



UNIVERSITY
OF
JOHANNESBURG

FACULTY OF SCIENCE

	Examiner	Moderator
Paper 1 30 marks		
Paper 2 70 marks		
EM/100		

	Examiner	Moderator
SM		
EM		
FM		

**DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS
NATIONAL DIPLOMA IN ENGINEERING:
ELECTRICAL**

MODULE **MAT3AW3** **ENGINEERING MATHEMATICS 3 (Paper 2)**
CAMPUS **DFC**

MAY EXAMINATION 2014

DATE: 31/5/2014

SESSION: 12:30 – 15:30

ASSESSOR:

MRS E KIRCHNER

MODERATOR:

DR Q VAN DER HOFF

DURATION: 3 HOURS

FULL MARKS: 100

SURNAME AND INITIALS	
STUDENT NUMBER	
CONTACT NUMBER	
LECTURER	

NUMBER OF PAGES: 15 PAGES

REQUIREMENTS: MATHEMATICS INFORMATION BOOKLET

Instructions:

- Please fill in your particulars on the front page.
- Answer all the questions in the space provided.
- Do not write in pencil. Pencil will not be marked.
- You may use the back of each page (i.e. the left-hand side) for **rough work OR to complete a question.**
- **Please indicate rough work as such.**
- Rough work will not be marked.
- One non programmable calculator is permitted.
- Information booklets may be used.
- **PLEASE CHECK THAT YOU HAVE RECEIVED 15 PAGES.**

QUESTION 1

1.1 Determine the following:

$$\mathcal{L}\{7tH(t-7)\}$$

(3)

1.2 Use the **Laplace transform** to solve the given differential equations, subject to the indicated initial conditions:

$$1.2.1 \quad x'' + 2x' + x = \frac{1}{2}t^4e^{-t}, \quad x(0) = 0 \text{ and } x'(0) = 0 \quad (3)$$

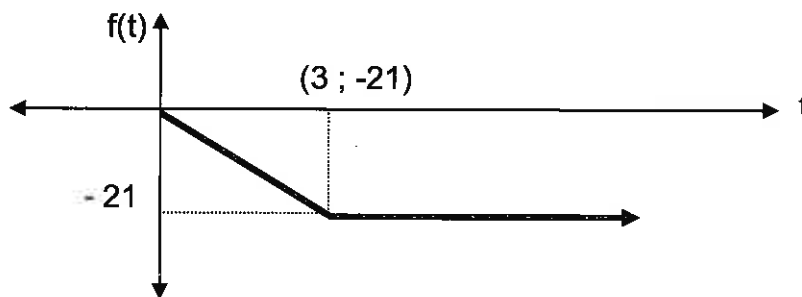
2.3 Given: $f(t) = 70[t - (t - 1)H(t - 1)]$

Sketch the graph of $f(t)$ for $t \geq 0$.

(2)

2.4 The graph below is defined analytically by

$$f(t) = \begin{cases} -7t & 0 \leq t < 3 \\ -21 & t \geq 3 \end{cases}$$



Express $f(t)$ in terms of Heaviside functions.

(3)

QUESTION 3

Find the general solution of the following differential equations, using **D-operator methods**.

$$3.1 \quad \frac{d^2 y}{dt^2} - 16y = 24e^{4t} \quad (7)$$

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

$$3.2 \quad \frac{d^2 x}{dt^2} + 12x = 2 \sin \sqrt{3} t \cos \sqrt{3} t \quad (5)$$

[illegible]

[illegible]

QUESTION 4

4.1 In an LRC series circuit the differential equation is given as

$$\frac{1}{2} \frac{d^2 q}{dt^2} + 10 \frac{dq}{dt} + \frac{q}{0,01} = 150$$

4.1.1 Use **D-operator methods** to find the charge q at any time t . (4)

4.1.2 What is the charge (q) after a long time? Discuss. (1)

[illegible]

- 4.2 An RC circuit has an emf given (in volts) by $200 \sin 2t$, a resistance of $50 \, \Omega$, a capacitance of $0,02 \, \text{F}$.

The model of the differential equation is given by

$$50 \frac{dq}{dt} + \frac{q}{0,02} = 200 \sin 2t$$

- 4.2.1 Use D-operator methods to find the general solution of the charge q at any time t** (7)

- 4.2.2 Find an equation for the current i (Hint: $i = \frac{dq}{dt}$) (1)

- #### 4.2.3 Rewrite the steady state current in the form $i = R \sin(2t \pm \alpha)$ (2)

[illegible]

4.3 Given the following system of simultaneous differential equations:

$$\frac{dy}{dt} + 2\frac{dx}{dt} + y - x = 25$$

$$2 \frac{dy}{dx} + x = 25e^t$$

Use D – operator methods to solve for y ONLY.

(11)

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

(6)

- 5.2** Determine a **half range Fourier sine series** to represent the function $f(x) = 5 - x$ ($0 \leq x \leq 5$) if it is **given** that $f(x) = f(x + 10)$

(6)

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

[12]

TOTAL: 100