UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE
FAKULTEIT NATUURWETENSKAPPE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	**ASMA2A1** SEQUENCES, SERIES AND VECTOR CALCULUS **(BSc Group)**
CAMPUS	**APK**
EXAM	**NOVEMBER 2015**

EXAMINER(S)	MR F SCHULZ
INTERNAL MODERATOR	MRS C DUNCAN
DURATION	2.5 HOURS
MARKS	50

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 11

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1

State and prove the Monotonic Sequence Theorem. [6]

Question 2

Test the following series for convergence or divergence: [11]

(2.1) $\sum_{n=1}^{\infty}(-1)^n \frac{(n!)^n}{n^{3n}}$ (4)

(2.2) $\sum_{n=1}^{\infty} \frac{\cos 2n}{1+3^n}$ (3)

(2.3) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/\sqrt{n}}}$ (4)

Question 3

Find a power series with radius of convergence $R = 2$ and interval of convergence $I = (3, 7]$. You must clearly prove that the chosen power series satisfies the criteria spesified above. [5]

Question 4

A function f is even if $f(x) = f(-x)$ for all x in the domain of f. Suppose that f is even and that f is equal to the sum of its Maclaurin series for all $|x| < R$. Prove that [3]

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(2n)}(0)}{(2n)!} x^{2n} \text{ for } |x| < R.$$

Question 5

Find a Maclaurin series for f and determine its radius of convergence: [4]

$$f(x) = (1 - 3x)^{-3}\,.$$

Question 6

Determine $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \cos^2 t, te^{t^2} \right\rangle$ and $\mathbf{r}(0) = \langle 1, 0, 1 \rangle$. [4]

Question 7

Reparametrize the curve with respect to arc length measured from the point where $t = 0$ in the direction of increasing t: [4]

$$\mathbf{r}(t) = \langle 2\sin t, 4, 2\cos t\rangle \,.$$

Question 8

State the definition of the curvature of a smooth curve C. [2]

Question 9

Show that if there is a $c \in \mathbb{R}$ such that $|\mathbf{r}(t)| = c$ for all t, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$. [3]

Question 10

Prove that the curvature of a curve C with vector function $\mathbf{r}(t)$ is given by the following formula:[4]

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Question 11

A particle moves with position function

$$\mathbf{r}(t) = \left\langle t^3, 1 - t^2, t + 7 \right\rangle .$$

Determine the normal component of the accelaration of the particle. [4]