Mathematics and Applied Mathematics

Calculus of One Variable Functions
SOLUTIONS

MAT1EA1\MAT1A1E: 03/06/2021

Time: 12H30—15H00   **Marks:** 15
Assessors: Mr. Chikore, Mr. Matsebula and Ms. Sebogodi
Moderator: Dr Robinson

Question 1 [3 mark(s)]
Simplify completely the trigonometric expression.
$$\frac{\cos^3 x - \sin^3 x}{\sin x - \cos x}.$$

Solution

$$\frac{\cos^3 x - \sin^3 x}{\sin x - \cos x} = -\frac{(\cos x - \sin x)\left(\cos^2 x + \sin x \cos x + \sin^2 x\right)}{\cos x - \sin x} = -(1 + \sin x \cos x) = -1 - \sin x \cos x \checkmark$$

Question 2 [4 mark(s)]
Find the inverse function and domain of f.
$$f(x) = x^2 - x, \quad x < 0.5.$$

Solution

$$\text{Let } y = x^2 - x \checkmark = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} \overset{\text{swap } x,y}{\Longrightarrow} x \checkmark = \left(y - \frac{1}{2}\right)^2 - \frac{1}{4} \implies y = \frac{1 \pm \sqrt{4x+1}}{2}$$

$$\therefore f^{-1}(x) \checkmark = \frac{1 - \sqrt{4x+1}}{2}, \quad D_{f^{-1}} \checkmark = \left\{ x \in \mathbb{R} \middle| x > -\frac{1}{4} \right\}.$$

Question 3 [8 mark(s)]
Find the values of the trigonometric functions of θ from the information given.
$\sec \theta = 7, \ \sin \theta < 0$.

   (a) $\sin \theta$ (3)
   (b) $\cos \theta$ (1)
   (c) $\tan \theta$ (2)
   (d) $\csc \theta$ (1)
   (e) $\cot \theta$ (1)

Solution

   (a) $\theta \checkmark \in \left(\frac{3\pi}{2}, 2\pi\right), \ \cos \theta = \frac{1}{7} \implies \cos^2 \theta = \frac{1}{49} \implies \sqrt{1 - \cos^2 \theta} = \sqrt{\frac{48}{49}} \implies |\sin \theta| \checkmark = \frac{4\sqrt{3}}{7}$

       $\therefore \sin \theta \checkmark = -\frac{4\sqrt{3}}{7}.$

   (b) $\cos \theta \checkmark = \frac{1}{7}$

(c) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} \checkmark = \left(-\dfrac{4\sqrt{3}}{7}\right) \times \dfrac{7}{1} \checkmark = -4\sqrt{3}$

(d) $\csc\theta \checkmark = -\dfrac{7}{4\sqrt{3}}$

(e) $\cot\theta \checkmark = -\dfrac{1}{4\sqrt{3}}$

$\checkmark$: half a mark.
$\checkmark$: one mark.