UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING: MINERAL SURVEYING/EXTRACTION METALLURGY

MODULE MNM31-1
NUMERICAL METHODS
CAMPUS DFC

DECEMBER EXAMINATION 2015

DATE: DECEMBER 2015

ASSESSOR **DR PG DLAMINI**

INTERNAL MODERATOR **DR M KHUMALO**

DURATION: 3 HRS **MARKS: 100 MARKS**

SURNAME & INITIALS:

STUDENT NUMBER:

COURSE:

CONTACT NO:

INSTRUCTIONS : ANSWER ALL QUESTIONS BY CREATING APPROPRIATE MATHEMATICA CODES
NO EXTERNAL STORAGE DEVICES ARE PERMITTED
NON-PROGRAMMABLE SCIENTIFIC CALCULATORS ALLOWED

REQUIREMENTS: FORMULA BOOKLET

Question 1

a) Plot the graph of the function $f(x) = \ln(x-1) + \cos(x-1)$, over the interval
$0 \leq x \leq 3$. [5]

b) Use the regula falsi the approximate the root of $\ln(x-1) + \cos(x-1) = 0$ accurate to within 10^{-7}, determining the number of iterations required. From the graph above choose appropriate values of a and b (Choose values of a and b greater than 1) [11]

Question 2

a) Use Newton Raphson method to find a solution accurate accurate to within 10^{-5} to the problem
$$e^{6x} + 1.441\, e^{2x} - 2.079\, e^{4x} - 0.333 = 0$$
Use Newton Raphson method with $t_0 = 1$. [10]

Question 3

a) A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in metres, and the speed is in metres per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

(i) Find the polynomial of highest possible degree that interpolates the speed. [3]

(ii) Find the polynomial of highest possible degree that interpolates the distance. [3]

(iii) Use the polynomials obtained in (i) and (ii) to predict the position of the car and the speed when t = 10s . [2]

(iv) Plot a graph time vs speed using the data points and the polynomial obtained in (i) that interpolates the speed on the same axes. Use plot range [60,80]. [5]

(v) Plot a graph of time vs distance using the data points and the polynomial obtained in (ii) that interpolates the distance on the same axes. [4]

Question 3

a) Solve the following system on linear equations using the Jacobi method. Terminate iterations when the infinity norm of the residual is

10^{-4}. Use the **ZERO** vector as starting values. [14]

$$4x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - x_3 = -5$$
$$x_1 + x_2 - 3x_3 = -9$$

b) Use the LinearSolve command to solve the system of equations in (a). [3]

Question 4

Solve the set of non-linear equations

$\ln(x^2 + y^2) - \sin(x\,y) = \ln 2 + \ln \pi$ and, $e^{x-y} + \cos(x\,y) = 0$

using Newton's method with starting values for $x_0 = 2$ and $y_0 = 1.5$. Terminate the method when $\|f(x)\|_\infty < 10^{-6}$. [12]

Question 5

a) Use trapezoidal rule to approximate

$$\int_1^{1.5} (t+2)^6 - (t+2)^2 \sin(2(t+1))\,dx$$

using 50 sub-intervals [8]

b) Use Simpson's rule to solve the same integral in (a) with 16 intervals. [8]

Question 6

a) Use Euler's method with a step size of $h = 0.2$ to find an approximate solution of the following IVP

$$y' = \frac{t\,y + y}{t\,y + t}, \qquad y(2) = 4$$

over $2 \le t \le 4$. Plot the solution. [12]