UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE MAT1A01
CALCULUS OF ONE-VARIABLE FUNCTIONS

CAMPUS APK

EXAM JUNE EXAM 2015

DATE 13/06/2015 **SESSION** 12:30 – 14:30

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DURATION 2 HOURS **MARKS** 70

SURNAME AND INITIALS ________________________________

STUDENT NUMBER ________________________________

CONTACT NR ________________________________

NUMBER OF PAGES: 1 + 12 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE NEXT TO IT AND INDICATE THIS CLEARLY.

Question 1 [8 marks]

For questions 1.1 – 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1) Which one of these curves has a vertical asymptote? [1]

a) $y = \tan^{-1} x$

b) $y = \sqrt{x}$

c) $y = \ln x$

d) $y = e^x$

e) None of the above.

1.2) The simplified form of $e^{\sqrt{\ln e}} + \sqrt{\ln e^e}$ is: [1]

a) $e + \sqrt{e}$

b) $2\sqrt{e}$

c) 2

d) $\sqrt{(\ln e)^2}$

e) None of the above.

1.3) The conditional proposition $p \to \neg q$ can be rewritten as an "or" formula as follows: [1]

a) $\neg p \vee q$

b) $p \vee q$

c) $\neg p \vee \neg q$

d) $p \vee \neg q$

e) None of the above.

1.4) $\dfrac{d}{dx}\big[\ln(2)\log_2(x^2)\big] =$ [1]

a) $\dfrac{2}{x}$

b) $\dfrac{\ln(2)}{2x}$

c) $\dfrac{4}{x\ln(2)}$

d) $\dfrac{2\ln(2)}{x\log 2}$

e) None of the above.

1.5) $\dfrac{d}{dx}\big[\arctan(\cot x)\big] =$ [1]

a) 1

b) $-\csc x$

c) -1

d) $\csc x$

e) None of the above.

1.6) Let f be a continuous function on the closed interval $[0{,}2]$. If $2 \leq f(x) \leq 4$, then the greatest possible value of $\displaystyle\int_0^2 f(x)\,dx$ is [1]

a) 0

b) 2

c) 4

d) 8

e) None of the above.

1.7) Which of the following is a tautology? [1]

a) $\neg A \wedge (\neg B \vee C)$

b) $\neg A \vee \neg B$

c) $\neg(\neg A \wedge A)$

d) $A \rightarrow (B \wedge C)$

e) None of the above.

1.8) Suppose that f is an integrable function and that $\int_0^1 f(x)\,dx = 2$, $\int_0^2 f(x)\,dx = 1$ and $\int_2^4 f(x)\,dx = 7$. Then $\int_0^4 f(x)\,dx =$ [1]

a) -1

b) 5

c) 8

d) 6

e) None of the above.

Question 2 [3 marks]

a) Solve for $x \in \mathbb{R}$: $\dfrac{x-3}{x-2} \leq 0$ [2]

b) Find all $x \in [0, \frac{\pi}{2}]$ that satisfy the inequality $\cos x < \sin x$. [1]

Question 3 [5 marks]

Let $z = 1 + i$ and $w = 1 - \sqrt{3}$.

a) Write z and w in polar form. [2]

b) Find $\dfrac{z}{w}$ and leave your answer in polar form. [1]

c) Find z^{10} and leave your answer in polar form. [2]

Question 4 [6 marks]

a) Translate the following sentence into first-order language: [2]

"All dogs wear hats only if not all dogs wear shoes".

b) Show that an implication $p \to q$ is logically equivalent to its contrapositive (without using truth tables). [2]

c) Use a direct proof to show that the square of any perfect square is a perfect square. [2]

Question 5 [4 marks]

a) Use transformations to sketch the graph of $y = -\cos(x + \pi)$ within the interval $[0, \pi]$. Show each step. [2]

b) Determine whether $f(x) = \dfrac{1}{\tan x}$ is even, odd or neither. [2]

Question 6 [4 marks]

Given: $f(x) = \ln(x^3 - 3)$

a) Show that f is a one-to-one function (without sketching the graph). [2]

b) Find $f^{-1}(x)$ [2]

Question 7 [7 marks]

Given:

$$f(x) = \begin{cases} 3x^2 & \text{if } x \leq 1 \\ 4 - x & \text{if } 1 < x \leq 4 \\ -1 & \text{if } 4 < x \leq 7 \\ (x-7)^2 - 1 & \text{if } x > 7 \end{cases}$$

a) Prove that f is continuous at $x = 1$ [3]

b) What kind of discontinuity is at $x = 4$? [1]

c) Show that f is differentiable at $x = 7$. [3]

Question 8 [3 marks]

Find the limit: $\lim_{x \to -\infty} \dfrac{\sqrt{4x^2 - 9}}{3x + 2}$ [3]

Question 9 [4 marks]

Prove the Product Rule of Differentiation: [4]

If $f(x)$ and $g(x)$ are both differentiable, then $\dfrac{d}{dx}\big[f(x).g(x)\big] = g(x).\dfrac{d}{dx}f(x) + f(x).\dfrac{d}{dx}g(x)$

Question 10 [3 marks]

Use the definition of the derivative to prove that $\dfrac{d}{dx}\sin x = \cos x$. [3]

(You DO NOT need to prove the special limits $\lim_{h\to 0} \dfrac{\sin h}{h} = 1$ and $\lim_{h\to 0} \dfrac{\cos h - 1}{h} = 0$.)

Question 11 [4 marks]

a) Find y' if $e^{x+y} = y^2 - \cos x$. [2]

b) Find $g'(t)$ if $g(t) = (\sec t + \tan t)^5$ [2]

Question 12 [2 marks]

Prove the following hyperbolic identity: $\cosh^2 x - \sinh^2 x = 1$ [2]

Question 13 [3 marks]

Evaluate the limit. Use L'Hospital's rule if necessary: $\lim_{x \to 0^+} (\tan 4x)^{3x}$ [3]

Question 14 [3 marks]

Find $f(x)$ if $f''(x) = -\cos x + 6$ and $f(0) = 3$ and $f(\pi) = 1$. [3]

Question 15 [4 marks]

a) State the Fundamental Theorem of Calculus **Part 1**. [2]

b) Using a) above, calculate: $\dfrac{d}{dx} \displaystyle\int_3^{x^2} \frac{1}{t}\, dt$ [2]

Question 16 [7 marks]

Evaluate the following integrals if they exist:

a) $\displaystyle\int \left(\frac{1-\sqrt{u}}{\sqrt{u}} + \sec^2 u \right) du$ [2]

b) $\displaystyle\int_1^3 x.e^{x^2}\, dx$ [2]

c) $\displaystyle\int_0^3 \mid x^2 - 4 \mid \, dx$ [3]