FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING: MINERAL SURVEYING/EXTRACTION METALLURGY

MODULE   MNM31-1
                 NUMERICAL METHODS

CAMPUS   DFC

JUNE 2014 EXAMINATION

DATE: 02/06/2014   **SESSION: 12:30 - 15:30**

ASSESSOR   **MS BP NTSIME**
                          MR PG DLAMINI

INTERNAL MODERATOR   **DR PRENTICE JSC**

DURATION: 3 HRS   **MARKS: 80 MARKS**

SURNAME & INITIALS:

STUDENT NUMBER:

COURSE:

CONTACT NO:

INSTRUCTIONS :   ANSWER ALL QUESTIONS BY CREATING APPROPRIATE
                 MATHEMATICA CODES
                  NO EXTERNAL STORAGE DEVICES ARE PERMITTED
                  NON-PROGRAMMABLE SCIENTIFIC CALCULATORS ALLOWED

REQUIREMENTS: FORMULA BOOKLET

Question 1

a) From the graph of $f(x) = 3x + \sin x - e^x$, find values of a and b such that $f(a) f(b) < 0$.

[5]

b) Use the following methods to find the root of f within a tolerance criterion $\left| f(x) \right| < 10^{-6}$, determining the number of iterations required

   (i) Regula Falsi method with the values of a and b as found above   [10]

(ii) the Newton-Raphson method with $x_0 = 0.5$ [10]

Question 2

a) (i) Use the built-in *Mathematica* solver to solve the following system of equations.

$$2x_1 - x_2 = 2$$ [3]

$$x_1 - 3x_2 + x_3 = -2$$

$$-x_1 + x_2 - 3x_3 = -6$$

  (ii) Compute the condition number of the matrix A with respect to the infinity norm. Is A ill-conditioned?

$$A = \begin{bmatrix} 4.5 & 3.1 \\ 1.6 & 1.1 \end{bmatrix}$$

[2]

b) Consider the data presented in the table below

x_i	f_i
0	0.0674
0.5	-0.9156
1.0	1.6253
1.5	3.0377
2.0	3.3535
2.	7.9409

   (i)   Find the polynomial of highest possible degree that interpolates f.
   (ii) Find the polynomial of degree 2, $P_2(x)$, that best fits the data in the least squares sense.
   (iii) Graph the interpolating polynomial, P_2 and the data points on the same axes.
[10]

Question 3

a) Solve the following system on linear equations using the Gauss Seidel method. Terminate iterations when the infinity norm of the residual is 10^{-6}. Use the ZERO vector as starting value.

$$4\, x_1 - x_2 - x_3 = 3$$ [10]

$$-2\, x_1 + 6\, x_2 + x_3 = 9$$

$$-x_1 + x_2 - 7\, x_3 = -6$$

Question 4

Solve the set of non-linear equations

$$x^3 + y = 1 \quad \text{and,} \quad y^3 - x = -1$$

using Newton's method with starting values for $x_0 = 0.5$ and $y_0 = 0.5$. Terminate the method when

$\| f(x) \|_\infty < 10^{-4}$.   [10]

Question 5

5.1 Use Simpson's rule to approximate

$$\int_1^7 \frac{\sqrt{x-1}}{x} \, dx$$

using 20 sub-intervals

[10]

5.2 Use Euler's method with a step size of $h = 0.2$ to find an approximate solution of the following IVP

$$y' + 2y = 2 - e^x \ , \qquad y(0) = 1$$

over $0 \le x \le 5$.   [10]