UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

BACHELOR OF ENGINEERING TECHNOLOGY
(Chemical, Civil, Electrical, Industrial, Mechanical)

CAMPUS: DFC
MODULE: ENGINEERING MATHEMATICS 2A - MATE2A2
ASSESSMENT: MAIN EXAM

03 JUNE 2021

ASSESSORS: DR PG DLAMINI, MR IK LETLHAGE & DR SM SIMELANE
MODERATOR: DR SR HERBST
MARKS: 65
TIME: 180 MINUTES

NUMBER OF PAGES: 2

REQUIREMENTS: NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS: Write your own answers down on paper using ink (**clearly showing all steps**), scan and upload as **a single pdf document** into Blackboard (Do not email us your solutions/work, it will not be marked).

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1. Show that the differential equation is exact and use the appropriate method to solve it. Justify all the steps taken and give full details. [5]

$$\left(\tan^2 x - \frac{y}{x^2+y^2}\right) dx + \left(\frac{x}{x^2+y^2} - e^y\right) dy = 0$$

2. Solve the initial value problem [5]

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \quad \text{subject to} \quad y(1) = 0$$

3. Identify the given differential equation and use the appropriate method to find its general solution. [6]

$$x\frac{dy}{dx} + y = \frac{x}{y^3}$$

4. Under favourable conditions, the rate of change of the population of a certain bacterium is directly proportional to the size of the population. Suppose that, on a particular day at 10h00, there are 100000 bacteria and that, by 11h00, the population had doubled in size.

(a) Model this phenomenon with a first order ordinary differential equation, using P to denote the population size. [1]

(b) Solve the equation in (a) and express P in terms of time t. [4]

(c) Calculate how long it will take for the population to reach 500000 bacteria. [2]

5. Given the function $f(t)$ as

$$f(t) = \begin{cases} t^2 & 0 \le t < 2 \\ t-1 & 2 \le t < 4 \\ -3 & t \ge 4 \end{cases}$$

(a) Sketch the graph of $f(t)$. [3]

(b) Express the function $f(t)$ in unit step/Heaviside form. Fully simplify. [2]

(c) Compute the Laplace transforms of $f(t)$. [5]

6. Determine [4]

$$\mathcal{L}^{-1}\left\{\frac{3s-2}{9s^2+25}\right\}$$

7. The system of differential equations for the charge on the capacitor $q(t)$ and current $i(t)$ in an electrical network is given by

$$R_1\,\frac{dq}{dt} + \frac{1}{C}\,q + R_1\,i_3 = E(t)$$
$$L\,\frac{di_3}{dt} + R_2\,i_3 - \frac{1}{C}\,q = 0$$

(a) Find the charge on the capacitor when $L = 1\ h$, $R_1 = 1\ \omega$, $R_2 = 1\ \omega$, $C = 1\ f$,

$$E(t) = \begin{cases} 0 & 0 \le t < 1 \\ 50\,e^{-t} & t \ge 1 \end{cases},$$

$i_3(0) = 0$ and $q(0) = 0$. [7]

(b) What is the charge on the capacitor at $t = 2$? [1]

8. Given the function

$$f(x) = \begin{cases} \cos x, & 0 \le x \le \dfrac{\pi}{2} \\ 0, & \dfrac{\pi}{2} \le x \le \pi \end{cases}$$

(a) Sketch the graph of $f(x)$, and then extend the sketch so that it becomes the graph of an odd function. [2]

(b) Find the half range Fourier sine series of $f(x)$. [8]

9. Show that the complex Fourier series of the function $f(x) = e^{-x}$, for $0 \le x \le 2$ with period 2, is given by.

$$\sum_{n=-\infty}^{\infty} \frac{1-e^{-2}}{2(1+in\pi)} e^{in\pi x}$$

[6]

10. Represent

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$$

as a Fourier cosine integral. [4]

TOTAL OF 65 MARKS AVAILABLE