UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

MODULE: **MAT2B4**

CAMPUS: **APK**

EXAM: **NOVEMBER 2015**

DATE:	**5 NOVEMBER 2015**	**SESSION: 8:00 - 10:00**
ASSESSOR(S):	**C MARAIS**	
INTERNAL MODERATOR:	**E JOUBERT**	
DURATION:	**2 HOURS**	**MARKS: 50**

SURNAME AND INITIALS:

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STUDENT NUMBER:

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CONTACT NR:

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NUMBER OF PAGES: 11 PAGES

INSTRUCTIONS: **ANSWER ALL THE QUESTIONS IN PEN**
YOU MAY USE A CALCULATOR
GOOD LUCK!

Question 1

Answer the following True or False questions and give a short justification/counter-example:

a) If G and G' are cyclic then $G \oplus G'$ is cyclic. [2]

TRUE	
FALSE	

b) If G is a finite cyclic group and m is a positive divisor of $|G|$, then G contains an element of order m. [2]

TRUE	
FALSE	

c) If p is prime, then $\mathbb{Z}_p$ is a group with respect to multiplication. [2]

TRUE	
FALSE	

d) If the order of a group, G, is 20, then $g^{20} = e$ for all $g \in G$. [2]

TRUE	
FALSE	

e) If $\varphi : G \to G'$ is a homomorphism with $G / Ker\varphi$ isomorphic to G', then φ is onto. [2]

TRUE	
FALSE	

Question 2

Prove that a subgroup H of G is normal in G if and only if $xHx^{-1} \subseteq H$ for all $x \in G$. [4]

Question 3

Consider the element $\alpha = (541)(3742)(1265) \in S7$.

a) Write α in disjoint cycle from. [1]

b) What is the order of α ? [1]

Question 4

a) List all the elements of the group $\mathbb{Z}/\langle 12 \rangle$ and hence find the order of $\mathbb{Z}/\langle 12 \rangle$. [3]

b) Does $\mathbb{Z}/\langle 12 \rangle$ have a subgroup of order 7? Why/why not? [2]

c) Show that $\mathbb{Z}/\langle 12\rangle$ is cyclic. [2]

d) Find all the elements of order 6 in $\mathbb{Z}/\langle 12\rangle$. [2]

Question 5

Let $H = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} : a,b \in \mathbb{R} \right\}$ where a and b, be a subset of $GL(2,\mathbb{R})$ (the group of all 2×2 matrices with entries from the real numbers with matrix addition as operation).

a) Show that H is a subgroup of $GL(2,\mathbb{R})$ using the Two-Step-Subgroup Test. [3]

b) Show that the map $\varphi : H \to (\mathbb{R},+)$ defined by $\varphi(A) = a$ where $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ is a group homomorphism. [2]

c) Find the kernel of φ. [2]

d) Find $\varphi^{-1}(3)$. [2]

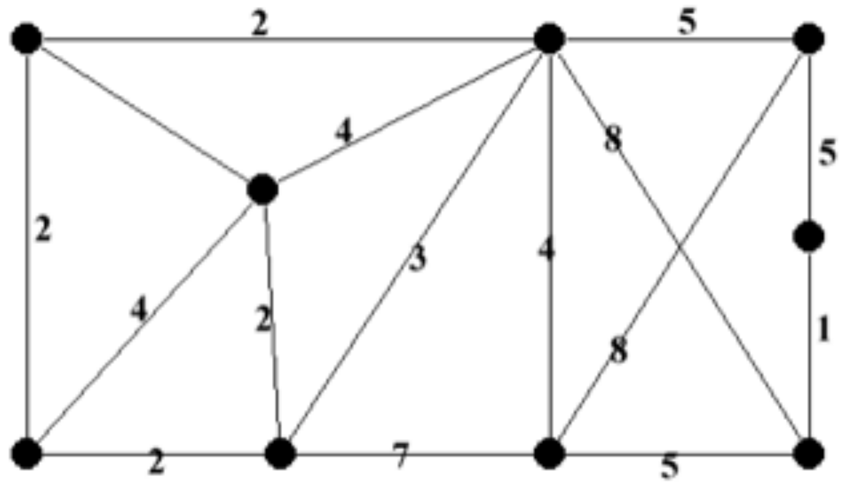
Question 6

a) Let φ be a homomorphism from a group G_1 to a group G_2. Prove that $Ker\varphi = \{e\}$ if and only if φ is one-to-one. [4]

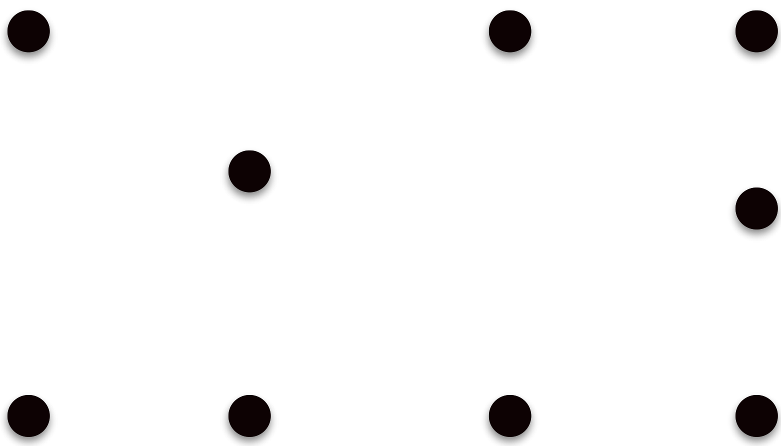
b) Let φ be a homomorphism from a group G_1 to a group G_2 such that $Ker\varphi = \{e, g\}$, and $\varphi(a) = b,\ a \in G_1,\ b \in G_2$. Show that there is another element in G_1 that maps onto b. [2]

Question 7

Consider the graph below and answer the following questions:



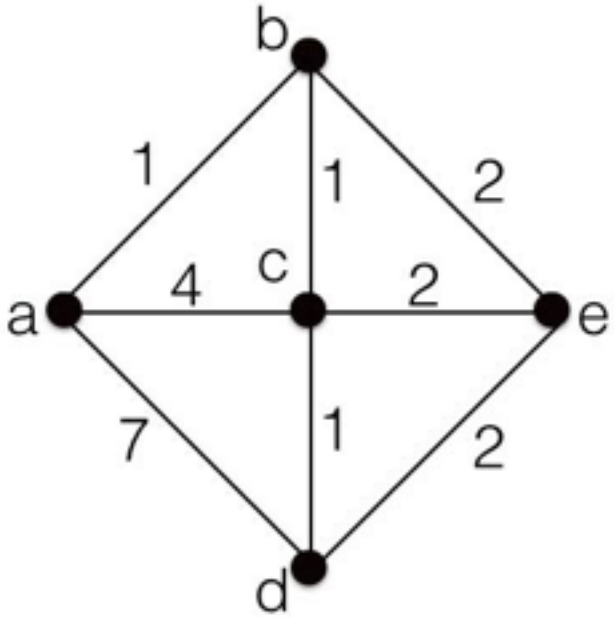
a) Use the Krushkal's algorithm to find a spanning tree of minimum weight. Draw your spanning tree in the space below with vertices as above. [4]



b) What is the weight of the resulting spanning tree? [1]

Question 8

Consider the graph below and answer the questions that follow:



a) Apply the Floyd-Marshall Algorithm to find the distance from a to every other vertex in the graph, showing your final answer in the table below. [4]

	a	b	c	d	e
a					
b					
c					
d					
e					

b) What is the shortest distance from a to e? Give the path to follow to obtain this distance. [2]