UNIVERSITY OF JOHANNESBURG

PROGRAM	:	NATIONAL DIPLOMA *INDUSTRIAL ENGINEERING TECHNOLOGY*
SUBJECT	:	**OPERATIONS RESEARCH**
CODE	:	**BOA 321**
DATE	:	SUMMER EXAMINATION 2015 16 NOVEMBER 2015
DURATION	:	(SESSION 2) 12:30 - 15:30
WEIGHT	:	40 : 60
TOTAL MARKS	:	102

EXAMINER	:	MRS STEENKAMP
MODERATOR	:	T. NENZHELELE
NUMBER OF PAGES	:	4 PAGES AND 1 ANNEXURES

INSTRUCTIONS	:	PLEASE ANSWER ALL THE QUESTIONS. IF ALL VALUES ARE NOT GIVEN MAKE LOGIC ASSUMPTIONS
REQUIREMENTS	:	STUDENTS MAY USE CALCULATORS GRAPH PAPER

QUESTION 1

From past experience we know that Intel produces motherboards in batches of 100. We know that 85% of batches are good batches and contain 2 % defective motherboards and 15 % of batches are bad batches and contain 30% defective motherboards. If one motherboard per batch is inspected use Bayes to calculate the following posterior probabilities: Given that G=batch is good B=batch is bad D= defective chip observed ND = non-defective chip is observed

1.1 $P(B|D) =$
1.2 $P(G|D) =$
1.3 $P(B|ND) =$
1.4 $P(G|ND) =$ **[8]**

QUESTION 2

A suburban specialty restaurant has developed a single drive-thru window. Customers order, pay, and pick up their food at the same window. Arrivals follow a Poisson distribution, while service times follow an exponential distribution. If the average number of arrivals is 6 per hour and the service rate is 2 every 15 minutes.

a) What type of queuing model is exhibited in this problem?
b) What is the average number of customers in the system?
c) What is the average number of customers waiting in line behind the person being served?
d) What proportion of the time is the server busy?
e) How much time will elapse (in hours) from the time a customer enters the line until he/she leaves the restaurant? **[10]**

QUESTION 3

A nuclear power company is deciding whether or not to build a nuclear power plant in Diablo Canyon or Roy Rogers City. The cost of building the power plant is \$10 million at Diablo and \$20 million at Roy Rogers City. If the company builds at Diablo, however, and an earthquake occurs at Diablo during the next five years, construction will be terminated and the company will lose \$10 million (and it will still have to build a power plant at Roy Rogers City). The company believes there is a 20% chance that an earthquake will occur at Diablo during the next five years. For \$1 million, a geologist can be hired to analyze the fault structure at Diablo Canyon. He will predict that an earthquake will occur or that an earthquake will not occur. The geologist's past record indicates that he will predict an earthquake 35% of the time and if he predicts an earthquake there is a 90% of there being an earthquake. If he predicts an earthquake will not occur there is a 75% chance of there being no earthquake. Draw a decision tree to minimize cost. What is the optimal solution? **[27]**

QUESTION 4

Johnny's apple shop sells home-made apple pies and freshly squeezed apple juice. Each apple pie requires 2 apples, and 1 apple yields 4 ounces of juice. Customer's use a self-service dispenser to pour apple juice in a container and are charged by the ounce at a rate of \$0.50 per ounce. The contribution to profit of the apple pie, factoring in the apples and remaining ingredients are \$2 per pie, and the contribution to profit of freshly squeezed apple juice if \$0.20 per ounce. In a given day, there must be at least 100 ounces of apple juice produced and at least 10 apple pies. The company has a supply of 60 apples per day. What is the optimal solution?

[15]

QUESTION 5

A firm producing a single product has three plants and four customers. The three plants will produce 3000, 5000 and 5000 units respectively, during the next time period. The firm has made a commitment to sell 4000 units to customer 1, 3000 units to customer 2 and at least 3000 to customer 3. Customer 4 wants to buy as many of the remaining units as possible. The cost associated with shipping a unit to different customers is given in the table below.

5.1 Formulate the transportation problem as a linear program.

5.2 Set up the initial table using the Northwest Corner rule

From	To Customer 1	2	3	4
Plant 1	R65	R63	R62	R64
Plant 2	R68	R67	R65	R62
Plant 3	R63	R60	R59	R60

[15]

QUESTION 6

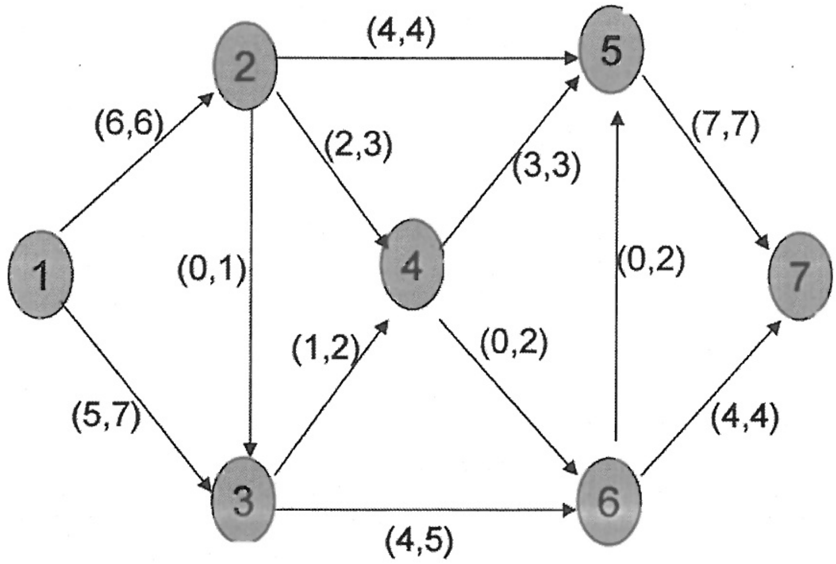
The following data consists of a matrix of transition probabilities (P) of three competing companies, and the initial market share $\pi(0)$. Assume that each state represents a company (Company 1, Company 2, Company 3, respectively) and the transition probabilities represent changes from one month to the next.

$$P = \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.7 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \qquad \pi(0) = (0.3, 0.6, 0.1)$$

Determine each of the three companies' estimated market shares in the next period.

[7]

QUESTION 7



Use max flow technique to determine the max flow through the network.

[10]

QUESTION 8

Peter Sellers sells barbeques. Depending on the quantity ordered, Peter offers the following discounts. The annual demand is 2000 units on average with a daily demand of 8 per day. The setup cost to produce the barbeque is $300. The daily production rate is 10 per day. He estimates the holding costs to be 10 % of the unit cost or $4 per year.

Quantity ordered		Price
From	To	
1	500	$ 45
501	1000	$ 42
1001	1500	$ 40

8.1. What is the optimal number of barbeques to produce at a time?
8.2 What would the optimal number be to order if your annual demand is 1000 and your ordering costs are $15?

[10]

TOTAL : 102
FULL MARKS : 100

Key Equations

(2-1) $0 \leq P(\text{event}) \leq 1$

A basic statement of probability.

(2-2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Probability of the union of two events.

(2-3) $P(A|B) = \dfrac{P(AB)}{P(B)}$

Conditional probability.

(2-4) $P(AB) = P(A|B)P(B)$

Probability of the intersection of two events.

(2-5) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' theorem in general form.

(2-6) $E(X) = \sum_{i=1}^{n} X_i P(X_i)$

An equation that computes the expected value (mean) of a discrete probability distribution.

(2-7) $\sigma^2 = \text{Variance} = \sum_{i=1}^{n} [X_i - E(X)]^2 P(X_i)$

An equation that computes the variance of a discrete probability distribution.

(2-8) $\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$

An equation that computes the standard deviation from the variance.

(2-9) Probability of r successes in n trials $= \dfrac{n!}{r!(n-r)!} p^r q^{n-r}$

A formula that computes probabilities for the binomial probability distribution.

(2-10) Expected value (mean) $= np$

The expected value of the binomial distribution.

(2-11) Variance $= np(1-p)$

The variance of the binomial distribution.

(2-12) $f(X) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

The density function for the normal probability distribution.

(2-13) $Z = \dfrac{X - \mu}{\sigma}$

An equation that computes the number of standard deviations, Z, the point X is from the mean μ.

(2-14) $f(X) = \mu e^{-\mu x}$

The exponential distribution.

(2-15) Expected value $= \dfrac{1}{\mu}$

The expected value of an exponential distribution.

(2-16) Variance $= \dfrac{1}{\mu^2}$

The variance of an exponential distribution.

(2-17) $P(X \leq t) = 1 - e^{-\mu t}$

Formula to find the probability that an exponential random variable (X) is less than or equal to time t.

(2-18) $P(X) = \dfrac{\lambda^x e^{-\lambda}}{X!}$

The Poisson distribution.

(2-19) Expected value $= \lambda$

The mean of a Poisson distribution.

(2-20) Variance $= \lambda$

The variance of a Poisson distribution.

(3-1) $\text{EMV(alternative } i) = \sum X_i P(X_i)$

An equation that computes expected monetary value.

(3-2) EVwPI $= \sum$(best payoff in state of nature i) $\times$ (probability of state of nature i)

An equation that computes the expected value with perfect information.

(3-3) EVPI = EVwPI − Best EMV

An equation that computes the expected value of perfect information.

(3-4) EVSI = (EV with SI + cost) − (EV without SI)

An equation that computes the expected value (EV) of sample information (SI).

(3-5) Efficiency of sample information $= \dfrac{\text{EVSI}}{\text{EVPI}} 100\%$

An equation that compares sample information to perfect information.

(3-6) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' theorem—the conditional probability of event A given that event B has occurred.

(3-7) Utility of other outcome $= (p)(1) + (1-p)(0) = p$

An equation that determines the utility of an intermediate outcome.

order quantity (EOQ).

(6-1) Average inventory level $= \dfrac{Q}{2}$

(6-2) Annual ordering cost $= \dfrac{D}{Q}C_o$

(6-3) Annual holding cost $= \dfrac{Q}{2}C_h$

(6-4) $\text{EOQ} = Q^* = \sqrt{\dfrac{2DC_o}{C_h}}$

(6-5) $TC = \dfrac{D}{Q}C_o + \dfrac{Q}{2}C_h$

Total relevant inventory cost.

(6-6) Average dollar level $= \dfrac{(CQ)}{2}$

(6-7) $Q = \sqrt{\dfrac{2DC_o}{IC}}$

EOQ with C_h expressed as percentage of unit cost.

(6-8) $\text{ROP} = d \times L$

Reorder point: d is the daily demand and L is the lead time in days.

Equations 6-9 through 6-13 are associated with the production run model.

(6-9) Average inventory $= \dfrac{Q}{2}\left(1 - \dfrac{d}{p}\right)$

(6-10) Annual holding cost $= \dfrac{Q}{2}\left(1 - \dfrac{d}{p}\right)C_h$

(6-11) Annual setup cost $= \dfrac{D}{Q}C_s$

(6-12) Annual ordering cost $= \dfrac{D}{Q}C_o$

(6-13) $Q^* = \sqrt{\dfrac{2DC_s}{C_h\left(1 - \dfrac{d}{p}\right)}}$

Optimal production quantity.

(6-14) Total cost $= DC + \dfrac{D}{Q}C_o + \dfrac{Q}{2}C_h$

Total inventory cost (including purchase cost).

Equations 6-15 to 6-20 are used when safety stock is required.

(6-15) ROP = (Average demand during lead time) + SS

General reorder point formula for determining when safety stock (SS) is carried.

(6-16) ROP = (Average demand during lead time) + $Z\sigma_{d\text{LT}}$

Reorder point formula when demand during lead time is normally distributed with a standard deviation of $\sigma_{d\text{LT}}$.

(6-17) $\text{ROP} = \bar{d}L + Z(\sigma_d\sqrt{L})$

Formula for determining the reorder point when daily demand is normally distributed but lead time is constant, where $\bar{d}$ is the average daily demand, L is the constant lead time in days, and σ_d is the standard deviation of daily demand.

(6-18) $\text{ROP} = d\bar{L} + Z(d\sigma_L)$

Formula for determining the reorder point when daily demand is constant but lead time is normally distributed, where $\bar{L}$ is the average lead time in days, d is the constant daily demand, and σ_L is the standard deviation of lead time.

(6-19) $\text{ROP} = \bar{d}\bar{L} + Z(\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2})$

Formula for determining reorder point when both daily demand and lead time are normally distributed; where $\bar{d}$ is the average daily demand, $\bar{L}$ is the average lead time in days, σ_L is the standard deviation of lead time, and σ_d is the standard deviation of daily demand.

(6-20) $\text{THC} = \dfrac{Q}{2}C_h + (\text{SS})C_h$

Total annual holding cost formula when safety stock is carried.

Equation 6-21 is used for marginal analysis.

(6-21) $P \geq \dfrac{\text{ML}}{\text{ML} + \text{MP}}$

Decision rule in marginal analysis for stocking additional units.

(14-1) $\pi(i) = (\pi_1, \pi_2, \pi_3, \ldots, \pi_n)$

Vector of state probabilities for period i.

(14-2) $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & & & & \vdots \\ P_{m1} & & \cdots & & P_{mn} \end{bmatrix}$

Matrix of transition probabilities, that is, the probability of going from one state into another.

(14-3) $\pi(1) = \pi(0)P$

Formula for calculating the state 1 probabilities, given state 0 data.

(14-4) $\pi(n + 1) = \pi(n)P$

Formula for calculating the state probabilities for the period $n + 1$ if we are in period n.

(14-5) $\pi(n) = \pi(0)P^n$

Formula for computing the state probabilities for period n if we are in period 0.

(14-6) $\pi = \pi P$

Equilibrium state equation used to derive equilibrium probabilities.

(14-7) $P = \left[\begin{array}{c|c} I & O \\ \hline A & B \end{array}\right]$

Partition of the matrix of transition for absorbing state analysis.

(14-8) $F = (I - B)^{-1}$

Fundamental matrix used in computing probabilities ending up in an absorbing state.

(14-9) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} \dfrac{d}{r} & \dfrac{-b}{r} \\ \dfrac{-c}{r} & \dfrac{a}{r} \end{bmatrix}$ where $r = ad - bc$

Inverse of a matrix with 2 rows and 2 columns.

λ = mean number of arrivals per time period
μ = mean number of people or items served per time period

Equations 12-1 through 12-7 describe operating characteristics in the single-channel model that has Poisson arrival and exponential service rates.

(12-1) L = average number of units (customers) in the system
$$= \frac{\lambda}{\mu - \lambda}$$

(12-2) W = average time a unit spends in the system (Waiting time + Service time)
$$= \frac{1}{\mu - \lambda}$$

(12-3) L_q = average number of units in the queue
$$= \frac{\lambda^2}{\mu(\mu - \lambda)}$$

(12-4) W_q = average time a unit spends waiting in the queue
$$= \frac{\lambda}{\mu(\mu - \lambda)}$$

(12-5) ρ = utilization factor for the system $= \frac{\lambda}{\mu}$

(12-6) P_0 = probability of 0 units in the system (i.e., the service unit is idle)
$$= 1 - \frac{\lambda}{\mu}$$

(12-7) $P_{n>k}$ = probability of more than k units in the system
$$= \left(\frac{\lambda}{\mu}\right)^{k+1}$$

Equations 12-8 through 12-12 are used for finding the costs of a queuing system.

(12-8) Total service cost $= mC_s$

where

m = number of channels

C_s = service cost (labor cost) of each channel

(12-9) Total waiting cost per time period $= (\lambda W)C_w$

C_w = cost of waiting

Waiting time cost based on time in the system.

(12-10) Total waiting cost per time period $= (\lambda W_q)C_w$

Waiting time cost based on time in the queue.

(12-11) Total cost $= mC_s + \lambda W C_w$

Waiting time cost based on time in the system.

(12-12) Total cost $= mC_s + \lambda W_q C_w$

Waiting time cost based on time in the queue.

Equations 12-13 through 12-18 describe operating characteris tics in multichannel models that have Poisson arrival and expc nential service rates, where m = the number of open channels.

(12-13)
$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu - \lambda}}$$

for $m\mu > \lambda$

Probability that there are no people or units in th system.

(12-14)
$$L = \frac{\lambda\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

Average number of people or units in the system.

(12-15)
$$W = \frac{\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

Average time a unit spends in the waiting line or bein serviced (namely, in the system).

(12-16)
$$L_q = L - \frac{\lambda}{\mu}$$

Average number of people or units in line waiting fc service.

(12-17)
$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

Average time a person or unit spends in the queue wai ing for service.

(12-18)
$$\rho = \frac{\lambda}{m\mu}$$

Utilization rate.

Equations 12-19 through 12-22 describe operating characteristics in single-channel models that have Poisson arrivals and constant service rates.

$$(12\text{-}19)\ L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

Average length of the queue.

$$(12\text{-}20)\ W_q = \frac{\lambda}{2\mu(\mu - \lambda)}$$

Average waiting time in the queue.

$$(12\text{-}21)\ L = L_q + \frac{\lambda}{\mu}$$

Average number of customers in the system.

$$(12\text{-}22)\ W = W_q + \frac{1}{\mu}$$

Average waiting time in the system.

Equations 12-23 through 12-28 describe operating characteristics in single-channel models that have Poisson arrivals and exponential service rates and a finite calling population.

$$(12\text{-}23)\ P_0 = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n}$$

Probability that the system is empty.

$$(12\text{-}24)\ L_q = N - \left(\frac{\lambda + \mu}{\lambda}\right)(1 - P_0)$$

Average length of the queue.

$$(12\text{-}25)\ L = L_q + (1 - P_0)$$

Average number of units in the system.

$$(12\text{-}26)\ W_q = \frac{L_q}{(N - L)\lambda}$$

Average time in the queue.

$$(12\text{-}27)\ W = W_q + \frac{1}{\mu}$$

Average time in the system.

$$(12\text{-}28)\ P_n = \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n P_0 \quad \text{for } n = 0, 1, \ldots, N$$

Probability of n units in the system.

Equations 12-29 to 12-31 are Little's Flow Equations, which can be used when a steady state condition exists.

$$(12\text{-}29)\ L = \lambda W$$

$$(12\text{-}30)\ L_q = \lambda W_q$$

$$(12\text{-}31)\ W = W_q + 1/\mu$$