FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	ASMA1B1 APPLICATIONS OF CALCULUS
CAMPUS	APK
ASSESSMENT	JUNE EXAM

DATE 07/06/2021 TIME 12:30

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INTERNAL MODERATOR DR TM SHUMBA

MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 6 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. ADHERE TO THE TERMS OF THE HONESTY DECLARATION.
5. DOWNLOAD AND COMPLETE AT HOME, SCAN AND UPLOAD A SINGLE PDF OF YOUR SOLUTION ON BLACKBOARD.

Question 1 [20 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 Consider the integral $\int \frac{\sqrt{16-x^2}}{x^3}\,dx$. Which of the following is the correct trigonometric substitution? (2)

(a) $x = 4\sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(b) $4x = \sin\theta,\ -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$

(c) $4x = \sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(d) $x = 4\sin\theta,\ -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$

(e) None of the above.

1.2 Give the polar coordinates of the point $(-1;\sqrt{3})$. (2)

(a) $\left(2;\frac{\pi}{3}\right)$

(b) $\left(-2;\frac{2\pi}{3}\right)$

(c) $\left(2;\frac{2\pi}{3}\right)$

(d) $\left(-2;\frac{\pi}{3}\right)$

(e) None of the above.

1.3 $\int_0^1 \sqrt{x^2+2x+1}\,dx = ?$ (2)

(a) $\frac{3}{2}$

(b) 1

(c) $-\frac{1}{2}$

(d) divergent

(e) None of the above.

1.4 Which of the following equations are linear?

(i) $x+5y-2z=1$ (ii) $x+3y+xz=2$ (iii) $x=-7y+3z$ (iv) $e^x-z=4$
(v) $\pi x+\sqrt{2}y+\frac{1}{3}z=7^{1/3}$ (2)

(a) (i), (iii) and (v)

(b) (ii) and (iii)

(c) (iii)

(d) (iii) and (iv)

(e) None of the above.

1.5 The second term in the expansion of $(2x-y)^4$ is: (2)

(a) $32x^3y$
(b) $4x^3y$
(c) $-8x^2y^2$
(d) $-32x^3y$
(e) None of the above.

1.6 Two numbers whose difference is 50 and whose product is a minimum are: (2)

(a) 30 and -20
(b) 25 and -25
(c) 70 and 20
(d) 25 and 25
(e) None of the above.

1.7 The correct trigonometric substitution for $\int \frac{x^3}{\sqrt{9+x^2}}\,dx$ leads to: (2)

(a) $\int 9\tan^3\theta\,d\theta$

(b) $\int 27\tan^3\theta\,d\theta$

(c) $\int 9\tan^3\theta\sec\theta\,d\theta$

(d) $\int 27\tan^3\theta\sec\theta\,d\theta$

(e) None of the above.

1.8 Consider the surface of revolution formed by revolving the curve $y = \frac{1}{x}$, $1 \leq x \leq a$, about the $x - axis$. The surface area is given by the definite integral: (2)

(a) $2\pi\int_1^a \frac{dx}{x}$

(b) $\int_1^a \frac{dx}{x^2}$

(c) $2\pi\int_1^a \frac{\sqrt{1+x^4}}{x^3}\,dx$

(d) $2\pi\int_1^a \sqrt{1+\frac{1}{x^4}}\,dx$

(e) None of the above.

1.9 $\sum_{k=0}^{n}\binom{n}{k}9^k$ is equal to: (2)

(a) 10^n

(b) 9^n

(c) 9^k

(d) 10^k

(e) None of the above.

1.10 The correct partial fraction decomposition for $\dfrac{x+1}{(x-1)^2(x^2+2)}$ is: (2)

(a) $\dfrac{A}{(x-1)^2}+\dfrac{Bx+C}{x^2+2}$

(b) $\dfrac{A}{(x-1)^2}+\dfrac{B}{x-1}+\dfrac{Cx+D}{x^2+2}$

(c) $\dfrac{Ax}{(x-1)^2}+\dfrac{Bx+C}{x^2+2}$

(d) $\dfrac{A}{(x-1)^2}+\dfrac{B}{x^2+2}$

(e) None of the above.

Question 2 [6 marks]

(a) **Set up, but do not evaluate**, an integral for the length of the curve below, simplifying as far as possible. (2)

$$y=\ln(1-x^2),\quad 0\le x\le \frac{1}{4}.$$

(b) The curve $y=\frac{1}{4}(x^2-2\ln x)$, $1\le x\le 4$, is rotated about the y-axis. Find the area of the resulting surface. (4)

Question 3 [5 marks]

Use the disk or washer method to find the volume of the solid obtained by rotating the region bounded by $y=1-x^2$ and $y=0$ about the line $y=2$. Make a clear sketch.

Question 4 [6 marks]

Evaluate each of the following integrals if they exist.

(a) $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx.$ (3)

(b) $\displaystyle\int_3^2 \frac{dx}{\sqrt{3-x}}.$ (3)

Question 5 [7 marks]

(a) Solve the differential equation shown below: (4)

$$y' = \frac{1+y^2}{(1+x^2)xy}, \quad x > 0.$$

(b) Solve the initial value differential equation shown below: (3)

$$\frac{dy}{dx} + (\sec^2 x)y = \sec^2 x, \quad y(\frac{\pi}{4}) = 0.$$

Question 6 [7 marks]

(a) Sketch the parametric curve below and indicate with an arrow the direction in which the curve is traced as t increase. (3)

$$x(t) = \frac{2}{t},\ y(t) = \frac{4}{t^2},\ 1 \leq t \leq 4.$$

(b) **Set up, but do not evaluate**, an integral for the arc length of the curve: (2)

$$x(t) = \frac{2}{t}, \quad y(t) = \frac{4}{t^2}, \quad 1 \leq t \leq 4.$$

(c) **Set up, but do not evaluate**, an integral for the surface area obtained by rotating the curve shown below about the $y - axis$. (2)

$$x = 3t^2, \quad y = 2t^3, \quad 0 \leq t \leq 5.$$

Question 7 [5 marks]

Expand the expression $\left(\sqrt{y} - \frac{x}{3}\right)^4$ using the **Binomial Theorem**. Simplify all coefficients and exponents.

Question 8 [9 marks]

(a) Give a system of linear equations corresponding to the given augmented matrix: (2)

$$\begin{bmatrix} 3 & 0 & 0 & 5 \\ 0 & 1 & 4 & 0 \\ 0 & -2 & -1 & 7 \end{bmatrix}$$

(b) Find the augmented matrix for the given system of linear equations: (1)

$$x_1 - x_5 = 7$$
$$x_2 = -1$$

(c) Solve the sytsem below using **Gauss–Jordan Elimination**: (6)

$$x + z = 6$$
$$-3y + z = 7$$
$$2x + y + 3z = 15$$

Question 9 [5 marks]

Consider the matrices shown below:

$$B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

Find the value of K such that:

$$B^{-1}C = \frac{1}{K}\begin{bmatrix} 10 & 18 & 18 \\ 24 & 8 & 40 \end{bmatrix}$$