UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS	
MODULE	**MAT2EB1\MAT1A2E**
CAMPUS	**APK**
ASSESSMENT	**Examination** (ONLINE DUE TO COVID-19)

DATE 2021/11/04   **TIME 12H30—15H30**

ASSESSOR(S)   **MR T CHIKORE**
                                                 MR L MATSEBULA
                                                 MS M SEBOGODI
MODERATOR   **DR ROBINSON**

DURATION   **SECTION A: 75 MINUTES**
                                                 SECTION B: 75 MINUTES

                                                 MARKS 75

SURNAME AND INITIALS ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

STUDENT NUMBER ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

NUMBER OF PAGES: 3 PAGES, INCLUDING COVER PAGE

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. YOU HAVE TO COMPLETE BOTH SECTION A AND B!
   YOU MUST START WITH SECTION A. SECTION B WILL
   ONLY BE AVAILABLE AT 13H00.
5. THE SUBMISSION LINK ON BB WILL BE TAKEN DOWN AT 15H30.
6. DO NOT EMAIL YOUR ANSWERS TO THE LECTURERS,
   THEY WILL NOT BE MARKED. YOU HAVE BEEN GIVEN TWO
   ATTEMPTS TO UPLOAD SECTION B ON BLACKBOARD.
   ONLY THE LAST ATTEMPT WILL BE MARKED

Mathematics and Applied Mathematics

Calculus of One Variable Functions
Examination
SECTION B

MAT2EB1\MAT1A2E: 2021-11-04

Time: 12H30—15H30   **Marks:** 35
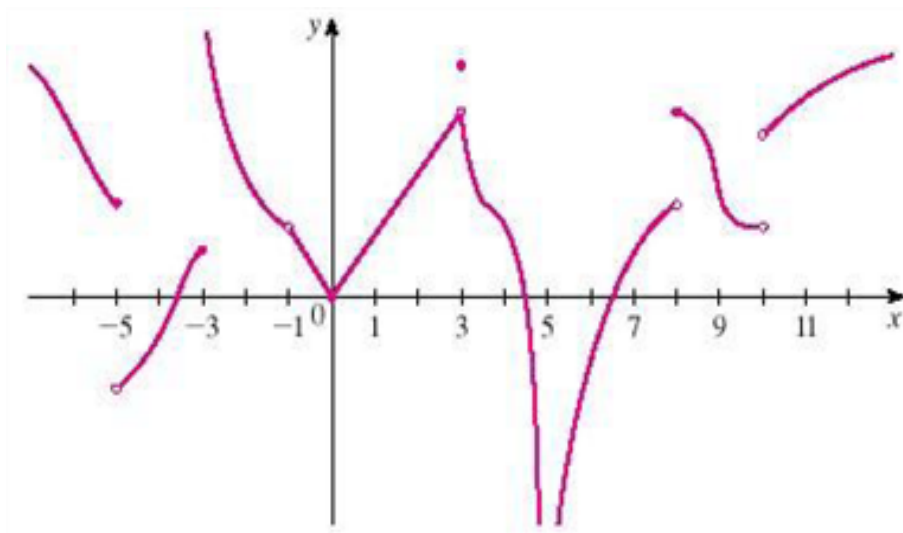**Assessors:** Mr. Chikore, Mr. Matsebula and Ms. Sebogodi
Moderator: Dr Robinson

Figure 1: The graph of $f(x)$.

Question 1 [10 mark(s)]
Use Figure 1 to answer the following questions. Do not justify.

   (a) Does the limit from the left of f at −5 exist?   (2)

   (b) Does the limit from the right of f at −5 exist?   (2)

   (c) Does the limit of f at −5 exist?   (2)

   (d) Is f continuous from the left at −5?   (2)

   (e) Is f continuous from the right at −5?   (2)

Question 2 [4 mark(s)]
Find the derivative of the function
$$y = \frac{2x}{4-x}$$

Question 3 [17 mark(s)]
Evaluate the following limits.

(a) $\lim\limits_{x \to \infty} \dfrac{4x^2 + 2}{x\sqrt{5x^2 + 1}}$ (4)

(b) $\lim\limits_{x \to \infty} \dfrac{\sinh(3x)}{7e^{3x}}$ (4)

(c) $\lim\limits_{x \to 0} \dfrac{3^x - 4^x}{x}$ (4)

(d) For the following question, use $f(x) = |2x - 8|$ to evaluate the following limits.

   (I) $\lim\limits_{x \to 4^+} \dfrac{f(x) - f(4)}{x - 4}$ (2)

   (II) $\lim\limits_{x \to 4^-} \dfrac{f(x) - f(4)}{x - 4}$ (2)

   (III) Does the value $f'(4)$ exist? Justify your answer. (1)

Question 4 [4 mark(s)]
Use mathematical induction to prove the following proposition.

$$\sum_{i=0}^{n-3} 4^{i+3} = \frac{4}{3}(4^n - 16), \quad n \geq 3$$