UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	**ASMA2A1** SEQUENCES, SERIES AND VECTOR CALCULUS **(Engineering Group)**
CAMPUS	**APK**
EXAM	**NOVEMBER 2015**

EXAMINER(S)	MR F SCHULZ
INTERNAL MODERATOR	MRS C DUNCAN
DURATION	2.5 HOURS
MARKS	50

SURNAME AND INITIALS ____

STUDENT NUMBER ____

CONTACT NUMBER ____

NUMBER OF PAGES: 1 + 12

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1

Find the limit of the given sequence if it exists: [4]

$$a_n = \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)}{2 \cdot 6 \cdot 10 \cdots (4n-2)} \cdot \sqrt{n}.$$

Question 2

Determine whether the given statement is true or false, and motivate your answer clearly: If $a_n \to 0$ as $n \to \infty$, then $\sum a_n$ is convergent. [3]

Question 3

Test the following series for convergence or divergence: [11]

(3.1) $\sum_{n=1}^{\infty}(-1)^n\frac{(n!)^n}{n^{3n}}$ (4)

(3.2) $\sum_{n=1}^{\infty}\frac{\cos 2n}{1+3^n}$ (3)

(3.3) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/\sqrt{n}}}$ (4)

Question 4

Prove the following: If $\sum a_n$ is absolutely convergent, then it is convergent. [4]

Question 5

Find a Maclaurin series for the given function: [3]

$$f(x) = \begin{cases} \frac{1-\cos x}{x^2} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0. \end{cases}$$

Question 6

Find a Maclaurin series for f and determine its radius of convergence: [4]

$$f(x) = (1 - 3x)^{-3}.$$

Question 6

Determine $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \cos^2 t, te^{t^2} \right\rangle$ and $\mathbf{r}(0) = \langle 1, 0, 1 \rangle$. [4]

Question 7

Reparametrize the curve with respect to arc length measured from the point where $t = 0$ in the direction of increasing t: [4]

$$\mathbf{r}(t) = \langle 2\sin t, 4, 2\cos t \rangle .$$

Question 8

State the definition of the curvature of a smooth curve C. [2]

Question 9

Show that if there is a $c \in \mathbb{R}$ such that $|\mathbf{r}(t)| = c$ for all t, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$. [3]

Question 10

Prove that the curvature of a curve C with vector function $\mathbf{r}(t)$ is given by the following formula:[4]

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Question 11

A particle moves with position function

$$\mathbf{r}(t) = \left\langle t^3, 1 - t^2, t + 7 \right\rangle .$$

Determine the normal component of the accelaration of the particle. [4]