UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

PHYSICS	**AUCKLAND PARK KINGSWAY CAMPUS**

PHY8X09

EXAMINATION
11 JUNE 2021
8:30-10:30

PHY8X09

EXAMINER: **Prof H Winkler**

EXTERNAL EXAMINER: Prof I Loubser
North-West Univ.

TIME: 2 ½ HOURS **MARKS: 100**

Please read the following instructions carefully:

ANSWER ALL QUESTIONS: 1-4

Constants: $c = 3\times10^{8}$ m.s^{-1} $h = 6.626\times10^{-34}$ J.s $k = 1.38\times10^{-23}$ J.K^{-1}
$m_e = 9.31\times10^{-31}$ kg $m_{\mathrm{H}} = 1.67\times10^{-27}$ kg $q_e = 1.6\times10^{-19}$ C
$G = 6.67\times10^{-11}$ N.m^{2}.kg^{-2} 1 eV = 1.6×10^{-19} J
$a = 7.56\times10^{-16}$ J.m^{-3}.K^{-4} $\sigma = 5.67\times10^{-8}$ J.m^{-2}.K^{-1}.s^{-1}
$R = 1.2\times10^{-15}\,\mathrm{m}\times A^{1/3}$ $\lambda_{\max}T = 0.29$ cm.K
$M_\odot = 2.0\times10^{30}$ kg $R_\odot = 7.0\times10^{8}$ m $L_\odot = 3.83\times10^{26}$ W
1 AU = 1.50×10^{11} m 1 pc = 3.086×10^{16} m
$E(\mathrm{2p+2n}\rightarrow\mathrm{1He}) = 26.731$ MeV

Formulae:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)}a^3 \qquad m_1+m_2 = \frac{P}{2\pi G}\frac{(v_{1r}+v_{2r})^3}{\sin^3 i} \qquad F = \frac{dE}{dA\,dt} \qquad I \propto \frac{1}{D^2}$$

$$I_\lambda = \frac{\partial I}{\partial\lambda} = \frac{E_\lambda d\lambda}{dA\,dt\cos\theta d\Omega\,d\lambda} \qquad u_\lambda = \frac{4\pi}{c}\langle I_\lambda\rangle \qquad \langle I_\lambda\rangle = \frac{1}{4\pi}\int_{all\ angles} I_\lambda d\Omega$$

$$F_\lambda d\lambda = \int_{all\ angles} I_\lambda d\lambda\cos\theta\,d\Omega \qquad F = \int_{all\ angles} I\cos\theta\,d\Omega \qquad p_\lambda = \frac{1}{c}\int_{all\ angles} I_\lambda\cos^2\theta\,d\Omega$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp\left(hc/\lambda kT\right)-1} \qquad u(T) = \frac{4\pi}{c}\int_0^\infty B_\lambda(T)d\lambda \qquad u(T) = aT^4 \qquad F = \sigma T^4$$

$$P = \frac{1}{3}u \qquad PV = NkT \qquad dI_\lambda = -\kappa_\lambda\rho I_\lambda\,ds \qquad d\tau_\lambda = -\kappa_\lambda\rho\,ds \qquad \frac{n_1}{n_2} = \frac{g_1}{g_2}\exp\left(\frac{h\nu}{kT}\right)$$

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda \qquad S_\lambda = j_\lambda/\kappa_\lambda \qquad \cos\theta\frac{dI_\lambda}{d\tau_{\lambda,v}} = I_\lambda - S_\lambda \qquad d = \sqrt{N}\ell$$

$$\frac{dP}{dr} = -G\frac{M_r\rho}{r^2} = -\rho g \qquad \frac{dM_r}{dr} = 4\pi r^2\rho \qquad U \sim -\frac{3}{5}\frac{GM_c^2}{R_c} \qquad P = \frac{\rho kT}{\mu m_H}$$

$$\left.\frac{dT}{dr}\right|_{adiabatic} = \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} \qquad m_\lambda = -2.5\log_{10}\left(\frac{I_\lambda}{I_{\lambda,m=0}}\right) \qquad \tau_\lambda = 0.921A_\lambda$$

$$v_s = \sqrt{\frac{\gamma P}{\rho}} \qquad \Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}} \qquad E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} \qquad E = \frac{3}{2}kT$$

$$P_{\min} = 2\pi\sqrt{\frac{R^3}{GM}} \qquad R_S = \frac{2GM}{c^2} \qquad v_{esc} = \sqrt{\frac{2GM}{R}}$$

Potentially useful mathematical identities:

$$\int_{-\infty}^{\infty}\frac{\sin^2 x}{x^2}dx = 2\pi \qquad a^2 = b^2 + c^2 - 2bc\cos\theta \qquad \int_0^\infty \frac{dx}{x^5\left(\exp\left(1/x\right)-1\right)} = \frac{\pi^4}{15}$$

$$V_{sphere} = \frac{4}{3}\pi r^3 \qquad A_{sphere} = 4\pi r^2 \qquad \left\langle\sin^2 x\right\rangle = \tfrac{1}{2} \qquad e^x = 1 + x + \frac{1}{2}x^2 + \ldots$$

QUESTION 1 **[30]**

a) A particular star with a mass 0.140 times the mass of the Sun and known to be 1.75 pc away is seen to oscillate about its mean position with an amplitude of 0.025 arcseconds with a period of 26 years. The oscillation is believed to be caused by a planet in a near-circular orbit around this star. Determine the distance from the planet to the star and the planet's mass. [Hint: use as an approximation the fact that the mass of the star is much larger than the mass of the planet] (8)

b) Show that for small wavelengths Planck's Law simplifies to Wien's Law

$$B_\lambda \simeq \frac{2hc^2}{\lambda^5}\exp\left(-\frac{hc}{\lambda kT}\right)$$

Furthermore confirm that this means that $B_\lambda \to 0$ as $\lambda \to 0$. (4)

c) Define the specific energy density, and confirm that this is equal to $u_\lambda = \frac{4\pi}{c}\langle I_\lambda \rangle$. (7)

d) A photon is moving from its origin in a uniform medium, scattering at random in a random direction. If in time t it moves an average distance d from the origin, how long will it take, on average, to reach a distance $2d$ from the origin? (4)

e) Given the radiative transfer equation form $\cos\theta \frac{dI_\lambda}{d\tau_{\lambda,v}} = I_\lambda - S_\lambda$, show that, for a grey atmosphere, $\cos\theta \frac{dI}{d\tau_v} = I - S$, and hence that $\frac{dF_{rad}}{d\tau_v} = 4\pi\left(\langle I \rangle - S\right)$.

[I and S are defined by $I = \int_0^\infty I_\lambda d\lambda$ and $S = \int_0^\infty S_\lambda d\lambda$]. (7)

QUESTION 2 **[20]**

a) A star has a density whose dependence on the distance r to the centre of the star is given by

$$\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$$

where ρ_0 is the density at the star's centre, and R is the outer stellar radius.
Determine an expression for the mass inside a distance r from the star's centre in terms of ρ_0 and R. Hence obtain the expression for the total mass M of the star. (7)

b) You wish to test a (wrong) theory that the energy of stars solely comes from the gravitational energy released while the interstellar cloud condenses.
Show that the total gravitational energy released due to this process for a star of mass M, radius R and uniform density would be given by

$$E \sim -\frac{3}{10}\frac{GM^2}{R}. \quad (9)$$

c) Show that the criterion for convection $\left|\frac{dT}{dr}\right|_{actual} > \left|\frac{dT}{dr}\right|_{adiabatic} \left[= \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}\right]$

may equally be expressed as $\frac{d\ln P}{d\ln T} < \frac{\gamma}{\gamma - 1}$ (4)

QUESTION 3 **[25]**

a) Comparing two stars, one with $B–V > 0$ and one with $B–V < 0$, which is the redder of the two, and why? (3)

b) We know that the radial speed of particles at the outer edge of a gravitationally collapsing interstellar cloud of initial radius r_0 and density ρ_0 is given by the expression

$$\frac{dr}{dt} = -\left[\frac{8\pi}{3}G\rho_0 r_0^2\left(\frac{r_0}{r} - 1\right)\right]^{1/2}$$

Describe the procedure and assumptions you would use to determine the time required to collapse the cloud to a star. NB: do not do any calculations. (5)

c) What is the Hertzsprung-Russel (HR) diagram? Hence briefly discuss the evolution of stars with reference to this diagram. (10)

d) Given that, assuming near-constant density, the pressure as a function of distance from a star's centre is given by $P(r) = \frac{2}{3}\pi G\rho^2\left(R^2 - r^2\right)$, and that the speed of sound in a gas with adiabatic ratio γ is $v_s = \sqrt{\frac{\gamma P}{\rho}}$, confirm that the period of oscillation for a pulsating star is approximately $\Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}}$. (7)

QUESTION 4 **[25]**

a) Given the relation $\frac{dP}{dr} = -G\frac{M_r\rho}{r^2}$, the stellar boundary condition $P = 0$ when $r = R$ and assuming constant density,

i) Show that $P(r) = \frac{2}{3}\pi G\rho^2\left(R^2 - r^2\right)$ (5)

ii) The pressure and temperature at the centre of a white dwarf of radius $R = 6.37\times10^6$ m, constant density $\rho = 1.84\times10^9$ kg.m^{-3} and a carbon ($\mu = 12.000$) core. (5)

b) One way to imagine a neutron star is as a huge atomic nucleus with $\sim10^{57}$ nucleons. What would be its radius under this simplistic model? (3)

c) Show that the minimum rotational period of a pulsar is $P_{\min} = 2\pi\sqrt{\dfrac{R^3}{GM}}$. (4)

d) In no more than two sentences each, describe the following terms:
i) The Schwarzschild radius
ii) An interacting binary
iii) An accretion disk
iv) A supernova (8)

END