \cup F) = \frac{7}{10}$ , find  $P(F|E)$ .

**Question 11** [8]

Differentiate each of the following functions:

11.1  $f(x) = (1 - 4x + 7x^5)^{30}$  [2]

11.2  $g(x) = \sqrt[3]{8x^2 - 3} \cdot \ln(x^3 - 1)$  [3]

11.3  $h(x) = \frac{4x^3 - 7x}{5x^2 + 2}$  [3]

**Question 12** [5]

If the total revenue and total cost is given by:

$$TR = 200Q - 4Q^2 \quad \text{and} \quad TC = \frac{Q^3}{3} - 12Q^2 + 164Q + 100$$

12.1 Find an expression for the marginal profit. [2]

12.2 Determine the maximum profit. [3]

**Question 13** [3]

Given the consumption function:

$$C = 3 + \sqrt{I} + 2\sqrt[3]{I}$$

13.1 Find the marginal propensity to consume. [1]

13.2 Find the marginal propensity to consume if  $I = 2$ . [1]

13.3 Find the marginal propensity to save if  $I = 2$ . [1]

**Question 14** [4]

Determine the equation of the tangent line at the point  $x = 2$  on the curve:

$$f(x) = 3x^2 - 4x + \frac{2}{x}$$

**Question 15****[8]**

Given the function

$$f(x) = \frac{x^3}{3} - 3x^2 + 8x$$

- 15.1 Determine the coordinates of the turning points. [2]
- 15.2 Determine whether the turning points are maximum or minimum. [2]
- 15.3 Determine the intervals along which the function is increasing and decreasing. [2]
- 15.4 Determine the coordinate of the point of inflection. [1]
- 15.5 Determine the intervals of curvature. [1]

<b>End of assessment – Total 75 marks</b>
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