UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

	Examiner	Moderator
Paper 1 30 Marks		
Paper 2 70 Marks		
EM/100		

	Examiner	Moderator
SM		
EM		
FM		

DEPARTMENT OF APPLIED PHYSICS AND ENGINEERING MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING:
ELECTRICAL ENGINEERING

CAMPUS: **DFC**

MODULE: **MAT3AW3** ENGINEERING MATHEMATICS 3

NOVEMBER EXAMINATION 2015
(PAPER 2)

DATE **06/11/2015** **SESSION** **08:30 - 11:30**

ASSESSOR **DR PG DLAMINI**

MODERATOR **DR Q VAN DER HOFF**

DURATION **3 HOURS** **MARKS** **100**

SURNAME AND INITIALS: ____________________

STUDENT NUMBER: ____________________

LECTURER: ____________________

CONTACT NUMBER: ____________________

NUMBER OF PAGES: 14

REQUIREMENTS : INFORMATION BOOKLET(AS ISSUED TO YOU IN THE TEST)
NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS : Please fill in your particulars on the front page.
Answer all the questions in the space provided.
Do not write in PENCIL. Pencil will not be marked.
You may use the back of each page (i.e. the left-hand side) for rough work OR to complete a question, if needed.
Rough work will not be marked.
PLEASE CHECK THAT YOU HAVE RECEIVED 14 PAGES

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1. Determine the following

(a) $L\{\cos 3t \cos 2t\}$ [3]

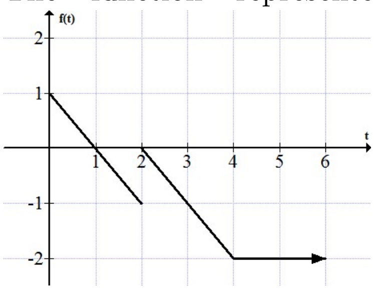
(b) $L\{te^{t-1}H(t-1)\}$ [3]

(c) $L^{-1}\left\{\dfrac{e^{2-4p}}{2p-1}\right\}$ [3]

(d) $\dfrac{1}{D^2+2D+1}\{12e^{-x}\}$ [3]

2. (a) Sketch the graph of the function $f(t) = [H(t-2) - H(t-5)]e^{-\frac{t}{4}}$ for $t \geq 0$. [3]

(b) The function represented by the graph below is defined analytically as



$$f(t) = \begin{cases} 1-t & 0 \leq t < 2 \\ 2-t & 2 \leq t < 4 \\ -2 & t \geq 4 \end{cases}$$

(i) Express $f(t)$ in terms of Heaviside functions. [2]

(ii) Find $L\{f(t)\}$ [4]

3. Determine the unique solutions of the following differential equations by using the **Laplace transform**, subject to the indicated initial conditions:

(a) $y'' + 2y' + y = 4\sin t, \qquad y(0) = -2, y'(0) = 1$ [9]

(b) $y'' + 2y' + 3y = e^{-t} + \delta(t - 3\pi), \qquad y(0) = y'(0) = 0$ [10]

4. Consider the motion of an object m attached to the end of a spring that is subject to a damping force. Assume that the damping force is proportional to the velocity of the mass and act in the direction opposite to the motion. Then the governing equation obtained by applying Newton's second law is given by

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)$$

where c and k are called the damping and spring constant respectively. Let $m = 1, c = 2.5$ and $k = 1$ and an external force $f(t) = e^{3t}$.

(a) Use the **Laplace transform** to solve the equation. [9]

(b) Use **D-operators** to solve the equation. [7]

(c) Discuss the solution as t approaches ∞ [2]

5. Find the general solutions of the following differential equations, using **D-operators**.

(a) $y'' - 2y' = 6e^{2x} - 4\sin x,$ [8]

(b) $(D^2 + D + 12)y = t^2 e^t$, [7]

6. Use **D-operators** to solve the following system of differential equations for $\boldsymbol{y}$ only.

$$(D^2 + 1)x + (D - 1)y = 1$$
$$(D + 1)x + (D^2 - 1)y = 2$$

[9]

7. Find a Fourier series for the following **even** function

$$f(t) = \begin{cases} t + \frac{\pi}{2} & -\pi \le t \le 0 \\ -t + \frac{\pi}{2} & 0 < t \le \pi \end{cases} \quad ; \qquad f(t) = f(t + 2\pi) \qquad [10]$$

8. Perform a numerical harmonic analysis on the following data, round off all calculations to two decimal places and present a **Fourier approximation** of $f(t)$ up to the **first harmonics**. [7]

t	0	0.25	0.5	1.0	1.5	2.0
$f(t)$	0	1.6	3.1	4.2	4.5	4.02

THE END