



UNIVERSITY
OF
JOHANNESBURG

DEPARTMENT OF ECONOMICS AND
ECONOMETRICS
EKONOMIE EN EKONOMETRIE
DEPARTEMENT

FINAL ASSESSMENT: JUNE 2014
FINALE ASSESSERING: JUNIE 2014

Course / Kursus: Econometrics / *Ekonometrie* 3A (EKM3A01)
Examiners / Eksaminatore: Mrs/Mev M Pretorius
Internal Moderator: Ms/Me M Wilson
External Moderator: Mr/Mnr GT Kashalala (University of Pretoria)
Time / Tyd: 3 hours/ure
Marks / Punte: 100

Instructions / Instruksies:

1. Answer all the questions. / *Beantwoord al die vrae.*
2. This paper consists of 4 pages / *Hierdie vraestel bestaan uit 4 bladsye.*

SECTION / AFDELING A

[40]

1. Differentiate each of the following:
Differensieër elk van die volgende:

(21)

- a) $f(x) = 3x^2 - 9x + 7x^{\frac{2}{5}} - 3x^{1/2}$
- b) $f(x) = x^2\sqrt{x^2 - 16}$
- c) $f(x) = \sqrt[5]{13x^2 - 5x + 8}$
- d) $f(x) = e^{5x^2 - 7x - 13}$
- e) $y = \frac{x}{\ln x}$
- f) $3x^4 - 7y^5 - 86 = 0$ (implicit differentiation / *implisiete differensiasie*)
- g) $g(x) = (x^3 - 2)(x^2 - 3)(8x - 5)$ (logarithmic differentiation / *logaritmiese differensiasie*)

2. The demand and total cost functions of a good are:
Die vraag en totale koste funksies van 'n produk is: (8)

$$P = 1400 - 7.5Q \text{ and } TC = Q^3 - 6Q^2 + 140Q + 750$$

respectively.
afsonderlik.

- a) Find expressions for TR, MR, MC and profit.
Vind uitdrukkings vir TR, MR, MC en wins.
- b) Determine the value of Q which maximises profit.
Bepaal die waarde van Q wat wins maksimeer.
- c) Give the maximum amount of profit that can be made.
Gee die maksimum bedrag wins wat gemaak kan word.
3. A new car depreciates in value by 3 percent a month for the first year. What is the book value of a R60 000 car at the end of the first year?
'n Nuwe motor depresieër in waarde teen 3 persent 'n maand vir die eerste jaar. Wat is die boekwaarde van 'n motor wat R60 000 kos teen die einde van die eerste jaar? (3)
4. You won R10 000 in a competition and want to invest your winnings over the following 5 years. If you have to choose between the following banks and their offers, which one will you choose and why? (Show all your calculations).
Jy het R10 000 gewen in 'n kompetisie en wil jou wengeld belê oor die volgende 5 jaar. Indien jy tussen die volgende banke en wat hulle aanbied moet kies, watter een sal jy kies en hoekom? (Wys al jou berekeninge). (8)
- a) Bank A, 4.75% interest, compounded annually.
Bank A, 4.75% rente, jaarliks bereken.
- b) Bank B, 4.7% interest, compounded semi-annually.
Bank B, 4.7% rente, twee-jaarliks bereken.
- c) Bank C, 4.6% interest, compounded continuously.
Bank C, 4.6% rente, aaneenlopend bereken.

SECTION / AFDELING B

[20]

1. Use Cramer's rule to solve the unknowns in the system of linear equations given below:
Maak gebruik van Cramer se reël om die onbekendes op te los in die onderstaande sisteem van linêre vergelykings: (10)

$$8x + 10z = 7y + 15$$

$$2x + 3y + 8z = 7$$

$$5y + 9 = 4x + 2z$$

2. Given / Gegee:

(10)

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -3 & 1 \\ -2 & 3 & 5 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$$

Calculate (if possible):
Bereken (indien moontlik):

- a) AC
- b) CA
- c) A + D'
- d) (AD)'

SECTION / AFDELING C

[16]

1. Use the substitution method to integrate the following definite integral:
Maak gebruik van die substitusie metode om die volgende definitiewe integraal te integreer:

(6)

$$\int_0^3 8x(2x^2 + 3)dx$$

2. Determine the following integral, using integration by parts.
Bepaal die volgende integraal deur van integrasie van dele gebruik te maak:

(4)

$$\int 4x(x + 1)^3 dx$$

3. Given the following demand and supply functions:
Gegee die volgende vraag- en aanbodfunksies:

(6)

$$P_D = -50Q + 2000$$

$$P_S = 10Q + 500$$

- a) Find the consumers' surplus.
Bepaal die verbruikerssurplus.
- b) Find the producers' surplus.
Bepaal die produsentesurplus.

SECTION / AFDELING D

[24]

1. Given the following first-order differential equation:
Gegee die volgende eerste-orde differensiële vergelyking:

(8)

$$\frac{dy}{dt} - 2ty = e^{t^2}$$

- a) Use the formula for a general solution to solve the equation.
Gebruik die formule vir 'n algemene oplossing om die vergelyking op te los.
- b) Determine the definite solution for the equation given that $y(0) = 1$.
Bepaal die spesifieke oplossing vir die vergelyking gegee dat $y(0) = 1$.

2. Given the following first-order difference equation:
Gegee die volgende eerste-orde verskil vergelyking: (8)

$$x_t + 3x_{t-1} + 8 = 0 \text{ and/en } x_0 = 16$$

- a) Solve the equation.
Los die vergelyking op.
- b) Check your answer by using $t = 0$ and $t = 1$.
Ondersoek jou antwoord deur van $t = 0$ en $t = 1$ gebruik te maak.
- c) Comment on the nature of the time path.
Lewer kommentaar oor die aard van die tydkoers.

3. Given the following second-order difference equation:
Gegee die volgende tweede-orde verskil vergelyking: (8)

$$y_t + 7y_{t-1} - 8y_{t-2} = 45$$

Find the:
Bepaal die:

- a) Particular solution.
Spesifieke oplossing.
- b) Complementary function.
Komplementêre funksie.
- c) General solution.
Algemene oplossing.

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SUPPLEMENTARY ASSESSMENT: JULY 2014
AANVULLENDE ASSESSERING: JULIE 2014

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SECTION / AFDELING A

[40]

1. Differentiate each of the following:
Differensieër elk van die volgende:

(21)

a) $y = (5x - 1)(3x + 4)^3$

b) $f(x) = x\sqrt{6x + 1}$

c) $y = e^{\ln(2x+1)}$

d) $y = x^3 2^x$

e) $y = \ln \frac{x^3}{(2x+5)^2}$

f) $2x^4 + 7x^3 + 8y^5 = 136$ (implicit differentiation / *implisiete differensiasie*)

g) $g(x) = \frac{(3x^5-4)(2x^3+9)}{(7x^4-5)}$ (logarithmic differentiation / *logaritmiëse differensiasie*)

2. Maximise the following total profit function
Maksimeer die volgende totale wins funksie (8)

$$\pi = -Q^2 + 11Q - 24$$

by:
deur:

- a) Finding the critical values.
Die kritiese waardes te vind.
- b) Testing the second-order conditions.
Die tweede-orde kondisies te toets.
- c) Calculating the maximum profit.
Die maksimum wins te bereken.
3. Costs C of a government program escalates from 5.39 billion in 2001 to 5.77 billion in 2011. Find the annual rate of growth.
Die koste C van 'n regeringsprojek het van 5.39 biljoen in 2001 tot 5.77 biljoen in 2011 toegeneem. Vind die jaarlikse groeikoers. (3)
4. At 11 percent annual interest rate, which of the following has the largest present value:
Teen 11 persent jaarlikse rentekoers, watter van die volgende het die grootste huidige waarde: (8)
- a) R850 two years from now.
R850 twee jaar van nou af.
- b) R400 after the first year and R400 after the second year.
R400 na die eerste jaar en R400 na die tweede jaar.
- c) R400 now and R390 from two years from now.
R400 nou en R390 twee jaar van nou af.

SECTION / AFDELING B

[20]

1. Use matrix inversion to solve the unknowns in the system of linear equations given below:
Maak gebruik van matriks-omkering om die onbekendes op te los in die onderstaande sisteem van linêre vergelykings: (10)

$$\begin{aligned}8x + 10z &= 7y + 15 \\2x + 3y + 8z &= 7 \\5y + 9 &= 4x + 2z\end{aligned}$$

2. Given / Gegee:

(10)

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -3 & 1 \\ -2 & 3 & 5 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$$

Calculate (if possible):

Bereken (indien moontlik):

- a) AB
- b) BA
- c) $A' - D$
- d) $(CD)'$

SECTION / AFDELING C

[16]

1. Use integration by parts to integrate the following definite integral:

Maak gebruik van die integrasie deur dele metode om die volgende definitiewe integraal te integreer:

(6)

$$\int_2^5 \frac{3x}{(x+1)^2} dx$$

2. Determine the following integral, using the substitution method.

Bepaal die volgende integraal deur die substitusie metode te gebruik:

(4)

$$\int 10x(x^2 + 3)^4 dx$$

3. Given the following demand and supply functions:

Gegee die volgende vraag- en aanbodfunksies:

(6)

$$Q_D = -0.5P + 70$$

$$Q_S = 0.7P - 50$$

a) Find the consumers' surplus.

Bepaal die verbruikerssurplus.

b) Find the producers' surplus.

Bepaal die produsentesurplus.

SECTION / AFDELING D

[24]

1. Given the following first-order differential equation:
Gegee die volgende eerste-orde differensiële vergelyking:

(8)

$$\frac{dy}{dt} - 6y = 18$$

- a) Use the formula for a general solution to solve the equation.
Gebruik die formule vir 'n algemene oplossing om die vergelyking op te los.
b) Determine the definite solution for the equation given that $y(0) = 1$.
Bepaal die spesifieke oplossing vir die vergelyking gegee dat $y(0) = 1$.

2. Given the following first-order difference equation:
Gegee die volgende eerste-orde verskil vergelyking:

(8)

$$y_t - y_{t-1} = 17$$

- a) Solve the equation.
Los die vergelyking op.
b) Check your answer by using $t = 0$ and $t = 1$.
Onderzoek jou antwoord deur van $t = 0$ en $t = 1$ gebruik te maak.
c) Comment on the nature of the time path.
Lewer kommentaar oor die aard van die tydkoers.

3. Given the following second-order difference equation:
Gegee die volgende tweede-orde verskil vergelyking:

(8)

$$y_t - 10y_{t-1} + 25y_{t-2} = 8$$

Find the:
Bepaal die:

- a) Particular solution.
Spesifieke oplossing.
b) Complementary function.
Komplementêre funksie.
c) General solution.
Algemene oplossing.

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