UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

BACHELOR OF ENGINEERING TECHNOLOGY
(Chemical, Civil, Electrical, Industrial, Mechanical)

CAMPUS: DFC
MODULE: ENGINEERING MATHEMATICS 2A - MATE2A2
ASSESSMENT: SUPPLEMENTARY EXAM

JULY 2021

ASSESSORS: DR PG DLAMINI, MR IK LETLHAGE & DR SM SIMELANE
MODERATOR: DR SR HERBST
MARKS: 65
TIME: 180 MINUTES

NUMBER OF PAGES: 2

REQUIREMENTS: NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS: Write your own answers down on paper using ink (**clearly showing all steps**), scan and upload as **a single pdf document** into Blackboard (Do not email us your solutions/work, it will not be marked).

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1. Show that the differential equation, given below, is exact and use the appropriate method to solve it. Justify all the steps taken and give full details. [5]

$$(y^2 \cos x - 3x^2 y - 2x)\, dx + (2y \sin x - x^3 + \ln y)\, dy = 0$$

2. Solve the initial value problem [5]

$$\left(x^3 + 3xy^2\right) \frac{dy}{dx} = y^3 + 3x^2 y \quad \text{subject to} \quad y(2) = 1$$

3. Identify the given differential equation and use the appropriate method to find its general solution. [6]

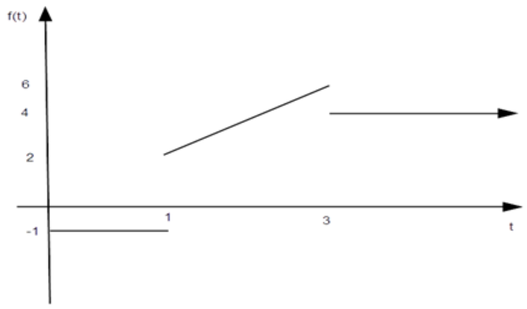
$$x\frac{dy}{dx} + y = x^3 y^2$$

4. Newton's Law of Heating/Cooling states that the rate of change of the temperature of an object is directly proportional to the difference between the temperature of the object and the ambient temperature, that is, the temperature of the surroundings.

(a) Use a first order ordinary differential equation to model Newton's Law as stated above. [1]

(b) A can of soft drink, with temperature $24°C$, is put in a freezer, where the temperature is $-7°C$. Ten minutes later, the temperature of the soft drink is measured and it is found to be $-1°C$. Use the differential equation in (a) to calculate the temperature of the soft drink 15 minutes after it was put in the freezer. [5]

5. Given the function $f(t)$ as



(a) Express $f(t)$ as a piecewise defined function. [3]

(b) Express the function $f(t)$ in unit step/Heaviside form. Fully simplify. [1]

(c) Compute the Laplace transforms of $f(t)$. [4]

6. Determine [5]

$$\mathcal{L}^{-1}\left\{\frac{2s+4}{2s^2+2s+1}\right\}$$

7. An arbitrary system is modeled by

$$y'' + 2y' - 3y = e^{-3(t-2)}\, u(t-2)$$

(a) Solve the differential equation above for $y(t)$ if $y(0) = y'(0) = 1$. [8]

(b) Is the system in steady or transient state? Discuss. [1]

8. Given

$$f(x) = \begin{cases} 2, & -2 \le x \le 0 \\ x, & 0 \le x \le 2 \end{cases}$$

and $f(x) = f(x+4)$

(a) Find the trigonometric Fourier series of $f(x)$ and present your answer as a sum of the first four harmonics. [12]

(b) Find the complex Fourier series of $f(x)$. [6]

9. Find the Fourier transform of

$$f(x) = \begin{cases} e^{2ix}, & \text{if} \quad x < |1| \\ 0, & \text{otherwise.} \end{cases}$$

Express your answer in trigonometric form. [4]

TOTAL OF 66 MARKS AVAILABLE