UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING: MINERAL SURVEYING/EXTRACTION METALLURGY

MODULE MNM31-1
NUMERICAL METHODS

CAMPUS DFC

NOVEMBER EXAMINATION 2015

DATE: 10/11/2015 **SESSION: 08:30-11:30**

ASSESSOR **DR PG DLAMINI**

INTERNAL MODERATOR **DR M KHUMALO**

DURATION: 3 HRS **MARKS: 100 MARKS**

SURNAME & INITIALS:

STUDENT NUMBER:

COURSE:

CONTACT NO:

INSTRUCTIONS : ANSWER ALL QUESTIONS BY CREATING APPROPRIATE MATHEMATICA CODES
NO EXTERNAL STORAGE DEVICES ARE PERMITTED
NON-PROGRAMMABLE SCIENTIFIC CALCULATORS ALLOWED

REQUIREMENTS: FORMULA BOOKLET

Question 1

a) Plot the graph of the function $f(x) = e^x + 2^{-x} + 2\cos x - 6$, over the interval $1 \le x \le 3$. [5]

b) Use the bisection method the approximate the root of $e^x + 2^{-x} + 2\cos x - 6 = 0$ accurate to within 10^{-5}, determining the number of iterations required. From the graph above choose appropriate values of a and b [11]

Question 2

a) Find an approximation for t, accurate to within 10^{-4}, for the population equation

$$1\,564\,000 = 1\,000\,000\,e^t + \frac{435\,000}{t}(e^t - 1)$$

Use Newton Raphson method with $t_0 = 1$. Determine the number of iterations rquired. [10]

Question 3

a) It is suspected that the high amounts of tannin in mature oak leaves inhibit the growth of the winter moth larvae that extensively damage these trees in certain years. The following table lists the average weight of a sample of larvae at times in the first 28 days after birth. The sample was reared on young oak leaves.

Day	0	6	10	13	17	20	28
Sample average weight (mg)	6.67	17.33	42.67	37.33	30.10	29.31	28.74

(i) Find the polynomial of highest possible degree that interpolates the average weight of the sample. [3]

(ii) Use the polynomial obtained in (i) to approximate the average weight after 12 and 25 days. [2]

(iii) Find the polynomial of degree 4, $P_4(x)$, that best fits the data in the least squares sense. [3]

(iv) Use the polynomial obtained in (iii) to approximate the average weight after 12 and 25 days. [2]

(iii) Graph the data points and the polynomials obtained in (i) and (iii) on the same axes. Use different colors for the plots. Use plot range [-40,50]. [7]

Question 3

a) Solve the following system on linear equations using the Gauss Seidel method. Terminate iterations when the infinity norm of the residual is 10^{-4}. Use the **ZERO** vector as starting value. [14]

$$3x_1 - x_2 + x_3 = 1$$
$$3x_1 + 6x_2 + 2x_3 = 0$$
$$3x_1 + 3x_2 + 7x_3 = 4$$

b) Use the inverse method to solve the system of equations in (a). [3]

Question 4

Solve the set of non-linear equations

$x(1 - x) + 4y = 12$ and, $(x - 1)^2 + (2y - 3)^2 = 25$

using Newton's method with starting values for $x_0 = 0$ and $y_0 = 0$. Terminate the method when $\| f(x) \|_\infty < 10^{-5}$. [12]

Question 5

a) Use trapezoidal rule to approximate

$$\int_1^{1.5} x^2 \ln x \, dx$$

using 30 sub-intervals [8]

b) Use Simpson's rule to solve the same integral in (a) with 16 intervals. [8]

Question 6

a) Use Euler's method with a step size of $h = 0.1$ to find an approximate solution of the following IVP

$$y' = t^{-2}(\sin 2t - 2ty), \qquad y(1) = 2$$

over $1 \le t \le 2$. Plot the solution. [12]