UNIVERSITY OF JOHANNESBURG

MAT3AW3 PAPER 1 October 2015 Time: 90 minutes Total =33 Mark: —

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Type your Surname, Initials and Student number in the orange box below :

Read the instructions handed out carefully. Use only **built-in syntax** and the Help Files to answer the following:

1. A small airplane can reach an altitude h (measured in meters) in time t (in minutes) according to the formula

$h = h_{max} - \frac{h_{max}}{10^{\frac{t}{50}}}$ where

$h_{max} = 5486\,m$ and describes the absolute ceiling of the plane.

1.1. Calculate the altitude reached by the airplane after 4.5 minutes. **(4)**

1026.81

1.2. Plot a graph of time against altitude over an interval $0 \le t \le 80$. Add gridlines to your graph. **(3)**

1.3. Use your graph to read off the approximate height of the airplane after an hour. **(1)**

2. Use the **inverse matrix method** to solve the following:

$$2p - 3q - r + 2s + 3t = 4$$
$$4p - 4q - r + 4s + 11t = 4$$
$$-5q - 2r + 2s - t = 9$$
$$2q + r + 4t = -5$$
$$-p + 2r + s + 3t = 1$$

Give your answer in decimals and in matrix form. **(6)**

Complete the following:

Hence $p =$; $q =$; r = ; s = and t =

3. The driven pendulum can be modelled with the **second order** differential equation
$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 12\cos t$
with initial conditions $x(0) = 0.75$;
$x'(0) = 0.25$

$$x''_i = f(t_i, x_i, x_i')$$

$$x_{i+1} = x_i + h\, x_i' + \frac{h^2}{2} x''_i$$

$$x'_{i+1} = x'_i + h\, x''_i$$

3.1. Write an Euler module for a second order differential equation. **(6)**

3.2. Produce Euler values over the domaine $0 \le t \le 15$ with step size $h = 0.1$. **DO NOT DISPLAY THE VALUES!** **(5)**

3.3. Plot the time-displacement graph. Label your axes **and** title your graph "Driven pendulum". **(5)**

3.4. Plot the displacement-velocity phase portrait, title your graph "Phase Portrait". **(3)**