UNIVERSITY OF JOHANNESBURG

PROGRAM	:	NATIONAL DIPLOMA *INDUSTRIAL ENGINEERING TECHNOLOGY*
SUBJECT	:	**OPERATIONS RESEARCH III**
CODE	:	**BOA 321**
DATE	:	SUMMER SSA EXAMINATION 2015 11 DECEMBER 2015
DURATION	:	(SESSION 1) 08:00 - 11:00
WEIGHT	:	40 : 60
TOTAL MARKS	:	102

EXAMINER	:	MRS STEENKAMP
MODERATOR	:	T. NENZHELELE
NUMBER OF PAGES	:	4 PAGES AND 1 ANNEXURES

INSTRUCTIONS	:	PLEASE ANSWER ALL THE QUESTIONS. IF ALL VALUES ARE NOT GIVEN MAKE LOGIC ASSUMPTIONS
REQUIREMENTS	:	STUDENTS MAY USE CALCULATOR

QUESTION 1

A post office has a single line for customers waiting for the next available postal clerk. There are two postal clerks who work at the same rate. The arrival rate of customers follows a Poisson distribution, while the service time follows an exponential distribution. The average arrival rate is three per minute and the average service rate is two per minute for each of the two clerks.

1.1. What type of queuing model is exhibited in this problem?
1.2. What is the average length of the line?
1.3. How long does the average person spend waiting for a clerk?
1.4. What proportion of time are both clerks idle?

[11]

QUESTION 2

If 20% of qualified Engineers do not work in the field of Engineering after 10 years.

2.1 What is the probability that you will find a qualified Engineer, not working in the field, in a company with 5 qualified Engineers that have been working for more than 10 years.

2.2 What is the probability of finding two Engineers not working in the field in the same company?

[8]

QUESTION 3

I am a contestant on the TV show *Remote Jeopardy* which works as follows. I am first asked a question about Stupid Videos. If I answer correctly I earn \$ 100. I believe I have an 80% chance of answering such a question correctly. If I answer incorrectly the game is over and I win nothing. If I answer correctly, I may leave the show with \$100 or I could go on and answer a question on stupid TV shows. If I answer correctly I earn another \$300, but if I answer incorrectly, I lose all previous earnings and am sent home. My chance of answering correctly is 60%. If I answer the stupid TV shows correctly, I may leave with my earnings or I can go on and answer a question about Statistics. If I answer this question correctly, I earn another \$ 500 there is a 50%, but if I answer it incorrectly, I lose all previous earnings and am sent home. Draw a decision tree that can be used to maximize my expected earnings.

[20]

QUESTION 4

Truckco manufactures two types of trucks: truck 1 and truck 2. Each truck must go through a painting and assembly shop. It takes 5hr to paint type 1 trucks and 4.5hr to paint type 2 truck; 2800hr is available in the painting shop. It takes 3hr to assemble truck1 engines and 2 hr to assemble type 2 engines. The assembly shop has 3000hr available. Each type 1 truck contributes \$300 to profit, and each type 2 truck contributes $500 to profit. Formulate a LP that will maximize Truckco's profit, solve graphically.

[14]

QUESTION 5

Pipeline fluid flows are indicated below. Draw the network and determine the maximum flow from Node 1 to Node 4.

From Node	To Node	Fluid Flow
1	3	200
3	1	0
1	2	150
2	1	50
2	3	100
3	2	100
3	4	150
4	3	50
2	4	100
4	2	50

[10]

QUESTION 6

The University of JHB has had steady enrollments over the past five years. The school has its own book store called UJBooks but there are three other private book stores close to campus. Testcraft, Van Schaik and Ferndale textbooks. The University is concerned about the large number of students who are switching to private bookstores. One of the Vice Dean's has decided to give a student time to look into the problem. The following matrix with transitional probabilities was obtained

	UJBooks	Testcraft	Van Schaik	Ferndale textbooks
UJBooks	0.5	0	0.2	0.3
Testcraft	0.25	0.6	0.15	0
Van Schaik	0.1	0.1	0.7	0.1
Ferndale textbooks	0.05	0.1	0.05	0.8

At present all four bookstores have equal market share. What will market share be in the next period?

[12]

QUESTION 7

The Ayatollah Oil Company controls three oil fields. Field 1 can produce up to 40 million barrels of oil per day and field 2 can produce up to 50 million barrels of oil per day and field 3 can produce 10 million barrels of oil per day. Ayatollah sells oil to three countries: England, Japan and Netherlands. The shipping cost is shown in the table below. Each day England is willing to buy up to 50 million barrels of oil, and Japan is willing to buy up to 30 million barrels of oil and Netherlands is willing to buy 20 million barrels of oil

7.1 Formulate a balanced transportation problem using the Northwest corner rule to minimize Ayatollah's transport costs.

7.2 Formulate the transportation problem as a linear programming problem.

To

From	England	Japan	Netherlands
Field 1	$1	$2	$3
Field 2	$2	$3	$1
Field 3	$3	$ 1	$ 2

[15]

QUESTION 7

A department store sells 10 000 cameras per year. The store orders cameras from a regional warehouse. Each time an order is placed an ordering cost of $15 is incurred. The cost of holding is estimated to be 20% of the cost of the camera. The following quantity discounts are offered to the company.

0-100	100
101 – 150	90
151 - 200	80

Determine the optimal EOQ to minimize the cost to the company.

[12]

TOTAL : 102
FULL MARKS : 100

Key Equations

(2-1) $0 \leq P(\text{event}) \leq 1$

A basic statement of probability.

(2-2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Probability of the union of two events.

(2-3) $P(A|B) = \dfrac{P(AB)}{P(B)}$

Conditional probability.

(2-4) $P(AB) = P(A|B)P(B)$

Probability of the intersection of two events.

(2-5) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' theorem in general form.

(2-6) $E(X) = \sum_{i=1}^{n} X_i P(X_i)$

An equation that computes the expected value (mean) of a discrete probability distribution.

(2-7) $\sigma^2 = \text{Variance} = \sum_{i=1}^{n} [X_i - E(X)]^2 P(X_i)$

An equation that computes the variance of a discrete probability distribution.

(2-8) $\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$

An equation that computes the standard deviation from the variance.

(2-9) $\text{Probability of } r \text{ successes in } n \text{ trials} = \dfrac{n!}{r!(n-r)!} p^r q^{n-r}$

A formula that computes probabilities for the binomial probability distribution.

(2-10) $\text{Expected value (mean)} = np$

The expected value of the binomial distribution.

(2-11) $\text{Variance} = np(1-p)$

The variance of the binomial distribution.

(2-12) $f(X) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

The density function for the normal probability distribution.

(2-13) $Z = \dfrac{X - \mu}{\sigma}$

An equation that computes the number of standard deviations, Z, the point X is from the mean μ.

(2-14) $f(X) = \mu e^{-\mu x}$

The exponential distribution.

(2-15) $\text{Expected value} = \dfrac{1}{\mu}$

The expected value of an exponential distribution.

(2-16) $\text{Variance} = \dfrac{1}{\mu^2}$

The variance of an exponential distribution.

(2-17) $P(X \leq t) = 1 - e^{-\mu t}$

Formula to find the probability that an exponential random variable (X) is less than or equal to time t.

(2-18) $P(X) = \dfrac{\lambda^X e^{-\lambda}}{X!}$

The Poisson distribution.

(2-19) $\text{Expected value} = \lambda$

The mean of a Poisson distribution.

(2-20) $\text{Variance} = \lambda$

The variance of a Poisson distribution.

(3-1) $\text{EMV(alternative } i) = \sum X_i P(X_i)$

An equation that computes expected monetary value.

(3-2) $\text{EVwPI} = \sum(\text{best payoff in state of nature } i) \times (\text{probability of state of nature } i)$

An equation that computes the expected value with perfect information.

(3-3) $\text{EVPI} = \text{EVwPI} - \text{Best EMV}$

An equation that computes the expected value of perfect information.

(3-4) $\text{EVSI} = (\text{EV with SI} + \text{cost}) - (\text{EV without SI})$

An equation that computes the expected value (EV) of sample information (SI).

(3-5) $\text{Efficiency of sample information} = \dfrac{\text{EVSI}}{\text{EVPI}} 100\%$

An equation that compares sample information to perfect information.

(3-6) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' theorem—the conditional probability of event A given that event B has occurred.

(3-7) $\text{Utility of other outcome} = (p)(1) + (1-p)(0) = p$

An equation that determines the utility of an intermediate outcome.

order quantity (EOQ).

(6-1) Average inventory level $= \dfrac{Q}{2}$

(6-2) Annual ordering cost $= \dfrac{D}{Q} C_o$

(6-3) Annual holding cost $= \dfrac{Q}{2} C_h$

(6-4) $\text{EOQ} = Q^* = \sqrt{\dfrac{2DC_o}{C_h}}$

(6-5) $TC = \dfrac{D}{Q} C_o + \dfrac{Q}{2} C_h$

Total relevant inventory cost.

(6-6) Average dollar level $= \dfrac{(CQ)}{2}$

(6-7) $Q = \sqrt{\dfrac{2DC_o}{IC}}$

EOQ with C_h expressed as percentage of unit cost.

(6-8) $\text{ROP} = d \times L$

Reorder point: d is the daily demand and L is the lead time in days.

Equations 6-9 through 6-13 are associated with the production run model.

(6-9) Average inventory $= \dfrac{Q}{2}\left(1 - \dfrac{d}{p}\right)$

(6-10) Annual holding cost $= \dfrac{Q}{2}\left(1 - \dfrac{d}{p}\right) C_h$

(6-11) Annual setup cost $= \dfrac{D}{Q} C_s$

(6-12) Annual ordering cost $= \dfrac{D}{Q} C_o$

(6-13) $Q^* = \sqrt{\dfrac{2DC_s}{C_h\left(1 - \dfrac{d}{p}\right)}}$

Optimal production quantity.

Equation 6-14 is used for the quantity discount model.

(6-14) Total cost $= DC + \dfrac{D}{Q} C_o + \dfrac{Q}{2} C_h$

Total inventory cost (including purchase cost).

Equations 6-15 to 6-20 are used when safety stock is required.

(6-15) ROP = (Average demand during lead time) + SS

General reorder point formula for determining when safety stock (SS) is carried.

(6-16) ROP = (Average demand during lead time) + $Z\sigma_{dLT}$

Reorder point formula when demand during lead time is normally distributed with a standard deviation of σ_{dLT}.

(6-17) $\text{ROP} = \bar{d}L + Z(\sigma_d \sqrt{L})$

Formula for determining the reorder point when daily demand is normally distributed but lead time is constant, where $\bar{d}$ is the average daily demand, L is the constant lead time in days, and σ_d is the standard deviation of daily demand.

(6-18) $\text{ROP} = d\bar{L} + Z(d\sigma_L)$

Formula for determining the reorder point when daily demand is constant but lead time is normally distributed, where $\bar{L}$ is the average lead time in days, d is the constant daily demand, and σ_L is the standard deviation of lead time.

(6-19) $\text{ROP} = \bar{d}\,\bar{L} + Z\left(\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2}\right)$

Formula for determining reorder point when both daily demand and lead time are normally distributed; where $\bar{d}$ is the average daily demand, $\bar{L}$ is the average lead time in days, σ_L is the standard deviation of lead time, and σ_d is the standard deviation of daily demand.

(6-20) $\text{THC} = \dfrac{Q}{2} C_h + (\text{SS})C_h$

Total annual holding cost formula when safety stock is carried.

Equation 6-21 is used for marginal analysis.

(6-21) $P \geq \dfrac{\text{ML}}{\text{ML} + \text{MP}}$

Decision rule in marginal analysis for stocking additional units.

(14-1) $\pi(i) = (\pi_1, \pi_2, \pi_3, \ldots, \pi_n)$

Vector of state probabilities for period i.

(14-2) $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & & & & \vdots \\ P_{m1} & & \cdots & & P_{mn} \end{bmatrix}$

Matrix of transition probabilities, that is, the probability of going from one state into another.

(14-3) $\pi(1) = \pi(0)P$

Formula for calculating the state 1 probabilities, given state 0 data.

(14-4) $\pi(n + 1) = \pi(n)P$

Formula for calculating the state probabilities for the period $n + 1$ if we are in period n.

(14-5) $\pi(n) = \pi(0)P^n$

Formula for computing the state probabilities for period n if we are in period 0.

(14-6) $\pi = \pi P$

Equilibrium state equation used to derive equilibrium probabilities.

(14-7) $P = \left[\begin{array}{c|c} I & O \\ \hline A & B \end{array}\right]$

Partition of the matrix of transition for absorbing state analysis.

(14-8) $F = (I - B)^{-1}$

Fundamental matrix used in computing probabilities ending up in an absorbing state.

(14-9) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} \dfrac{d}{r} & \dfrac{-b}{r} \\ \dfrac{-c}{r} & \dfrac{a}{r} \end{bmatrix}$ where $r = ad - bc$

Inverse of a matrix with 2 rows and 2 columns.

λ = mean number of arrivals per time period
μ = mean number of people or items served per time period

Equations 12-1 through 12-7 describe operating characteristics in the single-channel model that has Poisson arrival and exponential service rates.

(12-1) L = average number of units (customers) in the system

$$= \frac{\lambda}{\mu - \lambda}$$

(12-2) W = average time a unit spends in the system (Waiting time + Service time)

$$= \frac{1}{\mu - \lambda}$$

(12-3) L_q = average number of units in the queue

$$= \frac{\lambda^2}{\mu(\mu - \lambda)}$$

(12-4) W_q = average time a unit spends waiting in the queue

$$= \frac{\lambda}{\mu(\mu - \lambda)}$$

(12-5) ρ = utilization factor for the system $= \frac{\lambda}{\mu}$

(12-6) P_0 = probability of 0 units in the system (i.e., the service unit is idle)

$$= 1 - \frac{\lambda}{\mu}$$

(12-7) $P_{n>k}$ = probability of more than k units in the system

$$= \left(\frac{\lambda}{\mu}\right)^{k+1}$$

Equations 12-8 through 12-12 are used for finding the costs of a queuing system.

(12-8) Total service cost $= mC_s$

where

m = number of channels

C_s = service cost (labor cost) of each channel

(12-9) Total waiting cost per time period $= (\lambda W)C_w$

C_w = cost of waiting

Waiting time cost based on time in the system.

(12-10) Total waiting cost per time period $= (\lambda W_q)C_w$

Waiting time cost based on time in the queue.

(12-11) Total cost $= mC_s + \lambda W C_w$

Waiting time cost based on time in the system.

(12-12) Total cost $= mC_s + \lambda W_q C_w$

Waiting time cost based on time in the queue.

Equations 12-13 through 12-18 describe operating characteristics in multichannel models that have Poisson arrival and exponential service rates, where m = the number of open channels.

$$(12\text{-}13)\ P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu - \lambda}}$$

for $m\mu > \lambda$

Probability that there are no people or units in the system.

$$(12\text{-}14)\ L = \frac{\lambda\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

Average number of people or units in the system.

$$(12\text{-}15)\ W = \frac{\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

Average time a unit spends in the waiting line or being serviced (namely, in the system).

$$(12\text{-}16)\ L_q = L - \frac{\lambda}{\mu}$$

Average number of people or units in line waiting for service.

$$(12\text{-}17)\ W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

Average time a person or unit spends in the queue waiting for service.

$$(12\text{-}18)\ \rho = \frac{\lambda}{m\mu}$$

Utilization rate.

Equations 12-19 through 12-22 describe operating characteristics in single-channel models that have Poisson arrivals and constant service rates.

(12-19) $L_q = \dfrac{\lambda^2}{2\mu(\mu - \lambda)}$

Average length of the queue.

(12-20) $W_q = \dfrac{\lambda}{2\mu(\mu - \lambda)}$

Average waiting time in the queue.

(12-21) $L = L_q + \dfrac{\lambda}{\mu}$

Average number of customers in the system.

(12-22) $W = W_q + \dfrac{1}{\mu}$

Average waiting time in the system.

Equations 12-23 through 12-28 describe operating characteristics in single-channel models that have Poisson arrivals and exponential service rates and a finite calling population.

(12-23) $P_0 = \dfrac{1}{\sum_{n=0}^{N} \dfrac{N!}{(N-n)!}\left(\dfrac{\lambda}{\mu}\right)^n}$

Probability that the system is empty.

(12-24) $L_q = N - \left(\dfrac{\lambda + \mu}{\lambda}\right)(1 - P_0)$

Average length of the queue.

(12-25) $L = L_q + (1 - P_0)$

Average number of units in the system.

(12-26) $W_q = \dfrac{L_q}{(N - L)\lambda}$

Average time in the queue.

(12-27) $W = W_q + \dfrac{1}{\mu}$

Average time in the system.

(12-28) $P_n = \dfrac{N!}{(N-n)!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 \quad \text{for } n = 0, 1, \ldots, N$

Probability of n units in the system.

Equations 12-29 to 12-31 are Little's Flow Equations, which can be used when a steady state condition exists.

(12-29) $L = \lambda W$

(12-30) $L_q = \lambda W_q$

(12-31) $W = W_q + 1/\mu$