UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE:	**MATCXB1**
COURSE:	**ENGINEERING MATHEMATICS 1**
CAMPUS:	**DFC**
ASSESSMENT:	**MAIN EXAMINATION**

DATE: **2 NOVEMBER 2021**

TIME: **8:30 - 11:30.**

ASSESSOR: **MR M.P. SELOANE**

INTERNAL MODERATOR **MR E.Z. MORAPELI**

DURATION: **180 MINUTES**

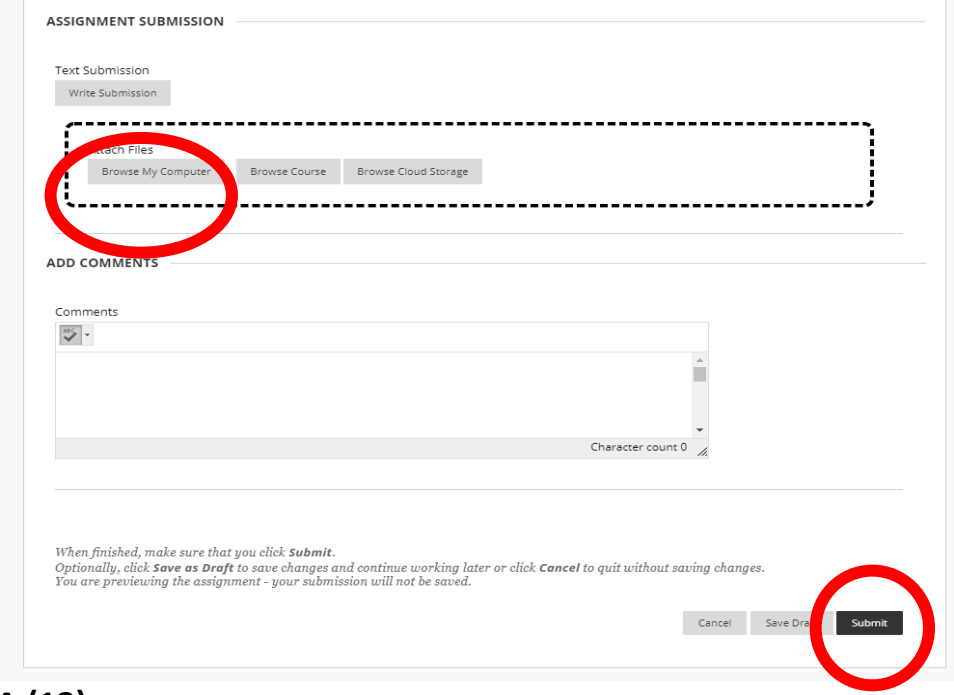
MARKS: **60**

NUMBER OF PAGES: **3 PAGES (INCLUDING FRONT PAGE AND ONLINE INSTRUCTIONS).**

ONLINE INSTRUCTIONS:

- **Write the complete solutions of the questions on paper. The steps, where applicable will be marked.**
- **Write your student number, surname, and initials on all pages.**
- **Use either blue or black pen.**
- **Non-programmable scientific calculators are allowed.**
- **The complete solutions must be in your own handwriting.**
- **All pages must be together, in sequential order, and please number the pages.**
- **Scan your work and save this as a pdf file on your device.**
- **Use the following file name when you save your work and upload your answer sheet: surname and initials_studentnumber_MAIN EXAMINATION**
- **Submit this as one pdf file on uLink: click on word "MAIN EXAMINATION". To submit you will get a screen like below – click on "Browse my Computer", then go to the file on your device and select, then click "Submit". You cannot upload a photo. You cannot upload page-by-page, only ONE pdf document. Submissions via e-mail if experiencing difficulties with bb will be accepted. If you upload the wrong file, or no file, the test cannot be marked, and you will get zero.**

- **You have <u>unlimited</u> submission opportunities before the deadline but only the last submission will be mark.**
- **If you experience any problems when submitting your test send an e-mail to Mr M P Seloane (pseloane@uj.ac.za) immediately.**
- **No late submission can be accepted.**



SECTION A (12)

CHOOSE ONE CORRECT ANSWER FROM THE ANSWERS GIVEN. WRITE DOWN ONLY THE LETTER CORRESPONDING TO YOUR CHOSEN ANSWER.

1. If one root of $ax^2 + bx + c = 0$ is $x = 1 + \sqrt{-9}$, then the other root is:

   A. $1 - 3j$ B. $1 + 3j$ C. -4 D. None of these

2. The rectangular form of $ln|2j|$ is:

   A. $1{,}1 + 0{,}69j$ B. $ln2 + 90j$ C. $1{,}57j + 0{,}69$ D. None of these

3. The value of $\lim\limits_{x \to 2} \frac{ln(3-x)}{2-x}$ is equal to:

   A. ∞ B. 1 C. 2 D. None of these

4. The derivative of $4 \, . \, e^{x^2-3}$ is:

A $4 \, . \, e^{x^2-3}$ B $8 \, . \, e^{x^2-3}$

C $8x \, . \, e^{x^2-3}$ D $4 \ln(x^2 - 3)$

5. If the velocity of an object is given by $v(t) = 3t^2 - 12t + 3$, then the expression of displacement is:

A. $3t^3 - 12t^2 + 3t + C$ B. $t^3 - 3t^2 + 3t + C$

C. $6t - 12 + C$ D. None of these

6. $\int \frac{2}{\sqrt{1-x}} dx$ is equal to:

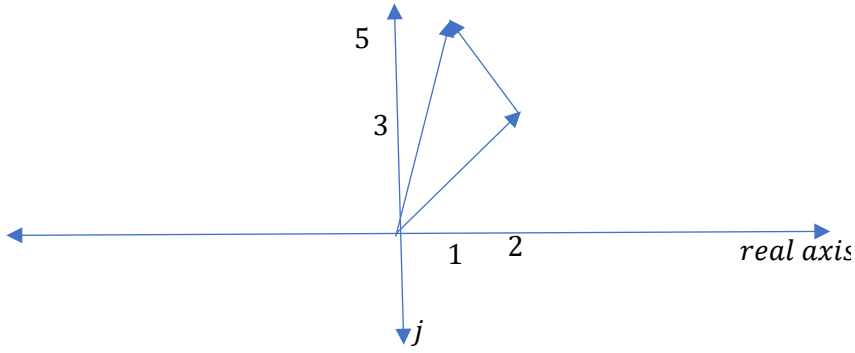
A. $-4\sqrt{1-x} + C$ B. $4\sqrt{1-x} + C$ C. $-\sqrt{(1-x)^3} + C$ D. None of these

(2 X 6 = 12)

SECTION B (48)

SHOW ALL IMPORTANT STEPS AND LEAVE ANSWERS WITH TWO DECIMAL PLACES, WHERE APPLICABLE

1. Given $z_1 = 3j + 2$ and $z_2 = 2j - 1$, use the Argand diagram to show that their sum, $z_1 + z_2$, is equal to $1 + 5j$. (3)



2. Simplify: $\frac{1}{e^{2j}} + 4 + 1{,}3\angle 60^{\circ}$, leave answer in exponential form. (4)

$\frac{1}{e^{2j}} + 4 + 1{,}3\angle 60^{\circ} = -0{,}4161 - 0{,}9093j + 4 + 0{,}65 + 1{,}126j$ √√

$= 4{,}234 + 0{,}2167j$ √

$= 4{,}24e^{0{,}5j}$ √

3. If $z_1 = -3j;\ z_2 = (\cos(30^{\circ}) - j\sin(30^{\circ}))$ and $z_3 = 3e^{4j}$, use De Moivre's theorem and evaluate: $\frac{(\overline{z_2})^2(z_1)^4}{(z_3)^3}$. leave your answer in rectangular form. (5)

$\frac{(\overline{z_2})^2(z_1)^4}{(z_3)^3} = \frac{\left(1\angle\frac{\pi}{6}\right)^2(3\angle -1{,}571)^4}{(3\angle 4)^3}$ √

$= \frac{(3)^4}{(3)^3}\angle\left(2\left(\frac{\pi}{6}\right) + 4(-1{,}571) - 3(4)\right)$ √

$= 3\angle -17{,}237$ √

$= -0{,}13 + 3j$ √√

4. Find all the roots of $z^3 + 3j - 1 = 0$. Leave your answer in rectangular form. (6)

$z^3 + 3j - 1 = 0$

$z = (1 - 3j)^{\frac{1}{3}}$ √

$= (3{,}1623\angle -1{,}249)^{\frac{1}{3}}$ √

$= (3{,}1623)^{\frac{1}{3}}\angle\frac{1}{3}(-1{,}249 + 2\pi);\ k = 0; 1; 2$ √

$\therefore\ root_1 = (3{,}1623)^{\frac{1}{3}}\angle -0{,}4163$

$= 1{,}34 - 0{,}59j$ √

$\therefore\ root_2 = (3{,}1623)^{\frac{1}{3}}\angle 1{,}6781$

$= -0{,}16 + 1{,}46j$ √

$\therefore\ root_3 = (3{,}1623)^{\frac{1}{3}}\angle 3{,}7725$

$= -1{,}19 - 0{,}87j$ √

5. Find $f'(x)$ using the definition if $f(x) = 3 - 4x^2$. (4)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(3-4(x+h)^2)-(3-4x^2)}{h} \quad \surd$$

$$= \lim_{h\to 0} \frac{(3-4x^2-8xh-4h^2)-(3-4x^2)}{h} = \lim_{h\to 0} \frac{-4h(2x-4h)}{h} = -8x \quad \surd\surd\surd$$

6. Determine $\frac{dy}{dx}$ given that:

6.1 $y = \ln\left(3^x.\sqrt{x^2-2x}\right)$. (4)

$$y = xln3 + \frac{1}{2}ln(x^2-2x) \quad \surd$$

$$\therefore \frac{dy}{dx} = ln3 + \frac{2x-2}{2(x^2-2x)} \quad \surd$$

$$= ln3 + \frac{x-1}{x^2-2x} \quad \surd\surd$$

6.2 $y = ln(x^2) - cos^2(2x)$. (3)

$$y = ln(x^2) - cos^2(2x) = 2lnx - (\cos 2x)^2 \quad \surd$$

$$\therefore \frac{dy}{dx} = \frac{2}{x} - 2(\cos 2x) \times -2\sin 2x = \frac{2}{x} + 2\sin 4x \quad \surd\surd$$

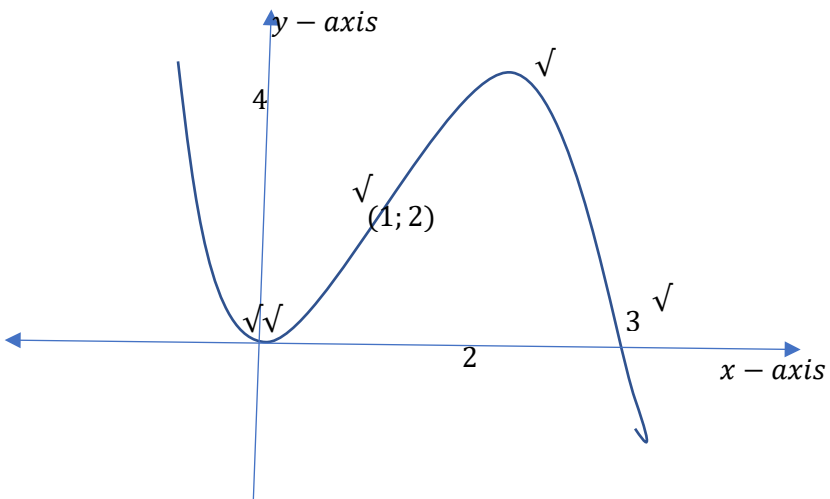
7. Show that $f'''(x) = \frac{2f\prime(x)}{x^2}$ if $f(x) = x^3 - 2ln(x) + 4$. (5)

$$\therefore f'(x) = 3x^2 - \frac{2}{x} \quad \surd$$

$$\therefore f''(x) = 6x + \frac{2}{x^2} \quad \surd \qquad \text{and } f'''(x) = 6 - \frac{4}{x^3} \quad \surd$$

$$\frac{2f\prime(x)}{x^2} = \frac{2\left(3x^2-\frac{2}{x}\right)}{x^2} = 6 - \frac{4}{x^3} = f'''(x) \quad \surd\surd$$

8. Sketch the graph of $f(x) = x^3 + 3x^2$. Clearly show all turning points, point of inflection, and intercepts with the axes. (5)



9. Determine:

9.1 $\int_0^2 \frac{3}{x+1}\,dx.$ (2)

$\int_0^2 \frac{3}{x+1}\,dx = 3ln|x+1| = 3\big(ln(3) - ln(1)\big) = 3{,}3$ square units $\surd\surd$

9.1 $\int \sin(x)cos^2(x).$ (3)

$$\int \sin(x)cos^2(x) = \int \sin(x)\,(\cos(x))^2 dx$$
$$= -\frac{(\cos(x))^3}{3} + C \qquad \surd\surd\surd$$

9.2 $\int \frac{x^2-6x+5}{x-3}\,dx.$ (4)

$\frac{x^2-6x+5}{x-3} = x - 3 - \frac{4}{x-3}$ $\surd\surd$

$\therefore \int \frac{x^2-6x+5}{x-3}\,dx = \int \left(x - 3 - \frac{4}{x-3}\right)$

$= \frac{(x-3)^2}{2} 4ln(x-3)$ $\surd\surd$

END - OF - EXAMINATION