UNIVERSITY OF JOHANNESBURG

PROGRAM	:	NATIONAL DIPLOMA **MECHANICAL ENGINEERING**
SUBJECT	:	**STRENGTH OF MATERIALS 3**
CODE	:	**SOM 312**
DATE	:	MAIN EXAMINATION 5 JUNE, 2015
DURATION	:	(SESSION 1) 08:30 – 11:30 HRS
WEIGHT	:	50:50
TOTAL MARKS	:	93
FINAL MARKS	:	100
ASSESSOR	:	A. MASHAMBA
MODERATOR	:	P. STACHELHAUS
NUMBER OF PAGES	:	6 PAGES + 2 ANNEXURE
REQUIREMENTS	:	HOT-ROLLED SECTION STEEL TABLES BOOKLET.

INSTRUCTIONS

1. ANSWER ALL QUESTIONS.
2. SHOW ALL CALCULATIONS AND DRAW APPROPRIATE SKETCHES.
3. ANSWERS WITHOUT UNITS WILL BE IGNORED.
4. ALL DIMENSIONS ARE IN mm UNLESS STATED OTHERWISE.
5. SOME HELPFUL FORMULAS ARE PROVIDED IN THE ANNEXURE.
6. FOR VALUES NOT SUPPLIED, REASONABLE ENGINEERING ASSUMPTIONS SHOULD BE MADE.

QUESTION 1

A copper rod 40 mm in diameter and a steel rod 25 mm in diameter are rigidly fixed to unyielding walls such that the two rods share a common longitudinal axis as shown in Figure Qn 1. There is a gap of 0.22 mm between the free ends of the copper and steel rods as shown. The copper rod is initially 600 mm in length and the steel rod is initially 400 mm in length.

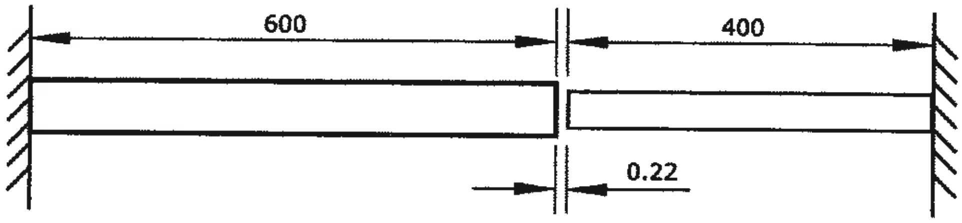

Figure Qn 1

If the two rods are then evenly heated to a temperature increase of 60 °C, calculate;

a) the stress experienced in the copper rod, (5)

b) the stress experienced in the steel rod and (5)

c) the final length of the copper rod after the 60 °C temperature increase. (4)

$E_{steel} = 205\ GPa,\ E_{copper} = 95\ GPa,\ \alpha_{steel} = 12 \text{ x } 10^{-6}/°C,$

$\alpha_{copper} = 20 \text{ x } 10^{-6}/°C$ **[14]**

QUESTION 2

A mass of 1000 kg is lowered at a constant velocity of 0.18 m/s by means of a steel rope of diameter 20 mm as shown in Figure Qn 2. When the steel rope is 4 m long, the lowering of the mass is suddenly stopped.

a) Calculate;

i. the instantaneous stress experienced by the steel rope, (5)

ii. the instantaneous elongation of the steel rope and (3)

iii. the instantaneous strain energy in the steel rope. (3)

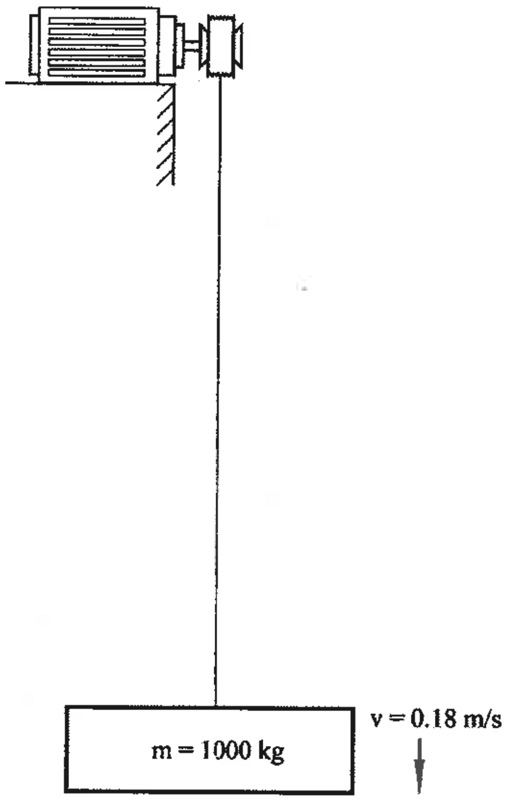

Figure Qn 2

b) In a separate scenario, a mass of 5 kg is firmly attached to the free-end of a 4 metres long steel rope of 20 mm diameter. The other end of the 4 metres long steel rope is firmly attached to an unyielding support. The 5 kg mass is then dropped through a height of 4 metres (free falling) before the rope becomes taut. Calculate;

i. the instantaneous stress experienced by the steel rope, (3)

ii. instantaneous elongation of the steel rope, and (3)

iii. the instantaneous strain energy in the steel rope. (3)

$E_{steel} = 200\ GPa$

[20]

QUESTION 3

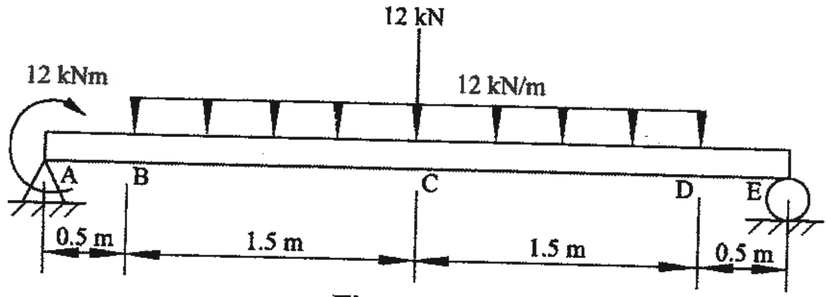

Figure Qn 3

For the beam shown in Figure Qn 3:

a) Calculate the vertical reactions at point A and E; (2)

b) Sketch the shear force diagram for the entire beam; and (5)

c) Sketch the bending moment diagram for the entire beam (8)

[15]

QUESTION 4

Two standard $180 \times 70\ mm$ Channels are welded together to make a built-up cross-section as shown in Figure Qn 4. The vertical Y-Y centroidal axis of the built-up cross-section passes through the vertical centroid of the two channels. Use the Section Steel Tables provided to calculate;

a) the position of the horizontal X-X centroid axis from the bottom, (4)

b) the Second Moment of Area, I_X about the X-X centroid axis, and (8)

c) the Second Moment of Area, I_Y about the Y-Y centroid axis. (4)

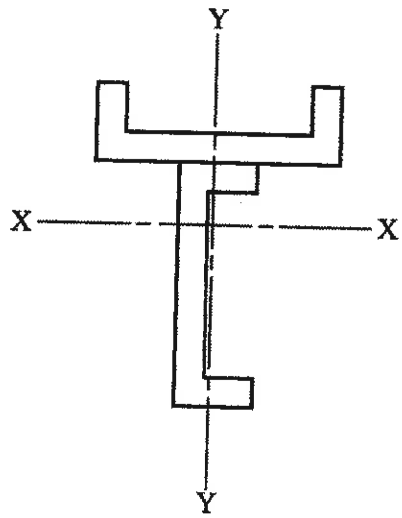

Figure Qn 4

[16]

QUESTION 5

A cantilever beam is 1.5 m long and is subjected to a uniformly distributed load of 20 kN/m and an eccentric longitudinal tensile load of 110 kN as shown in Figure Qn 5(a). The beam has a rectangular cross-section that is 100 mm wide and 200 mm deep as shown in Figure Qn 5(b). The longitudinal tensile point load of 110 kN is positioned on the Y-Y centroid axis but is 50 mm above the X-X centroidal axis of the beam's cross-section as illustrated in Figure Qn 5(b).

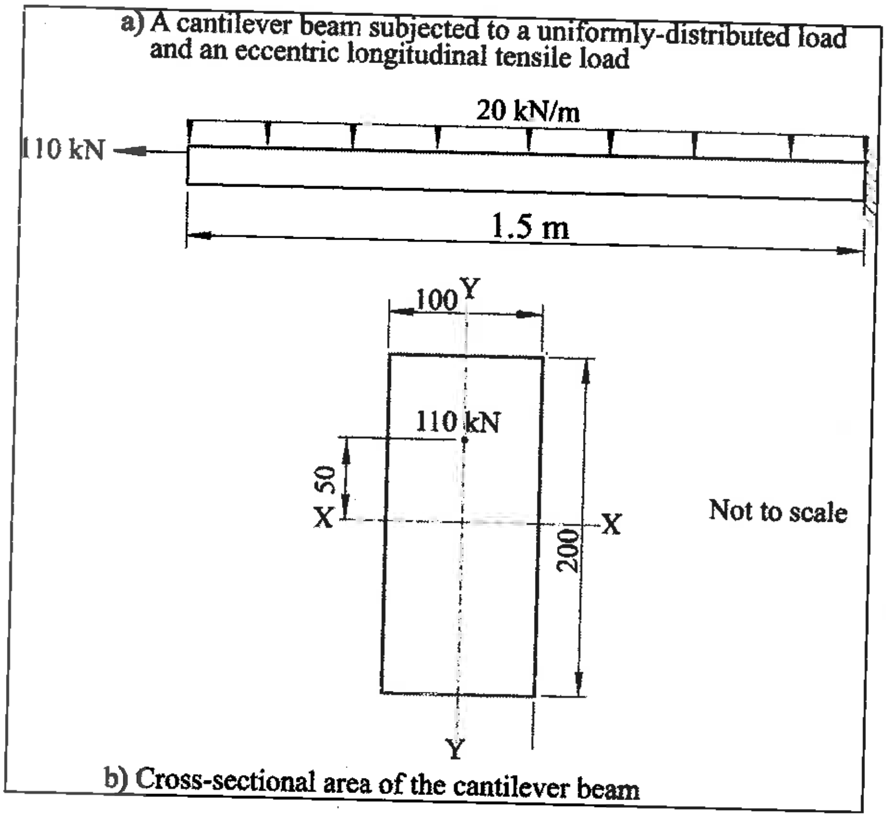

Figure Qn 5

Calculate;

a) the maximum bending moment experienced by the cantilever beam due to the 20 kN/m uniformly distributed load only, (3)

b) the maximum tensile stress induced in the cantilever beam due to the 20 kN/m uniformly distributed load only, (3)

c) the maximum stress induced in the cantilever beam due to the 110 kN eccentric longitudinal tensile load, and (5)

d) the maximum resultant stress experienced by the cantilever beam due to the uniformly distributed load and the eccentric longitudinal tensile load. (3)

[14]

QUESTION 6

A built-up wooden beam is made from two $50 \times 150\,mm$ rectangular flanges and one $50 \times 200\,mm$ rectangular web as shown in Figure Qn 6. The top and bottom flanges are each attached to the web by a single row of round mild steel nails that are 6 mm in diameter and 100 mm in length as shown. The second moment of area of the built-up beam about the horizontal centroid (I_X) is $270.833 \times 10^6\,mm^4$. The built-up beam is to carry a uniformly distributed load, which exerts a maximum shear force (V) of 4 kN. If the allowable shear stress (f) of each nail is 22 MPa, calculate the maximum spacing (pitch) of the nails that will be adequate to resist failure of the beam due to shearing of the nails.

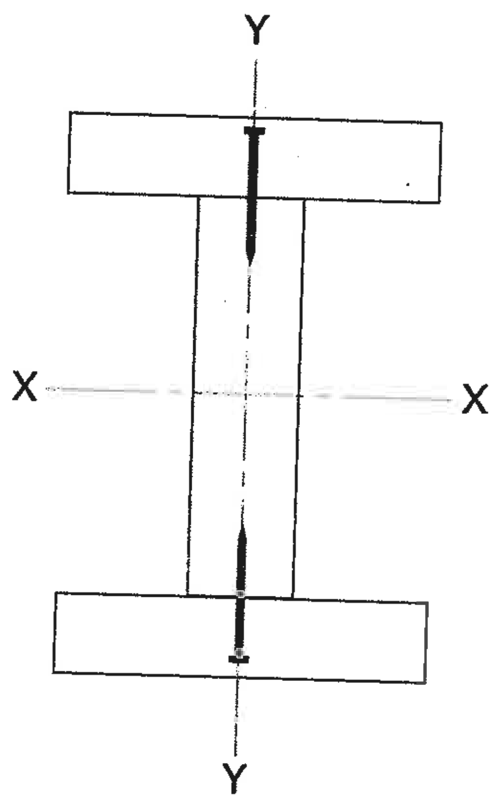

Figure Qn 6

[14]

TOTAL MARKS = 93

FINAL MARKS = 100

	Total rivet strength: $R = \frac{\pi d^2}{4} \times f \times n \times c = Q = \frac{VA\bar{y}}{I}$ Pitch of the rivets: $p = \frac{unit\ length \times number\ of\ rows\ of\ rivets}{n}$
6. Shear forces and Bending Moments	Sum of all vertical shear forces: $\sum V_y = 0$ Sum of all bending moments about point A: $\sum M_A = 0$
7. Close-Coiled Helical Springs	Shear stress (torsion): $\tau = \frac{8WD}{\pi d^3}$ Spring extension: $\delta = \frac{8WD^3 n}{Gd^4}$ Strain energy: $U = \frac{1}{2} W\delta = \frac{16T^2 Dn}{Gd^4} = \frac{4W^2 D^3 n}{Gd^4} = \frac{\tau^2}{4G} \times wire\ volume$ Stiffness: $S = \frac{Gd^4}{8D^3 n}$ Two springs in series: $\delta = \delta_1 + \delta_2,\ U = U_1 + U_2, S = \frac{S_1 S_2}{S_1 + S_2}$ Two springs in parallel: $\delta = \delta_1 = \delta_2, U = U_1 + U_2, S = S_1 + S_2$

ANNEXTURE 1: FORMULA SHEET

1. Temperature Stresses	Free expansion: $\Delta l = l\alpha\Delta T$ Length change due force: $\Delta l = \frac{\sigma l}{E}$
2. Strain Energy	Stress due to gradually applied load, P: $\sigma = \frac{P}{A}$ Stress due to suddenly applied load, P: $\sigma = \frac{2P}{A}$ Stress due to impact load, P: $\sigma = \frac{P}{A}\left[1 \pm \sqrt{1 + \frac{2AhE}{Pl}}\right]$ General strain energy: $U = \frac{\sigma^2}{2E} \times volume\ of\ material$ Strain energy, when P is gradually applied: $U = \frac{1}{2}P\Delta l$ Strain energy, when P is suddenly applied: $U = P\Delta l$ Strain energy, when P falls from a height h: $U = P(h + \Delta l)$ Impact loads on structures: $\frac{\delta}{\delta_s} = \frac{\sigma}{\sigma_s} = \left[1 \pm \sqrt{1 + \frac{2h}{\delta_s}}\right]$ Weight moving at a constant velocity: $\frac{\delta}{\delta_s} = \frac{\sigma}{\sigma_s} = \left[1 + \sqrt{\frac{v^2}{g\delta_s}}\right]$
3. Second Moment of Area	Distance to centroid of cross-section with n sub-cross-sections: $\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + \cdots + A_n\bar{y}_n}{A_1 + A_2 + \cdots + A_n}$ Rectangular: $I_X = \frac{bd^3}{12}, I_Y = \frac{db^3}{12}$ Cylindrical (solid): $I_X = I_{YY} = \frac{\pi D^4}{64}, J = \frac{\pi D^4}{32}$ Cylindrical (hollow): $I_X = I_{YY} = \frac{\pi(D^4 - d^4)}{64}, J = \frac{\pi(D^4 - d^4)}{32}$ Parallel axis theorem: $I_{NA} = I_X + Ah^2$ Perpendicular axis theorem: $I_Z = I_X + I_Y$
4. Direct Stresses due to Bending	Bending moment equation: $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ Section modulus (elastic): $Z_e = \frac{I}{y_{max}} \left(= \frac{M_{max}}{\sigma_{max}}\right)$ Bending stress from eccentric longitudinal point loading, P: $\sigma = \frac{P}{A} \pm \frac{P\hat{x}x}{I_Y} \pm \frac{P\hat{y}y}{I_X}$
5. Shear Stresses due to Bending	Shear stress (general): $\tau = \frac{VA\bar{y}}{Ib}$ Shear stress (b x d rectangular cross-section): $\tau = \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right)$ Shear stress (maximum): $\tau = \frac{3V}{2bd} = 1.5 \times \tau_{mean}$