

MAFTOAI Special Exam 2021 Memo.

(a) $f(x) = e^{2x-1}$
 $x = e^{2y-1}$
 $\ln x = 2y - 1$ ✓
 $y = \frac{1}{2}(\ln x + 1)$ ✓ (2)

(b) $h(x) = \frac{x}{2} - \frac{7}{2}$
 $h(x^2 - 2) = 0$
 $\frac{x^2 - 2 - 7}{2} = 0$ ✓
 $\therefore (x+3)(x-3) = 0$
 $x = \pm 3$ ✓ (2)

(c) $f(x) = x^3 - 8$ $g(x) = x^2 - 4$
(i) $g \circ f = g(x^3 - 8)$ ✓
 $= (x^3 - 8)^2 - 4$ ✓
 $= x^6 - 16x^3 + 60$ ✓ (3)

(ii) $(gf)(x) = (x^2 - 4)(x^3 - 8) = 0$ ✓
 $\therefore (x+2)(x-2)(x^3 - 8) = 0$ ✓
 $x = -2, x = 2, x = 2$
 $\therefore x = \pm 2$ ✓ (3)

2. $L = \frac{k}{d^2}$
 $70 = \frac{k}{10^2}$ ✓
 $k = 7000$ ✓

(3)

$$\begin{aligned} \text{(ii)} \quad \log_2(\log_3 x) &= 1 \\ \log_3 x &= 2^1 \quad \checkmark \\ x &= 3^2 \\ \therefore x &= 9 \quad \checkmark \end{aligned}$$

(2)

5a. Since $r=5$, it touches

y-axis at $A(0;3)$ \checkmark

Eqn. of circle:

$$(x-5)^2 + (y-3)^2 = 25 \quad \checkmark$$

x-int:

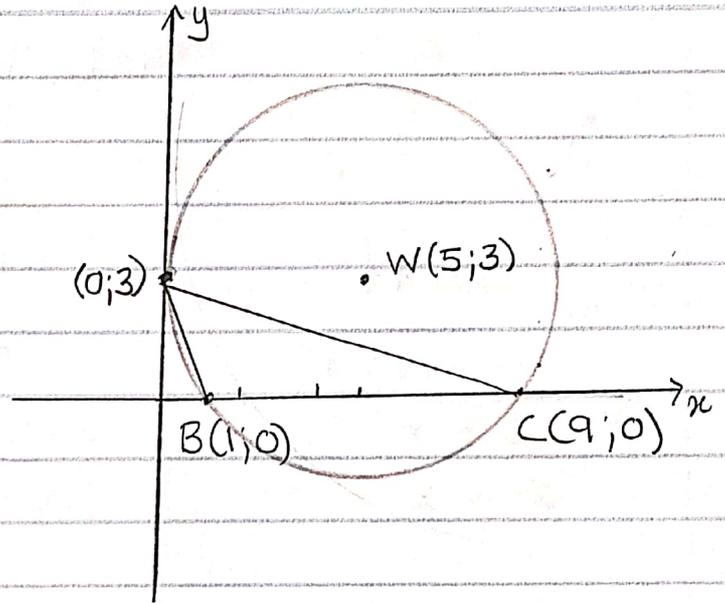
$$(x-5)^2 + (-3)^2 = 25 \quad \checkmark$$

$$x^2 - 10x + 25 + 9 - 25 = 0$$

$$(x-9)(x-1) = 0$$

$$\therefore x=1 \text{ and } x=9$$

$$\therefore B(1;0) \text{ and } C(9;0) \quad \checkmark$$



$$\text{Area } \triangle ABC = \frac{1}{2}bh$$

$$= \frac{1}{2}(8)(3)$$

$$= 12 \quad \checkmark$$

(4)

$$\text{(b)} \quad m_{DB} = 2 \quad \checkmark$$

$DB \perp AC$, diagonals of rhombus \checkmark

$$\therefore m_{AC} = -\frac{1}{2} \quad \checkmark$$

$$\text{Eqn of AC: } y-7 = -\frac{1}{2}(x-4) \quad \checkmark$$

$$y = -\frac{1}{2}x + 9 \quad \checkmark$$

(3)

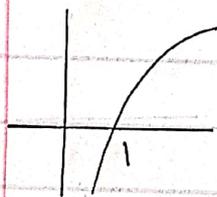
(4)

- 6a. • horizontal shift 2 units left ✓
- reflection in x-axis ✓
- vertical shift 1 unit up ✓

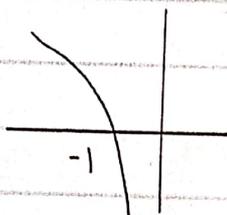
Correct order ✓

(2)

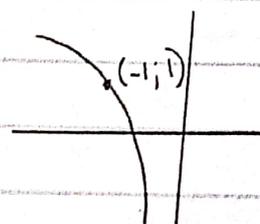
b. $\ln x$



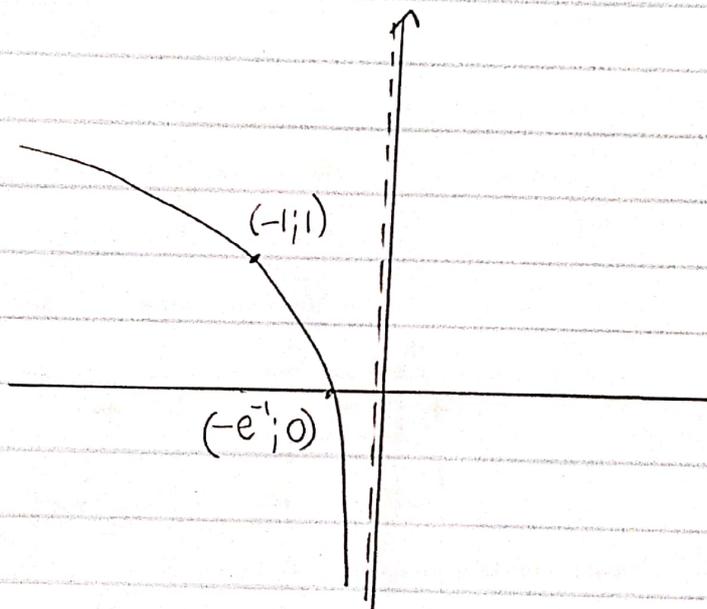
$\rightarrow \ln(-x)$



$\rightarrow 1 + \ln(-x)$



✓

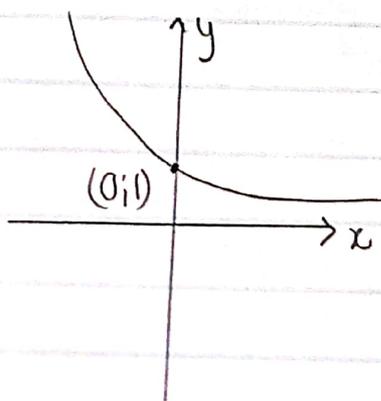


$$\begin{aligned} \ln(-x) &= -1 \\ -x &= e^{-1} \quad \checkmark \\ x &= -e^{-1} \end{aligned}$$

- x-int ✓
- extra point ✓
- asymptote ✓
- shape ✓

(3)

c(i)



(2)

(5)

(ii) Decreasing

(1)

(iii) $y = \log_{\frac{1}{3}} x$

(1)

(iv) $y = -5$

(1)

7. $f(x) = \frac{x^2 - 4}{x^2}$

a) $x \neq 0$, \therefore no y-int. ✓

$y = 0 \Rightarrow x = \pm 2$ ✓

(2)

b) $x = 0$

(1)

c) $y = 1$

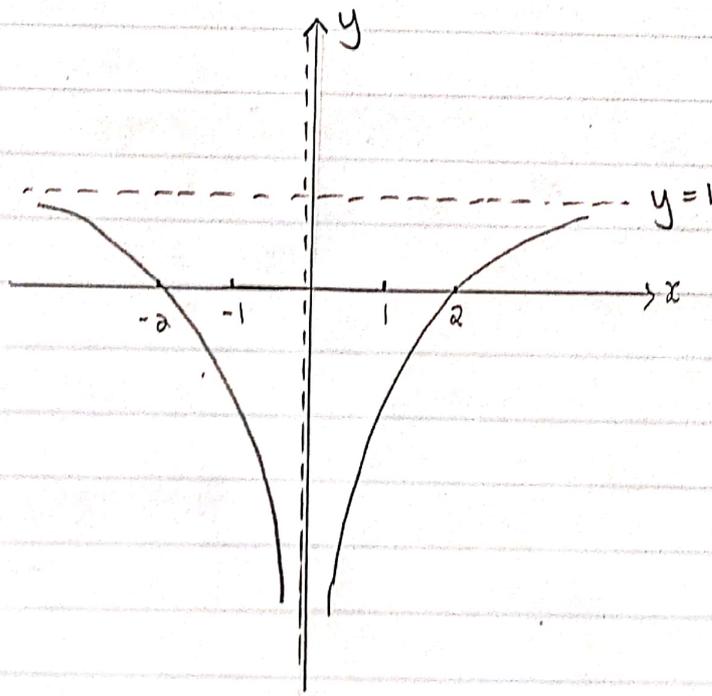
(1)

d)

		-a	0	a
x^2	+	+	+	+
$x+a$	-	+	+	+
$x-a$	-	-	-	+
$f(x)$	+	-	-	+

(1)

e)



x-int ✓

HA ✓

VA ✓

Shape ✓

(3)