UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE:	ADVANCED BIO & ENVIRO MATHS & STATS – MAT1D01
CAMPUS:	APK
ASSESSMENT:	FINAL SUMMATIVE ASSESSMENT
SECTION:	CALCULUS

DATE:	31 OCTOBER 2015
SESSION:	08:30 – 10:30
ASSESSORS:	MR ET CHISORO
INTERNAL MODERATOR:	MR V VAN APPEL
DURATION:	60 MINUTES

40

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

NUMBER OF PAGES: 9 (INCLUDING COVER PAGE)

INSTRUCTIONS:

- ANSWER ALL THE QUESTIONS IN PEN
- NO PENCIL OR TIPEX ALLOWED
- SHOW ALL THE NECCESARY CALCULATIONS CLEARLY
- IF FORMULAS ARE USED THEY MUST BE STATED AS MARKS ARE GIVEN TO THEM
- NON-PROGRAMMABLE SCIENTIFIC CALCULATORS ARE ALLOWED
- ROUND OFF TO TWO DECIMAL PLACES UNLESS OTHERWISE STATED
- THE QUESTIONS CAN BE ANSWERED IN ANY ORDER

Question 1

[2]

Find all the critical points of the following function:

$$f(x) = 1 + 2x - 2x^2$$

Question 2

[4]

Organic waste deposited in a lake at $t = 0$ decreases the oxygen content of the water. Suppose the oxygen content is $C(t) = t^3 - 30t^2 + 6000$ for $0 \leq t \leq 25$.

Find the maximum oxygen content during this time.

Question 3

[5]

3.1 Given the following two functions: [1]

$$x^3 \text{ and } 1000x$$

Which of the two functions approaches infinity faster as x approaches infinity?

Find the following limits:

3.2 [1]

$$\lim_{x\to\infty} x^{-0.25}$$

3.3 [1]

$$\lim_{x\to\infty} (1 - e^{-4x})$$

3.4 The following is a possible absorption function. What happens to the function as c approaches infinity? [2]
[Assume that all parameters take on positive values]

$$f(c) = \frac{5c}{1+c}$$

Question 4

[5]

Find the indefinite integrals of the following functions:

4.1 [2]

$$f(x) = x^2 - \sin(x)$$

4.2 [3]

$$f(x) = \frac{1}{x\ln(x)}$$

Question 5

[10]

Suppose an object is thrown from a height of $h = 100\ m$ with an initial velocity of $v = 5\ m/s$ (upward) to find its trajectory in a Jupiter gravitational field of strength $a = -22.88\ m/s^2$.

5.1 Find the velocity $v(t)$ of the object as a function of time. [2]

5.2 Find the position $p(t)$ of the object as a function of time. [2]

5.3 How high will the object get? [3]

5.4 How long will it take to pass the initial height of $100\ m$ on the way down? [2]

5.5 How fast will it be moving? [1]

Question 6

[4]

In Saskatchewan, the growth of the walleye fish can be described by the following differential equation:

$$\frac{dL}{dt} = 64.3e^{-1.19t}$$

where L is the length in cm, and t is the time in years.

6.1 Find the solution of the differential equation if $L(0) = 0$. [3]

6.2 Find the limit of size as t approaches infinity. [1]

Question 7

[6]

Compute the following definite integrals:

7.1 [3]

$$\int_1^8 \left(\frac{2}{\sqrt[3]{t}} + 3\right) dt$$

7.2 [3]

$$\int_0^1 (3e^x + 2x^3) dx$$

Question 8

[4]

Find the change in the amount of chemical produced between times $t = 5$ and $t = 10$ if the amount P follows the following differential equation:

$$\frac{dP}{dt} = 5e^{-2t}$$

with initial condition $P(0) = 2$ and t is measured in minutes and P in moles.

End of assessment – Total 40 marks